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FIXED POINT OF WEAKLY COMPATIBLE MAPS IN INTUTIONISTIC FUZZY METRIC SPACE SATISFYING A GENERAL CONTRACTIVE CONDITION OF INTEGRAL TYPE

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ABSTRACT

We prove fixed point theorem for weakly compatible maps in Intuitionistic fuzzy metric space satisfying a general contractive condition of Integral type.

Keywords: *Intuitionistic fuzzy metric space, weakly compactiable mapping common fixed point contractive condition.*

AMS mathematics subject classification: *47H10, 54H25.*

1. INTRODUCTION

Motivated by the potential applicability of fuzzy topology to quantum particle physics particularly in connection with both string and e^{∞} theory developed by El Naschie [6], [7], Park introduced and discussed in [21] a notion of intuitionistic fuzzy metric spaces which is based on the idea of intuitionistic fuzzy sets due to Atanassov [2] and the concept of fuzzy metric space given by George and Veeramani [11]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions. It has direct physics motivation in the context of the two-slit experiment as the foundation of E-infinity of high energy physics, recently studied by El Naschie [8], [9].

Alaca et al. [1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park [21] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [3] and Edelstein [5] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [10]. Turkoglu et al. [25] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β). Sharma and Tilwankar [24] and Kutukcu [18] proved fixed point theorems for multivalued mappings in intuitionistic fuzzy metric spaces.

Several authors [12], [13], [15], [23] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Branciari [4] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Sedghi et al. [22] established a common fixed point the- orem for weakly compatible mappings in intuitionistic fuzzy metric space satisfying a contractive condition of integral type. Muralisankar et al. [20] proved a common fixed point theorem in an intuitionistic fuzzy metric space for pointwise R-weakly commuting mappings using contractive condition of integral type and established a situation in which a collection of maps has a fixed point which is a point of discontinuity. In this paper, we prove some common fixed point theorems for six mappings by using contractive condition of integral type for class of weakly compatible maps in noncomplete intuitionistic fuzzy metric spaces, without taking any continuous mapping. We improve and extend the results of Muralisankar and Kalpana [20].

2. PRELIMINARIES

Definition 2.1: A binary operation *: [0, 1] x [0, 1] $\rightarrow$ [0, 1] is continuous t-norm if * is satisfying the following conditions:
(i) * is commutative and associative,
(ii) * is continuous,

(iii) $a * 1 = a$ for all $a \varepsilon [0, 1]$,
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$, a, b, c, d $\varepsilon [0, 1]$.

Definition 2.2: A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-conorm if $\lozenge$ is satisfying the following conditions:

(i) $\lozenge$ is commutative and associative,
(ii) $\lozenge$ is continuous,
(iii) $a \lozenge 0 = a$ for all $a \in [0, 1]$,
(iv) $a \lozenge b \le c \lozenge d$ whenever $a \le c$ and $b \le d$, a, b, c, d $\in [0, 1]$.

Remark 1: The concept of triangular norms (t-norms) and triangular conorms (t-conorms) are known as the axiomatic skeletons that we use for characterizing fuzzy intersections and unions, respectively. These concepts were originally introduced by Menger [19] in his study of statistical metric spaces. Several examples for these concepts were proposed by many authors [16], [26].

Definition 2.3: A 5-tuple $(X, M, N, *, \lozenge)$ is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, * is a continuous t-norm, $\lozenge$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions for all x, y, z $\in$ X and t, s > 0,

(i) $M(x, y, t) + N(x, y, t) \le 1$,
(ii) $M(x, y, 0) = 0$,
(iii) $M(x, y, t) = 1$ for all $t > 0$ if and only if $x = y$,
(iv) $M(x, y, t) = M(y, x, t)$,
(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(vi) $M(x, y, .) : [0, \infty) \to [0, 1]$ is left continuous,
(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all x, y in X,
(viii) $N(x, y, 0) = 1$,
(ix) $N(x, y, t) = 0$ for all $t > 0$ if and only if $x = y$,
(x) $N(x, y, t) = N(y, x, t)$,
(xi) $N(x, y, t) \lozenge N(y, z, s) \ge N(x, z, t + s)$,
(xii) N(x, y, .) : [0,1) ! [0, 1] is right continuous,
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2: Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form $(X, M, 1- M, *, \lozenge)$ such that t-norm □ and t-conorm $\lozenge$ are associated, i.e., $x \lozenge y = 1\ ((1\ x) * (1 - y))$ for all $x, y \in X$.

