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# LOGICAL OPERATORS AND WEAK LATTICE GRAPHS

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***ABSTRACT***

*We introduce the logical operators $\bar{L}$ , $\underline{L}$ and max – $\bar{L}$ , that is composition $\boxed{L}$ . Also we Introduce weak lattice, sub-weak lattice. Finally we obtain some properties.*

***Key words:*** *Set, Graphs, Logical operators, Weak lattice, Sub-week Lattice.*

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## 1. INTRODUCTION

In crisp graphs the concept of internally stable sets, denoted Int(R) and externally stable sets, denoted Ext(R) of a given graph$G = (X, R)$. A very important and interesting method for the determination of those sets uses of the algebraic formulation of these concepts [5, 7]. The properties of not external domination have been extended under some valued operators by Kitainik [4]. In section 3, we introduce the extension of composition law, weak lattice and sub-weak lattice in graphs. The structure of counterpart of the not externally dominated set denoted Ned($\rho$ ) is completely determined. Finally, we develop some properties on weak lattice by using the set Ned ($\rho$ ).

## 2. PRELIMINARIES

**Definition 2.1** [8]**:** A lattice is an algebraic system $(L, \wedge, \vee)$ with two binary operations $\wedge$and $\vee$on a non empty set L which are both idempotent, commutative, associative and satisfy the absorption laws.

**Example 2.1**[1]**:** The algebraic system ( $\wp(X), \wedge, \vee$ ) is a lattice under Zadeh's inclusion ( $\mu_1 \subseteq \mu_2 \Leftrightarrow (\forall x(\mu_1(x) \leq$ $\mu 2x$

**Definition 2.2**[8]**:** Let $(L, \wedge, \vee)$ be a lattice then the non empty subset S of the set $L$ is said to be sub lattice if it is closed under the operations $\wedge$ $and$ $\vee$and of $L$ , that is if $(a \wedge b) \in S$ and $(a \vee\ b) \in S, \forall\ a, b \in S$.

**Definition 2.3** [1]**:** Let $X$ be an arbitrary finite non empty set, $R$ a crisp relation defined on $X$ and $G = (X, R)$ is the associated directed graph. If $A\ \subseteq X$, the set of the elements of $X$ are dominated by A then the composition of A and R such that $A \circ R = \{y \in X | (\ \exists x A) x R y\}$.

**Definition 2.4** [1]: A subset A of a non empty set $X$ is said to be not externally dominated (Ned) if "no element in $A$ is dominated by an element in $\bar{A}$" $(\forall\ y)[y \in A \Rightarrow (\forall x \in \bar{A}) Not(xRy)]$.

**Note:** The set of the not externally dominated sets of the crisp graph $G = (X, R)$ for each $A\ \subseteq X$, is denoted by Ned(R).

Here $\bar{A}$ is the complement of $A$ in $X$ such that $\bar{A} = X - A$.

**Proposition 2.1** [1]**:** Let $G = (X, R)$ be a crisp loop free graph and $A \subseteq X$ we have $A$ is a Ned $\Leftrightarrow \bar{A} \circ R \subseteq \bar{A}\ \Leftrightarrow A \circ R^{-1} \subseteq A$.

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## 3. MAIN RESULTS

**Definition:** The logical operators $\bar{L}$ , $\underline{L}$ and N are defined as follows, let $\mu_1$ *and* $\mu_2$ be any two sets of $X$ then,$\forall\ x \in X$
i. $(\mu_1\,\bar{L}\ \mu_2) = \max_{x\in X} \mu_1(x) + \mu_2(x) - 1,0$
ii. $\left(\mu_1\,\underline{L}\,\mu_2\right) = \min_{x\in X} \mu_1(x) + \mu_2(x), 1, and$
iii. $N(\mu_1) = \overline{\mu_1(x)} = 1 - \mu_1(x).$

**Definition:** Let $\mu, \rho$ be a subset and a relation respectively defined on a non empty set $X$ and the composition $\boxed{L}$ , then the composition of $\mu$ *and* $\rho$ ( $\mu\,\boxed{L}\,\rho$) is defined as, for each $x \in X$,

$$(\mu\,\boxed{L}\,\rho)(x) = \max_{y\in x}[\,\mu(x) + \rho(x,y) - 1,0]$$

**Note:** In graph $G = (\ \mu\ , \rho)$ with underlying set $X$ where ( $\mu : X \to [0,1]$ $\rho : X \times X \to [0,1]$ then the above composition $\boxed{L}$ can be defined as, for each $a\epsilon X$, $\left(\mu(a)\boxed{L}\,\rho(a)\right) = \max_{b\epsilon X} Max[\,\mu(a) +\ \rho(a,b) - 1,0]$

