

**LOGICAL OPERATORS AND WEAK LATTICE GRAPHS**

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**ABSTRACT**

*We introduce the logical operators  $\bar{L}$ ,  $\underline{L}$  and  $\max - \bar{L}$ , that is composition  $\square \bar{L}$ . Also we Introduce weak lattice, sub-weak lattice. Finally we obtain some properties.*

*Key words: Set, Graphs, Logical operators, Weak lattice, Sub-week Lattice.*

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**1. INTRODUCTION**

In crisp graphs the concept of internally stable sets, denoted  $\text{Int}(R)$  and externally stable sets, denoted  $\text{Ext}(R)$  of a given graph  $G = (X, R)$ . A very important and interesting method for the determination of those sets uses of the algebraic formulation of these concepts [5, 7]. The properties of not external domination have been extended under some valued operators by Kitainik [4]. In section 3, we introduce the extension of composition law, weak lattice and sub-weak lattice in graphs. The structure of counterpart of the not externally dominated set denoted  $\text{Ned}(\rho)$  is completely determined. Finally, we develop some properties on weak lattice by using the set  $\text{Ned}(\rho)$ .

**2. PRELIMINARIES**

**Definition 2.1** [8]: A lattice is an algebraic system  $(L, \wedge, \vee)$  with two binary operations  $\wedge$  and  $\vee$  on a non empty set  $L$  which are both idempotent, commutative, associative and satisfy the absorption laws.

**Example 2.1**[1]: The algebraic system  $(\wp(X), \wedge, \vee)$  is a lattice under Zadeh's inclusion  $(\mu_1 \subseteq \mu_2 \Leftrightarrow (\forall x)(\mu_1(x) \leq \mu_2(x)))$

**Definition 2.2**[8]: Let  $(L, \wedge, \vee)$  be a lattice then the non empty subset  $S$  of the set  $L$  is said to be sub lattice if it is closed under the operations  $\wedge$  and  $\vee$  of  $L$ , that is if  $(a \wedge b) \in S$  and  $(a \vee b) \in S, \forall a, b \in S$ .

**Definition 2.3** [1]: Let  $X$  be an arbitrary finite non empty set,  $R$  a crisp relation defined on  $X$  and  $G = (X, R)$  is the associated directed graph. If  $A \subseteq X$ , the set of the elements of  $X$  are dominated by  $A$  then the composition of  $A$  and  $R$  such that  $A \circ R = \{y \in X | (\exists x \in A)xRy\}$ .

**Definition 2.4** [1]: A subset  $A$  of a non empty set  $X$  is said to be not externally dominated (Ned) if "no element in  $A$  is dominated by an element in  $\bar{A}$ "  $(\forall y)[y \in A \Rightarrow (\forall x \in \bar{A}) \text{Not}(xRy)]$ .

**Note:** The set of the not externally dominated sets of the crisp graph  $G = (X, R)$  for each  $A \subseteq X$ , is denoted by  $\text{Ned}(R)$ .

Here  $\bar{A}$  is the complement of  $A$  in  $X$  such that  $\bar{A} = X - A$ .

**Proposition 2.1** [1]: Let  $G = (X, R)$  be a crisp loop free graph and  $A \subseteq X$  we have  $A$  is a Ned  $\Leftrightarrow \bar{A} \circ R \subseteq \bar{A} \Leftrightarrow A \circ R^{-1} \subseteq A$ .

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### 3. MAIN RESULTS

**Definition:** The logical operators  $\bar{L}$ ,  $\underline{L}$  and  $N$  are defined as follows, let  $\mu_1$  and  $\mu_2$  be any two sets of  $X$  then,  $\forall x \in X$

- i.  $(\mu_1 \bar{L} \mu_2) = \max_{x \in X} \mu_1(x) + \mu_2(x) - 1, 0$
- ii.  $(\mu_1 \underline{L} \mu_2) = \min_{x \in X} \mu_1(x) + \mu_2(x), 1$ , and
- iii.  $N(\mu_1) = \overline{\mu_1(x)} = 1 - \mu_1(x)$ .

**Definition:** Let  $\mu, \rho$  be a subset and a relation respectively defined on a non empty set  $X$  and the composition  $\boxed{L}$ , then the composition of  $\mu$  and  $\rho$  ( $\mu \boxed{L} \rho$ ) is defined as, for each  $x \in X$ ,

$$(\mu \boxed{L} \rho)(x) = \max_{y \in X} [\mu(x) + \rho(x, y) - 1, 0]$$

**Note:** In graph  $G = (\mu, \rho)$  with underlying set  $X$  where  $(\mu : X \rightarrow [0,1] \rho : X \times X \rightarrow [0,1])$  then the above composition  $\boxed{L}$  can be defined as, for each  $a \in X$ ,  $(\mu(a) \boxed{L} \rho(a)) = \max_{b \in X} \text{Max}[\mu(a) + \rho(a, b) - 1, 0]$

**Proposition:** Let  $\rho_1, \rho_2 \in \wp(X \times X)$  and  $\mu_1, \mu_2 \in \wp(X)$  we have for any composition  $\boxed{L}$  the following axioms are hold