Example 1: Let (X, d) be a metric space. Define t-norm $a*b = \min\{a, b\}$ and t-conorm $a \lozenge b = \max(a, b\}$ and for all x, $y \in X$ and $t >0$,

$$M_d(x, y, t) = \frac{t}{t + d(x, y)},\ N_d(x, y, t) = \frac{d(x, y)}{t + d(x, y)} \tag{2.1}$$

Then $(X, M, N, *,\lozenge)$ is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric. On the other hand, note that there exists no metric on X satisfying (2.1).

Remark 3: In intuitionistic fuzzy metric space $(X, M, N, *,\lozenge)$, M(x, y .) is non-decreasing and N(x, y, .) is non-increasing for all $x, y \in X$.

Definition 2.4: Let $(X, M, N, *, \lozenge)$ be an intuitionistic fuzzy metric space. Then

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ (denoted by $\lim_{n\to\infty} x_n = x$) if, for all $t >0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1$, $\lim_{n\to\infty} N(x_n, x, t) = 0$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p,} x_n, t) = 1 \quad \lim_{n\to\infty} N(x_{n+p,} x_n, t) = 0$$

Remark 4: Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

Definition 2.5: ([1]). An intuitionistic fuzzy metric space (X, M, N, *,◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma 1: ([1]). Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X. If there exists a number $k \in (0, 1)$ such that

$$M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t),\ N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all t >0 and n = 1, 2,…. , then $\{y_n\}$ is a Cauchy sequence in X.

Lemma 2: ([1]). Let (X, M, N, *,◊) be an intuitionistic fuzzy metric space and for all $x, y \in X$, t > 0 and if for a number k 2 (0, 1),

$$M(x, y, kt) \geq M(x, y, t) \text{ and } N(x, y, kt) \leq N(x, y, t),$$

then x = y.

Definition 2.6: ([14]). Two self mappings S and T are said to be weakly compatible if they commute at their coincidence points, i.e., if Tu = Su for some $u \in X$, then TSu = STu.

In this paper, we prove some common fixed point theorems for four mappings by using contractive condition of integral type for class of weakly compatible maps in non complete intuitionistic fuzzy metric spaces, with- out taking any continuous mapping. We improve and extend the results of Muralisankar and Kalpana [20].

3. MAIN RESULT

Theorem 3.1: Let (X, M, N * ◊) be an insutitionistic fuzzy metric space with continous t-norm * and contintinuous t-norm ◊ defined by $t * t \geq t$ and $(1 – t) \lozenge (1 – t) \leq (1 – t)\ \forall\ t \in [0, 1]$.

Let A, B, S and T be mappings from X into itself such that

1) $S(x) \subseteq B(x)$, $T(x) \subseteq A(x)$
2) $\exists$ a constant $K \in (0, 1)$ such that

$$\int_0^{M(Sx,\ Ty)} \varphi(t)dt \geq \psi\left(\int_0^{m(x,\ y)} \varphi(t)dt\right) \tag{3.1}$$

and
$$\int_0^{N(Sx,Ty)} \varphi(t)dt < \psi \int_0^{N(x,\ y)} \varphi(t)dt \tag{3.2}$$

where $\phi : R^+ \to R^+$ is a lebesque – Integrable mapping which is summable non negative and such that

$$\int_0^{\in} \varphi(t)dt > 0 \text{ for each } \in > 0 \tag{3.3}$$

where
$$M = \min\{m(Ax, By), m(Sx, Ax), M(Ty, By), \frac{M(Sx, By) + M(Ty, Ax)}{2} \tag{3.4}$$

$$N = \max\{N(Ax, By), N(Sx, Ax), N(Ty, By) \frac{N(Sx, By)^2 + N(Ty, Ax)}{2} \tag{3.5}$$

for all $x, y \in X$, and t > 0 If are of A(x), B(x), S(x) or T(x) is complete subspace of X. Then