**Proposition:** Let $\rho_1, \rho_2 \epsilon \wp(X \times X)$ and $\mu_1, \mu_2 \epsilon \wp(X)$ we have for any composition $\boxed{L}$ the following axioms are hold
i. $\ominus\,\boxed{L}\rho_1\ = \ominus$
ii. $\mu_1\ \subseteq\ \ \mu_2 \Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\right) \subseteq \left(\mu_2\boxed{L}\,\rho_2\right)$
iii. $\rho_1\ \subseteq\ \ \rho_2 \Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\right) \subseteq \left(\mu_2\boxed{L}\,\rho_2\right)$
iv. $\mu_1\,\boxed{L}\,\left(\rho_1\,\boxed{L}\,\rho_2\right) = \left(\mu_1\,\boxed{L}\,\rho_1\right)\boxed{L}\,\rho_2$
v. $(\mu_1\,\bar{L}\ \mu_2)\,\boxed{L}\,\rho_1 \supseteq \left(\mu_1\,\boxed{L}\,\rho_1\right)\bar{L}\,\left(\mu_2\boxed{L}\,\rho_1\right)$
vi. $\mu_1\,\boxed{L}\,(\rho_1\,\bar{L}\ \rho_2) = \left(\mu_1\,\boxed{L}\,\rho_1\right)\bar{L}\,\left(\mu_1\boxed{L}\,\rho_2\right)$

**Proof:**

**I.** By the definition----- we have
$(\ominus\,\boxed{L}\,\rho_1(x)) =\ \max_{y\in X} Max\{0 + \rho_1\,(x,y) - 1,0\} = 0$, Since
$0 \le \rho_1\,(x,y) \le 1 \forall x, y \in X$

**II.** We know that $\mu_1\ \subseteq\ \ \mu_2 \Rightarrow \mu_1\,(x) \le\ \mu_2(x) for\ all\ x \in X, then\ \exists\ y \in X$, For any relation $\rho_1\,(x,y)$ such that $\mu_1\,(x) \ge \rho_1\,(x,y) and\ \mu_2\,(x) \ge \rho_1\,(x,y)$ we have

$\Rightarrow [\mu_1\,(x) + \rho_1\,(x,y)] \le [\mu_2\,(x) + \rho_1\,(x,y)\ ]$

$\Rightarrow [\mu_1\,(x) + \rho_1\,(x,y) - 1] \le [\mu_2\,(x) + \rho_1\,(x,y) - 1]$

$Max\ Max\ [\mu_1\,(x) + \rho_1\,(x,y) - 1,0] \le Max\ Max[\mu_2\,(x) \ge \rho_1\,(x,y) - 1, 0]$

$\Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right)(x) \le \left(\mu_2\,\boxed{L}\,\rho_1\ \right)(x)$

$\Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right) \subseteq \left(\mu_2\,\boxed{L}\,\rho_1\ \right)$

Therefore $\mu_1\ \subseteq\ \ \mu_2 \Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right) \subseteq \left(\mu_2\,\boxed{L}\,\rho_1\ \right)$

**III.** We know that $\rho_1\ \subseteq\ \ \rho_2 \Leftrightarrow \rho_1\,(x,y) \le \rho_2\,(x,y) for\ all\ x, y \in X$

$\Rightarrow [\mu_1\,(x) + \rho_1\,(x,y)] \le [\mu_2\,(x) + \rho_2\,(x,y)]$

Since $\mu_1\,(x) \ge \rho_1\,(x,y) and\ \mu_2\,(x) \ge \rho_1\,(x,y)$

$\Rightarrow [\mu_1\,(x) + \rho_1\,(x,y) - 1] \le [\mu_2\,(x) + \rho_2\,(x,y) - 1]$

$\Rightarrow Max\ Max\ [\mu_1\,(x) + \rho_1\,(x,y) - 1, 0] \le Max\ Max[\mu_2\,(x) + \rho_2\,(x,y) - 1, 0]$

$\Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right)(x) \le \left(\mu_2\,\boxed{L}\,\rho_2\ \right)(x)$

$\Rightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right) \subseteq \left(\mu_2\,\boxed{L}\,\rho_2\ \right)$

Therefore $\rho_1\ \subseteq\ \ \sigma \Longrightarrow \left(\mu_1\,\boxed{L}\,\rho_1\ \right) \subseteq \left(\mu_2\,\boxed{L}\,\rho_2\ \right)$

**IV.** Since composition $\boxed{L}$ is associative, we have $\left( \mu_1 \boxed{L} \left( \rho_1 \boxed{L} \rho_2 \right)\right) = \left(\left(\mu_1 \boxed{L} \rho_1 \right) \boxed{L} \rho_2 \right)$

Since the definition of composition $\boxed{L}$ , $\bar{L}$ and the properties of t – norm arrive immediately. Therefore we have (V) and (IV).

**Example:** Let $G = (\mu , \rho)$ be a graph where $X = \{a, b, c, d, e\}$. $\mu: X \to [0,1], \rho: X \times X \to [0,1]$ as defined as

$\mu(a) = 0.6, \mu(b) = 0.8, \mu(c) = 0.7, \mu(d) = 0.9, \mu(e) = 0.5,$

$\rho(a, b) = 0.4, \rho(b, c) = 0.7, \rho(c, d) = 0.5, \rho(b, d) = 0.6, \rho(e, d) = 0.5, \rho(a, e) = 0.3.$

i. Is trivial

ii. If $\mu(a) \leq \mu(b) \Longrightarrow 0.6 \leq 0.8$ consider $\rho(a, b) = 0.4$

$$\left(\mu(a)\boxed{L}\,\rho(a, b)\right) = \text{Max}\,Max[\,\mu(a) + \rho(a, b) - 1{,}0] = 0 \tag{1}$$

$$\left(\mu(b)\boxed{L}\,\rho(a, b)\right) = \text{Max}\,Max[\,\mu(b) + \rho(a, b) - 1{,}0] = 0.2 \tag{2}$$

From (1) and (2) we have $\left(\mu(a)\boxed{L}\,\rho(a, b)\right) \leq \left(\mu(b)\boxed{L}\,\rho(a, b)\right)$

iii. If $\rho(a, b) = 0.4, \rho(b, c) = \sigma(b, c) = 0.7\ \mu(a) = 0.6\ and\ \mu(b) = 0.8$

Now $\rho(b, c) \leq \sigma(b, c)$

$$\left(\mu(a)\boxed{L}\,\rho(a, b)\right) = \text{Max}\,Max[\,\mu(a) + \rho(a, b) - 1{,}0] = 0 \tag{3}$$

$$\left(\mu(a)\boxed{L}\,\sigma(a, b)\right) = \text{Max}\,Max[\,\mu(a) + \sigma(a, b) - 1{,}0] = 0.5 \tag{4}$$

From (3) and (4) we have $\left(\mu(a)\boxed{L}\,\rho(a, b)\right) \leq \left(\mu(b)\boxed{L}\,\sigma(a, b)\right)$

iv. If $\mu(a) = 0.6, \rho(a, e) = 0.3, \rho(a, d) = \sigma(a, d) = 0.5$
v. $\left(\mu(a)\boxed{L}\,\rho(a, e)\right) = \text{Max}\,Max[\,\mu(a) + \rho(a, e) - 1{,}0] = 0.1$

$\left(\rho(a, e)\boxed{L}\,\sigma(a, d)\right) = \text{Max}\,Max[\,\rho(a, e) + \sigma(a, d) - 1{,}0] = 0$

$(\mu(a)\boxed{L}\ (\rho(a, e)\boxed{L}\,\sigma(a, d)))$=0,

$\left(\left(\mu(a)\boxed{L}\,\rho(a, e)\right)\boxed{L}\,\sigma(a, d)\right) = 0$

Therefore, we get $(\mu(a)\boxed{L}\ (\rho(a, e)\boxed{L}\,\sigma(a, d))) = \left(\left(\mu(a)\boxed{L}\,\rho(a, e)\right)\boxed{L}\,\sigma(a, d)\right)$

v. If $\mu(b) = 0.8, \mu(c) = 0.7, \rho(c, d) = 0.7$

$\left(\mu(b)\ \bar{L}\,\mu(c)\right) = \text{Max}\,Max[\,\mu(b) + \mu(b) - 1{,}0] = 0.5$

$$\left(\left(\mu(b)\ \bar{L}\,\mu(c)\right)\boxed{L}\,\rho(c, d)\right) = 0.2 \tag{5}$$