1) A and S have a coincidence point and
2) B and T have a coincidence point

Further if S and A as well a T and B are weakly compactable, then (3) A, B, S and T have a unique common fixed points.

Proof: Let $x_0 \in X$ be an arbitrary point of X. From (i) we can construct a sequence $\{y_n\}$ in X as follows:

$$Y_{2n+1} = Sx_{2n} = Bx_{2n+1}, \quad y_{2n+2} = Tx_{2n+1} = Ax_{2n+2} \tag{3.6}$$

for all n = 0, 1, ….. Define $M_n = M\ (y_n\ y_{n+1})$. Suppose the $M_{2n} = 0$ for some n. Then $y_{2n} = y_{2n+1}$, that is $tx_{2n-1} = Ax_{2n}, = Sx_{2n} = Bx_{2n+1}$, and A and S have a coincidence point.

Similarly, if $M_{2n+1} = N_{2n+1}=0$, then B and T have a coincidence point. Assume that $M_n \neq N_n$ for each n.

Then, by (ii)

$$\int_0^{M(Sx_{2n},\,Tx_{2n+1})} \varphi(t)\,dt \geq \psi\left(\int_0^{M(x_{2n},\,x_{2n+1})} \varphi(t)dt\right) \textit{and} \int_0^{N(Sx_{2n},\,Tx_{2n+1})} \varphi(t)\,dt \leq \psi\left(\int_0^{N(x_{2n},\,x_{2n+1})} \varphi(t)dt\right)$$

where

$$M(x_{2n}, x_{2n+1}) = \min\{N(Ax_{2n}, Bx_{2n+1})\ N(Sx_{2n}, Ax_{2n})\ N(Tx_{2n+1}, Bx_{2n+1}), \frac{M(Sx_{2n}, Bx_{2n+1}) + M(Tx_{2n+1}, Ax_{2n})}{2} \tag{3.7}$$

$$= \min\{M_{2n}, M_{2n+1}\}$$

and $$N(x_{2n}, x_{2n+1}) = \max\{N(Ax_{2n}, Bx_{2n+1})\ .\ N(Sx_{2n}, Ax_{2n})\ N(Tx_{2n+1}, Bx_{2n+1}), \frac{N(Sx_{2n}, Bx_{2n+1}) + N(Tx_{2n+1}, Ax_{2n})}{2}$$

$$= \min\{N_{2n}, N_{2n+1}\}$$

$$\int_0^{M_{2n+1}} \varphi(t)\,d \geq \psi\left(\int^{\min\{M_{2n},\,M_{2n+1}\}} \varphi(t)\,dt\right) \tag{3.8}$$

and $$\int_0^{N_{2n+1}} \varphi(t)\,d \leq \psi\left(\int^{\max\{N_{2n},\,N_{2n+1}\}} \varphi(t)\,dt\right) \tag{3.9}$$

Now, if $M_{2n+1} \geq M_{2n}$ and $N_{2n+1} \leq N_{2n}$ for some n, then from (3.8) and (3.9) we have

$$\int_0^{M_{2n+1}} \varphi(t)dt \geq \left(\int_0^{M_{2n+1}} \varphi(t)dt\right) > \int_0^{M_{2n+1}} \varphi(t)dt, \tag{3.10}$$

$$\int_0^{N_{2n+1}} \varphi(t)dt \leq \left(\int_0^{N_{2n+1}} \varphi(t)dt\right) < \int_0^{N_{2n+1}} \varphi(t)dt,$$

which is a contradiction. Thus $M_{2n} > M_{2n+1}$ and $N_{2n} > N_{2n+1}$ for all n, and so, for (3.8) and (3.9) we have

$$\int_0^{M_{2n+1}} \varphi(t)dt \geq \psi\left(\int_0^{M_{2n}} \varphi(t)dt\right) \textit{and} \int_0^{N_{2n+1}} \varphi(t)dt \leq \psi\left(\int_0^{N_{2n}} \varphi(t)dt\right) \tag{3.11}$$