$\left(\mu(b)\boxed{L}\,\rho(c, b)\right) = 0.5, \left(\mu(c)\boxed{L}\,\rho(c, b)\right) = 0.4$

$$\left(\left(\mu(b)\boxed{L}\,\rho(c, b)\right)\bar{L}\left(\mu(c)\boxed{L}\,\rho(c, b)\right)\right) = \text{Max}\,Max[\,0.5 + 0.4 - 1{,}0] = 0 \tag{6}$$

From (5) and (6)

 

We have$\left((\mu(b)\,\overline{L}\,\mu(c))\,\boxed{L}\,\rho(c,d)\right) \geq \left(\left(\mu(b)\boxed{L}\,\rho(c,b)\right)\overline{L}\left(\mu(c)\boxed{L}\,\rho(c,b)\right)\right)$.

vi. If $\mu(d) = 0.9, \rho(b,d) = \sigma(b,d) = 0.6, \rho(e,d) = 0.5$

$\left(\mu(d)\boxed{L}\,(\rho(e,b)\overline{L}\sigma(d,b))\right) = 0$

$\left(\mu(d)\boxed{L}\,\rho(e,d)\right) = 0.4, \left(\mu(d)\boxed{L}\,\sigma(d,b)\right) = 0.5,$

$\left(\mu(d)\boxed{L}\,\rho(e,d)\right)\overline{L}\left(\mu(d)\boxed{L}\,\sigma(d,b)\right) = 0,$

Therefore, we have $\left(\mu(d)\boxed{L}\,(\rho(e,d)\overline{L}\sigma(d,b))\right) = \left(\mu(d)\boxed{L}\,\rho(e,d)\right)\overline{L}\left(\mu(d)\boxed{L}\,\sigma(d,b)\right)$.

From the above example we consider $\mu(b) = 0.8$, $\rho(b,d) = 0.6$, $\rho(b,c) = \sigma(b,c) = 0.7$

$\left(\rho(b,d)\overline{L}\sigma(b,c)\right) = 0.3$

$$\left(\mu(b)\,\boxed{L}\,\rho(b,d)\,\overline{L}\,\sigma(b,c)\right) = 0.1 \quad (7)$$

$\left(\mu(b)\,\boxed{L}\,\rho(b,d)\right) = 0.4, \left(\mu(b)\,\boxed{L}\,\sigma(b,c)\right) = 0.5$

$$\left(\mu(b)\,\boxed{L}\,\rho(b,d)\right)\overline{L}\left(\mu(b)\,\boxed{L}\,\sigma(b,c)\right) = 0 \quad (8)$$

From (7) and (8) we have $\left(\mu(b)\,\boxed{L}\,\left(\rho(b,d)\overline{L}\sigma(b,c)\right)\right) \geq \left(\mu(b)\,\boxed{L}\,\rho(b,d)\right)\,\overline{L}(\mu(b)\,\boxed{L}\,\sigma(b,c)))$.

Hence Property (vi). Does not hold

**Remarks:** the following axioms

i. $\left((\mu_1\,\underline{L}\,\mu_2\,)\,\boxed{L}\,\rho\right) = (\mu_1\,\boxed{L}\,\rho)\,\underline{L}\,(\mu_2\,\boxed{L}\,\rho)$

ii. $(\mu_1\,\boxed{L}\,(\rho\,\underline{L}\,\sigma)) = (\mu_1\,\boxed{L}\,\rho)\,\underline{L}\,(\mu_1\,\boxed{L}\,\sigma)$

But we get $\left((\mu_1\,\underline{L}\,\mu_2\,)\,\boxed{L}\,\rho\right) \geq (\mu_1\,\boxed{L}\,\rho)\,\underline{L}\,(\mu_2\,\boxed{L}\,\rho)$ and

$(\mu_1\,\boxed{L}\,(\rho\,\underline{L}\,\sigma)) \geq (\mu_1\,\boxed{L}\,\rho)\,\underline{L}\,(\mu_1\,\boxed{L}\,\sigma)$.