Similarly,

$$\int_0^{M_{2n}} \varphi(t)dt \geq \psi\left(\int_0^{M_{2n-1}} \varphi(t)dt\right) \textit{and} \int_0^{N_{2n}} \varphi(t)dt \leq \psi\left(\int_0^{N_{2n-1}} \varphi(t)dt\right) \tag{3.12}$$

In general, we have for all n = 1, 2 …

$$\int_0^{M} \varphi(t)dt \geq \psi\left(\int_0^{M_{2n-1}} \varphi(t)dt\right) \textit{and} \int_0^{N} \varphi(t)dt \leq \psi\left(\int_0^{N_{2n-1}} \varphi(t)dt\right) \tag{3.13}$$

From (3.13) we have

$$\int_0^{M} \varphi(t)dt \geq \psi\left(\int_0^{M_{n-1}} \varphi(t)dt\right) \leq \psi^2\left(\int_0^{M_{n-2}} \varphi(t)dt\right) \tag{3.14}$$

$$\vdots$$

$$\geq \psi^n\left(\int_0^{M_0} \varphi(t)dt\right)$$

and also N gives $\int_0^{N} \varphi(t)\, dt < \psi^n \int_0^{N} \varphi(t)dt$

and $\lim_{n\to\infty} \int_0^{M_n} \varphi(t)dt \geq \lim_{n\to\infty} \psi^n\left(\int_0^{M} \varphi(t)dt\right) = 0$

$$\lim_{n\to\infty} \int_0^{N_n} \varphi(t)dt \leq \lim_{n\to\infty} \psi^n\left(\int_0^{N} \varphi(t)dt\right)$$

and, taking the limit as $n \to \infty$ and using Lemma 1.2., we have

which, from (3.3), implies that

$$\lim_{n\to\infty} M_n = \lim_{n\to\infty} M(y_n, y_{n+1}) = 0 \text{ and } \lim_{n\to\infty} N_n = \lim_{n\to\infty} N(y_n, y_{n+1}) = 0 \tag{3.15}$$

We now show that $\{y_n\}$ is a Cauchy sequence. For this it is sufficient to show that $\{y_{2n}\}$ is a Cauchy sequence. Suppose that $\{y_{2n}\}$ is not a Cauchy sequence. Then there exists an $\varepsilon > 0$ such that for each even integer 2k there exist even integers 2m (k) > 2n (k) > 2k such that.

$$M(y_{2n}(k), y_{2m(k)}) > \varepsilon, \text{ and } N(y_{2n(k)}, y_{2m(k)} \leq \in \tag{3.16}$$

For every even integer 2k, let 2m(k) be the least positive integer exceeding 2n(k) satisfying (3.16) such that

$$M(y_{2n(k)}, y_{2m(k)-2}) > \varepsilon. \text{ and } N(y_{2n}(k), y_{2m}(k)-2) < \in \tag{3.17}$$

$$0 < \delta := \int_0^{\in} \varphi(t)dt \geq \int_0^{M(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt \geq \int_0^{M(y_{2n(k)}, y_{2m(k)}-2) + M_{2m(k)-2}M_{2m(k)-1}} \varphi(t)dt.$$

and $0 < \delta := \int_0^{\in} \varphi(t)dt \geq \int_0^{N(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt \geq \int_0^{N(y_{2n(k)}, y_{2m(k)}-2) + N_{2m(k)-2}N_{2m(k)-1}} \varphi(t)dt.$

Then by (3.15), (3.16) and (3.17), it follows that

$$\lim_{k\to\infty} \int_0^{M(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt = \delta \textit{ and } \lim_{k\to\infty} \int_0^{N(y_{2n(k)}, y_{2m(k)})} \varphi(t)dt = \delta \tag{3.18}$$

Also, by the triangular inequality,

$$\left|M(y_{2n(k)}, y_{2m(k)-1}) - M(y_{2n(k)}, y_{2m(k)})\right| \leq M_{2m(k)-1}, \tag{3.19}$$

$$\left|M(y_{2n(k)+1}, y_{2m(k)-1}) - M(y_{2n(k)}, y_{2m(k)})\right| \leq M_{2m(k)-1} + M_{2n(k)},$$

and so

$$\int_0^{\left|M(y_{2n(k)}, y_{2m(k)-1}) - M(y_{2n(k)}, y_{2m(k)})\right|} \varphi(t)dt \geq \int_0^{M_{2m(k)-1}} \varphi(t)dt \tag{3.20}$$

$$\int_0^{\left|M(y_{2n(k)+1}, y_{2m(k)-1}) - M(y_{2n(k)}, y_{2m(k)})\right|} \varphi(t)dt \geq \int_0^{M_{2m(k)-1}+M_{2n(k)}} \varphi(t)dt$$

Using (3.18) we get

$$\int_0^{M\left(y_{2n(k)},\, y_{2m(k)-1}\right)} \varphi(t)\, dt \to \delta \tag{3.21}$$

$$\int_0^{M\left(y_{2n(k)+1},\, y_{2m(k)-1}\right)} \varphi(t)\, dt \to \delta \tag{3.22}$$

as $k \to \infty$. Thus

and similarly $\int_0^{N\left(y_{2n(k)+1},\, y_{2m(k)-1}\right)} \varphi(t)\, dt \to \delta$

$$M(y_{2n(k)}, y_{2m(k)}) \ge M_{2n(k)} + M(y_{2n(k)+1}, y_{2m(k)}) \ge M_{2n(k)} + M(Sx_{2n(k)}, Tx_{2m(k)-1}), \tag{3.23}$$

and $N(y_{2n(k)}, y_{2m(k)}) \le N_{2n(k)} + N(y_{2n(k)+1}, y_{2m(k)}) \ge N_{2n(k)} + N(Sx_{2n(k)}, Tx_{2m(k)-1}),$

and so

$$\int_0^{M\left(y_{2n(k)}+y_{2m(k)}\right)} \varphi(t) dt \ge \int_0^{M_{2n(k)+M\left(Sx_{2n(k)-1}\right)}} \varphi(t) dt. \tag{3.24}$$

$$\int_0^{N\left(y_{2n(k)}+y_{2m(k)}\right)} \varphi(t) dt \ge \int_0^{N_{2n(k)+N+\left(Sx_{2n(k)}T_{2n(k)-1}\right)}} \varphi(t) dt.$$

Letting $k \to \infty$ on both sides of the last inequality, we have

$$\delta \le \lim_{k\to\infty} \int_0^{M\left(Sx_{2n(k)},\, Tx_{2m(k)-1}\right)} \varphi(t) dt \ge \lim_{k\to\infty} \psi\left(\int_0^{M\left(x_{2n(k)}, x_{nm(k)-1}\right)} \varphi(t) dt \right) \tag{3.25}$$

$$\delta \le \lim_{k\to\infty} \int_0^{N\left(Sx_{2n(k)},\, Tx_{2m(k)-1}\right)} \varphi(t) dt \le \lim_{k\to\infty} \psi\left(\int_0^{N\left(x_{2n(k)}, x_{nm(k)-1}\right)} \varphi(t) dt \right)$$

where

$$M(x_{2n(k)}, x_{2n(k)-1}) = \min\{M(y_{2n(k)}, y_{2m(k)-1})\ M_{2n(k)}, M_{2m(k)-1}, \frac{M\left(y_{2n(k)+1}, y_{2m(k)-1}\right) + M\left(y_{2n(k)}, y_{2m(k)}\right)}{2} \tag{3.26}$$

and $$N(x_{2n(k)}, x_{2n(k)-1}) = \max\{N(y_{2n(k)}, y_{2m(k)-1})\ N_{2n(k)}, N_{2m(k)-1}, \frac{N\left(y_{2n(k)+1}, y_{2m(k)-1}\right) + N\left(y_{2n(k)}, y_{2m(k)}\right)}{2}$$