**Definition:** a weak lattice is an algebraic system ( $W, \overline{L}, \underline{L}$ ) with two binary logical operators $\overline{L}$ $and$ $\underline{L}$ on non empty set $W$ which satisfies both commutative and associative laws.

i. Commutative laws: $(a\,\overline{L}\,b) = (b\,\overline{L}\,a)$ $\boldsymbol{and}$ $(a\,\underline{L}\,b) = (b\,\underline{L}\,a)$

ii. Associative law: $(a\,\overline{L}\,b)\overline{L}\,c = a\,\overline{L}\,(b\,\overline{L}\,c)$ $\boldsymbol{and}$ $(a\underline{L}\,b)\underline{L}\,c = a\,\underline{L}\,(b\,\underline{L}\,c)\ \forall\, a, b, and\ c\ \epsilon\ W$

**Remarks:** in this paper we consider an algebraic system $(\wp(X), \overline{L}\,, \underline{L})$ is a weak lattice under inclusion $\mu_1 \subseteq \mu_2 \Leftrightarrow (\forall\, X(\mu_1(x) \leq \mu_2(x))$. let $G = (\mu, \rho)$ be a graph, then $(\mu(x), \overline{L}\,, \underline{L})$ is a week lattice under condition for each $a, b\ \epsilon\ X \Longrightarrow \mu(a) \leq \mu(b)$ and the composition $\boxed{L}$.

**Example:** let $\mu_1$ $and$ $\mu_2$ be any sets of $(\wp(X)$ where $\mu_1(x) = 0.4$, $\mu_2(x) = 0.7$ $and$ $\mu_3(x) = 0.6$ $for\ each\ x\ \epsilon\ X$, then we have

1) Idempotent Laws:
i. $(\mu_1\,\overline{L}\,\mu_2\,) = 0 \neq \mu_1$

ii. $(\mu_1\,\underline{L}\,\mu_2\,) = 0.8 \neq \mu_1$ , Therefore, idempotent Laws are not satisfied.

 

2) Commutative Laws:
i. $(\mu_1 \overline{L}\, \mu_2) = 0.1 = (\mu_2 \overline{L}\, \mu_1)$

ii. $(\mu_1 \underline{L}\, \mu_2) = 1 = (\mu_2 \underline{L}\, \mu_1)$, Therefore, Commutative laws are satisfied.

3) Associative Laws:
i. $(\mu_1 \overline{L}\, \mu_2)\, \overline{L}\, \mu_3 = 0 = \mu_1 \overline{L}\, (\mu_2 \overline{L}\, \mu_3)$

ii. $(\mu_1 \underline{L}\, \mu_2)\, \underline{L}\, \mu_3 = 1 = \mu_1 \underline{L}\, (\mu_2 \underline{L}\, \mu_3)$, Therefore Associative laws satisfied.

4) Absorption Laws:
i. $\mu_1 \overline{L}\, (\mu_1 \underline{L}\, \mu_2) = 0.4 = \mu_1$

ii. $\mu_1 \underline{L}\, (\mu_1 \overline{L}\, \mu_2) = 0.5 \neq \mu_1$

Suppose if $\mu_1(x) = 0.2$, $\mu_2(x) = 0.6$ then we have $\mu_1 \overline{L}\, (\mu_1 \underline{L}\, \mu_2) = 0 \neq \mu_1$ therefore absorption laws are not satisfied. Hence $(\wp(X), \overline{L}, \underline{L})$ is a week lattice.

**Definition:** Let $(W, \overline{L}, \underline{L})$ is a weak lattice then the two non empty sets $S$ of the set $W$ is said to be sub – weak lattice if it is closed under the operations $\overline{L}, and\ \underline{L}$ that is if $(a\, \overline{L}\, b) \in S$ and $(a\, \underline{L}\, b) \in S, \forall\, a, b \in S$.

**Definition:** let $G = (\mu, \rho)$ be a graph without loops and with underlying set $X$ where $\mu: X \to [0,1]$, $\rho: X \times X \to [0,1]$ and $a, b \in X$ we shall say that in $G$ a is composition $\overline{L}$ Ned $\Leftrightarrow \left(\left(\overline{\mu(a)}\boxed{L}\rho(a,b)\right) \leq \overline{\mu(a)}\right) and \left(\mu(a)\boxed{L}\rho^{-1}(a,b) \leq \right.$ $\mu a$. We denote it by Ned $\rho, L$, the set of all sets satisfying the equivalent condition.

**Proposition of the set Ned $(\boldsymbol{\rho}, \boxed{L})$**

**Proposition:** let $G = (\mu, \rho)$ be a graph without loops and with underlying set $X$ where $\mu: X \to [0,1]$, $\rho: X \times X \to [0,1]$ and $a \in X$ the set Ned $(\rho, \boxed{L})$
i. Is a sub – weak lattice of $(\mu(X), \overline{L}, \underline{L})$

ii. Contains any constant k.1 of set $(\wp(X)$

**Proof:**
i. Let $a, b \in Ned\ (\rho, \boxed{L})$ then we have,
$$\left(\mu(a)\boxed{L}\rho^{-1}(a,b) \leq \mu(a)\right) and \left(\mu(b)\boxed{L}\rho^{-1}(a,b) \leq \mu(b)\right) \quad (9)$$

$$\left(\left(\overline{\mu(a)}\boxed{L}\rho(a,b)\right) \leq \overline{\mu(a)}\right) and \left(\left(\overline{\mu(b)}\boxed{L}\rho(b,a)\right) \leq \overline{\mu(b)}\right) \quad (10)$$

*To prove* $(\left(\mu(a)\overline{L}\mu(b)\right)\boxed{L}\,\rho^{-1}(a,b) \leq (\mu(a)\overline{L}\,\mu(b))$

Now $(\left(\mu(a)\overline{L}\mu(b)\right)\boxed{L}\,\rho^{-1}(a,b) \leq (\mu(a)\,\boxed{L}\rho^{-1}(a,b))\overline{L}(\mu(b)\boxed{L}\rho^{-1}(a,b))$

$$\leq \left(\mu(a)\overline{L}\,\mu(b)\right) \qquad from\ (9)$$

Similarly, we to prove that $\overline{\left(\mu(a)\overline{L}\mu(b)\right)}\boxed{L}\,\rho(a,b) \leq \overline{\left(\mu(a)\overline{L}\mu(b)\right)}$ $from\ (10)$

Same method for the operator $\underline{L}$, we have $\overline{\left(\mu(a)\underline{L}\mu(b)\right)}\boxed{L}\,\rho(a,b) \leq \overline{\left(\mu(a)\underline{L}\mu(b)\right)}$ and

$$\left(\left(\mu(a)\,\underline{L}\,\mu(b)\right)\boxed{L}\,\rho(a,b)\right) \leq \left(\mu(a)\,\underline{L}\,\mu(b)\right).$$

Hence Ned $(\rho, \boxed{L})$ is a sub-weak lattice of $(\mu(X), \overline{L}, \underline{L})$
ii. Let $k \in [0,1]\ \forall\ x \in X$ then $\exists\ y\ \in X$,
iii. $\left((k.1)\boxed{L}\,\rho^{-1}(x,y)\right) = Max\ Max\ [k + \rho^{-1}(x,y) - 1{,}0] \leq k since\ 0 \leq \rho(x,y) \leq$ (11)

And $\left((\overline{k}.1)\boxed{L}\,\rho(x,y)\right) = Max\ Max\left[\overline{k} + \rho(x,y) - 1{,}0\right] \leq \overline{k}$ (12)

From (11) and (12), we get condition for an element in Ned $(\rho, \boxed{L})$

Hence (ii) proved

Example: if $k\epsilon[0,1]$ and any relation $\rho$ of a non empty set $X$ such that $0 \leq \rho(x,y) \leq 1$ for all $x, y \in X$ consider $k = 0.6\ and\ \rho(x,y) =\ 0.6$ Then
$\left((k.1)\boxed{L}\,\rho^{-1}(x,y)\right) = Max\ Max\ \{0.6 + 0.6 - 1{,}0\} =\ 0.2$

$$= 0.2 \leq 0.6 = k \quad (13)$$

And $\left((k.1)\boxed{L}\,\rho(x,y)\right) = Max\ Max\ \left\{\overline{k} + \rho(x,y) - 1{,}0\right\}$

$= Max\ Max\ \{0.4 + 0.6 - 1{,}0\} = 0 \leq \overline{k} = 0.4$ (14)

From (13) and (14) we have the conditions (11) and (12) respectively

Hence the condition (ii) of proposition 3.2 is proved.

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