Combining (3.15), (3.16), (3.17), (3.18), (3.22), and (3.23) yields the following contradiction from (3.25).

$$\delta \le \psi(\delta) < \delta \tag{3.27}$$

Thus $\{y_{2n}\}$ is a Cauchy sequence and so $\{y_n\}$ is a Cauchy sequence.

Now, suppose that A(X) is complete. Note that the sequence $\{y_{2n}\}$ is contained in A(X) and has a limit in A(X). Call it u. Let $v \in A^{-1}u$. Then $Av = u$. We will use the fact that the sequence $\{y_{2n-1}\}$ also lconverges to u. To prove that $Sv = u$, let $r = d(Sv, u) > 0$. Then taking $x = v$ and $y = x_{2n-1}$ in (ii)

$$\int_0^{M(Sv,\, y_{2n})} \varphi(t) dt = \int_0^{M(Sv, Tx_{2n-1})} \varphi(t) dt \le \psi\left(\int_0^{M(v, x_{2n-1})} \varphi(t) dt \right) \tag{3.28}$$

where

$$M(v, x_{2n-1}) = \min\left\{ d(u, y_{2n-1}), d(Sv, u), d(y_{2n}, y_{2n-1}), \frac{d(Sv, y_{2n-1}) + d(y_{2n,} u)}{2} \right\} \tag{3.29}$$

Since $\lim_n M(Sv, y_{2n}) = r$, $\lim_n M(u, y_{2n-1}) = \lim_n M(y_{2n}, y_{2n-1}) = 0$, and $\lim_n [M(Sv, y_{2n-1}) + M(y_{2n}, u)] = r$, we may conclude that

$$\int_0^M \varphi(t)\, dt \geq \psi\left(\int_0^M \varphi(t)dt\right) > \int_0^M \varphi(t)dt,$$

and $$\int_0^N \varphi(t)\, dt \leq \psi\left(\int_0^N \varphi(t)dt\right) < \int_0^N \varphi(t)dt, \tag{3.30}$$

which is a contradiction. Hence from (2.2), Sv = u. This proves (1)

Since $S(X) \subseteq B(X)$, Sv = u implies that $u \in B(X)$. Let $w \in B^{-1} u$. Then Bw = u. By using the argument of the previous section, it can be easily verified that tw = u. This proves (3.2).

The same result holds if we assume that B(X) is complete instead of A(X).

Now if T(X) is complete, then by (i), $u \in T(X) \subseteq A(X)$. Similarly if S(X) is complete, then $u \in S(X) \subseteq B(X)$. Thus (1) and (2) are completely established.

To prove (3), note that S, A and T, B are weakly compatible and

$$U = Sv = Av = Tw = Bw, \tag{3.31}$$

Then

$$Au = ASv = SAv = Su,$$
$$By = Btw = TBw = Tu. \tag{3.32}$$

If $Tu \neq u$ then from (ii), (3.31) and (3.32)

$$\int_0^{M(u,\, Tu)} \varphi(t)dt = \int_0^{M(Sv,Tu)} \varphi(t)dt \leq \psi\left(\int_0^{M(v,\, u)} \varphi(t)dt\right)$$
$$= \psi\left(\int_0^{M(u,\, Tu)} \varphi(t)dt\right) < \int_0^{M(u,\, Tu)} \varphi(t)dt \tag{3.33}$$

and $$\int_0^{N(u,\, Tu)} \varphi(t)dt = \int_0^{N(Sv,Tu)} \varphi(t)dt \leq \psi\left(\int_0^{N(v,\, u)} \varphi(t)dt\right)$$
$$= \psi\left(\int_0^{N(u,\, Tu)} \varphi(t)dt\right) < \int_0^{N(u,\, Tu)} \varphi(t)dt$$

which is a contradiction. So Tu = u. Similarly Su = u. Then evidently from (2.32), u is a common fixed point of A, B, S, and T.

The uniqueness of the common fixed point follows easily from condition.

Corollary 1: Let (X, M, N * ◊) be an insutitionistic fuzzy metric space with continous t-norm * and contintinuous t-norm ◊ defined by $t * t \geq t$ and $(1 - t) \lozenge (1 - t) \leq (1 - t)$ $\forall\, t \in [0, 1]$.

Let A, B, S and T be mappings from X into itself such that

3) $S(x) \subseteq B(x)$, $T(x) \subseteq A(x)$
4) $\exists$ a constant $K \in (0, 1)$ such that

$$\int_0^{M(Sx,\, Ty)} \varphi(t)dt \geq \psi\left(\int_0^{m(x,\, y)} \varphi(t)dt\right) \tag{3.34}$$

and $$\int_0^{N(Sx,Ty)} \varphi(t)dt < \psi \int_0^{N(x,\, y)} \varphi(t)dt \tag{3.35}$$

where $\phi : R^+ \to R^+$ is a lebesque – Integrable mapping which is summable non negative and such that

$$\int_0^{\in} \varphi(t)dt > 0 \text{ for each } \in > 0 \tag{3.36}$$

where M = min {m (Ax, By), m (Sx, Ax), M(Ty, By), $\frac{M(Sx, By) + M(Ty, Ax)}{2}$ (3.37)

N = max {N (Ax, By), N(Sx, Ax), N(Ty, By) $\frac{N(Sx, By)^2 + N(Ty, Ax)}{2}$ (3.38)

for all x, y ∈ X, and t > 0 If are of A(x), B(x), S(x) or T(x) is complete subspace of X. Then

1) T and S have a coincidence point and
2) T and A have a coincidence point

Further if S and A as well a T and B are weakly compactable, then (3) A, B, S and T have a unique common fixed points.

Corollary 2: Let (X, M, N * ◊) be an insutitionistic fuzzy metric space with continous t-norm * and contintinuous t-norm ◊ defined by t * t ≥ t and (1 – t) ◊ (1 – t) ≤ (1 – t) ∀ t ∈ [0, 1].

Let A, B, S and T be mappings from X into itself such that

5) S(x) ⊆ B (x), T(x) ⊆ A (x)
6) ∃ a constant K ∈ (0, 1) such that

$$\int_0^{M(Sx,\,Ty)} \varphi(t)dt \geq \psi\left(\int_0^{m(x,\,y)} \varphi(t)dt\right) \tag{3.39}$$

and

$$\int_0^{N(Sx,Ty)} \varphi(t)dt < \psi \int_0^{N(x,\,y)} \varphi(t)dt \tag{3.40}$$

where $\phi : R^+ \to R^+$ is a lebesque – Integrable mapping which is summable non negative and such that

$$\int_0^{\in} \varphi(t)dt > 0 \text{ for each } \in > 0 \tag{3.41}$$

where

M = min {m (Ax, By), m (Sx, Ax), M(Ty, By), $\frac{M(Sx, By) + M(Ty, Ax)}{2}$ (3.42)

N = max {N (Ax, By), N(Sx, Ax), N(Ty, By) $\frac{N(Sx, By)^2 + N(Ty, Ax)}{2}$ (3.43)

for all x, y ∈ X, and t > 0 If are of A(x), B(x), S(x) or T(x) is complete subspace of X. Then

1) T and S have a coincidence point and
2) A and B have a coincidence point

Further if S and A as well a T and B are weakly compactable, then (3) A, B, S and T have a unique common fixed points.

REFERENCES

[1] Alaca C., Turkoglu D., Yildiz C., Fixed points in intuitionistic fuzzy metric spaces, Choas, Solitons & Fractals, 29(2006), 1073-1078.

[2] Atanassov K., Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1986), 87-96.

[3] Banach S., Theoriles Operations Linearies Manograie Mathematyezne, War- saw, Poland, 1932.

[4] Branciari A., A fixed point theorem for mappings satisfying a general con- tractive condition of integral type, International J. of Mathematics and Math- ematical Sciences, 29(2002), 531-536.

[5] Edelstein M., On fixed and periodic points under contractive mappings, J. London Math. Soc., 37(1962), 74-79.

[6] El Naschie M.S., On the uncertainty of Cantorian geometry and two-slit experiment, Choas, Solitons & Fractals, 9(1998), 517-529.

[7] El Naschie M.S., On the verifications of heterotic string theory and e(1) theory, Chaos, Solitons & Fractals, 11(2000), 397-407.

[8] El Naschie M.S., A review of E-infinity theory and the mass spectrum of high energy particle physics, Chaos, Solitons & Fractals, 19(2004), 209-236.

[9] El Naschie M.S., Fuzzy dodecahedron topology and E-infinity spacetime as a model for quantum physics, Chaos, Solitons & Fractals, 30(2006), 1025-1033.

[10] Grabiec M., Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, 27(1988), 385-389.

[11] George A., Veeramani P., On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64(1994), 395-399.

[12] Gregori V., Sapena A., On fixed point theorem in fuzzy metric spaces, Fuzzy Sets and Systems, 125(2002), 245-252.

[13] Hadzic O., Fixed point theory in probabilistic metric spaces, Navi Sad: Serbian Academy of Science and Arts, 1995.

[14] Jungck G., Rhoades B.E., Fixed point for set valued functions without continuity, Ind. J. Pure and Appl. Math., 29(3)(1998), 227-238.

[15] Karmosil O., Michalek J., Fuzzy metric and statistical metric spaces, Kybernetica, 11(1975), 326-334.

[16] Klement E.P., Mesiar R., Pap E., Triangular Norms, Kluwer Academic Pub. Trends in Logic 8, Dordrecht 2000.

[17] Kubiaczyk I., Sharma S., Common coincidence point in fuzzy metric space, J. Fuzzy Math., 11(2003), 1-5.

[18] Kutukcu S., Weak Compatibility and common coincidence points in intuitionist fuzzy metric spaces, Southeast Asian Bulletin of Mathematics, 32(2008), 1081-1089.

[19] Menger K., Statistical metrices, Proc. Nat. Acad. Sci., 28(1942), 535-537.

[20] Muralisankar S., Kalpana G., Common fixed point theorem in intuitionistic fuzzy metric spaces using general contractive condition of integral type, Int. J. Contemp. Math. Sciences, 4(11)(2009), 505-518.

[21] Park J.H., Intuitionistic fuzzy metric spaces, Choas, Solitons & Fractals, 22(2004), 1039-1046.

[22] Sedghi S., Shobe N., Aliouche A., Common fixed point theorems in intuitionistic fuzzy metric spaces through condition of integral type, Applied Mathematics and Information Sciences, 2(1)(2008), 61-82.

[23] Schweizer B., Sklar A., Statistical metric spaces, Pacific Journal Math., 10(1960), 314-334.

[24] Sharma S., Tilwankar P., Common fixed point theorem for multivalued mappings in intuitionistic fuzzy metric space, East Asian Mathematical Journal., Korea, 24(2008), 223-232.

[25] Turkoglu D., Alaca C., Yildiz C., Compatible maps and compatible maps of types (U3b1) and (U3b2) in intuitionistic fuzzy metric spaces, Demon- stratio Math., 39(2006), 671-684.

[26] Yager R.R., On a class of weak triangular norm operators, Information Sciences, 96(1-2) (1997), 47-78.

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