- i.  $\ominus \boxed{L} \rho_1 = \ominus$
- ii.  $\mu_1 \subseteq \mu_2 \Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_2)$
- iii.  $\rho_1 \subseteq \rho_2 \Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_2)$
- iv.  $\mu_1 \boxed{L} (\rho_1 \boxed{L} \rho_2) = (\mu_1 \boxed{L} \rho_1) \boxed{L} \rho_2$
- v.  $(\mu_1 \bar{L} \mu_2) \boxed{L} \rho_1 \supseteq (\mu_1 \boxed{L} \rho_1) \bar{L} (\mu_2 \boxed{L} \rho_1)$
- vi.  $\mu_1 \boxed{L} (\rho_1 \bar{L} \rho_2) = (\mu_1 \boxed{L} \rho_1) \bar{L} (\mu_1 \boxed{L} \rho_2)$

**Proof:**

**I.** By the definition----- we have

$$(\ominus \boxed{L} \rho_1(x)) = \max_{y \in X} \text{Max}\{0 + \rho_1(x, y) - 1, 0\} = 0, \text{ Since } 0 \leq \rho_1(x, y) \leq 1 \forall x, y \in X$$

**II.** We know that  $\mu_1 \subseteq \mu_2 \Rightarrow \mu_1(x) \leq \mu_2(x)$  for all  $x \in X$ , then  $\exists y \in X$ , For any relation  $\rho_1(x, y)$  such that  $\mu_1(x) \geq \rho_1(x, y)$  and  $\mu_2(x) \geq \rho_1(x, y)$  we have

$$\Rightarrow [\mu_1(x) + \rho_1(x, y)] \leq [\mu_2(x) + \rho_1(x, y)]$$

$$\Rightarrow [\mu_1(x) + \rho_1(x, y) - 1] \leq [\mu_2(x) + \rho_1(x, y) - 1]$$

$$\text{Max Max} [\mu_1(x) + \rho_1(x, y) - 1, 0] \leq \text{Max Max} [\mu_2(x) + \rho_1(x, y) - 1, 0]$$

$$\Rightarrow (\mu_1 \boxed{L} \rho_1)(x) \leq (\mu_2 \boxed{L} \rho_1)(x)$$

$$\Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_1)$$

$$\text{Therefore } \mu_1 \subseteq \mu_2 \Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_1)$$

**III.** We know that  $\rho_1 \subseteq \rho_2 \Leftrightarrow \rho_1(x, y) \leq \rho_2(x, y)$  for all  $x, y \in X$

$$\Rightarrow [\mu_1(x) + \rho_1(x, y)] \leq [\mu_2(x) + \rho_2(x, y)]$$

Since  $\mu_1(x) \geq \rho_1(x, y)$  and  $\mu_2(x) \geq \rho_1(x, y)$

$$\Rightarrow [\mu_1(x) + \rho_1(x, y) - 1] \leq [\mu_2(x) + \rho_2(x, y) - 1]$$

$$\Rightarrow \text{Max Max} [\mu_1(x) + \rho_1(x, y) - 1, 0] \leq \text{Max Max} [\mu_2(x) + \rho_2(x, y) - 1, 0]$$

$$\Rightarrow (\mu_1 \boxed{L} \rho_1)(x) \leq (\mu_2 \boxed{L} \rho_2)(x)$$

$$\Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_2)$$

$$\text{Therefore } \rho_1 \subseteq \rho_2 \Rightarrow (\mu_1 \boxed{L} \rho_1) \subseteq (\mu_2 \boxed{L} \rho_2)$$

IV. Since composition  $\bar{L}$  is associative, we have  $(\mu_1 \bar{L} (\rho_1 \bar{L} \rho_2)) = ((\mu_1 \bar{L} \rho_1) \bar{L} \rho_2)$

Since the definition of composition  $\bar{L}$ ,  $\bar{L}$  and the properties of t – norm arrive immediately. Therefore we have (V) and (IV).

**Example:** Let  $G = (\mu, \rho)$  be a graph where  $X = \{a, b, c, d, e\}$ .  $\mu: X \rightarrow [0,1], \rho: X \times X \rightarrow [0,1]$  as defined as

$$\mu(a) = 0.6, \mu(b) = 0.8, \mu(c) = 0.7, \mu(d) = 0.9, \mu(e) = 0.5,$$

$$\rho(a, b) = 0.4, \rho(b, c) = 0.7, \rho(c, d) = 0.5, \rho(b, d) = 0.6, \rho(e, d) = 0.5, \rho(a, e) = 0.3.$$

i. Is trivial

ii. If  $\mu(a) \leq \mu(b) \Rightarrow 0.6 \leq 0.8$  consider  $\rho(a, b) = 0.4$

$$(\mu(a) \bar{L} \rho(a, b)) = \text{MaxMax}[\mu(a) + \rho(a, b) - 1, 0] = 0 \tag{1}$$

$$(\mu(b) \bar{L} \rho(a, b)) = \text{MaxMax}[\mu(b) + \rho(a, b) - 1, 0] = 0.2 \tag{2}$$

From (1) and (2) we have  $(\mu(a) \bar{L} \rho(a, b)) \leq (\mu(b) \bar{L} \rho(a, b))$

iii. If  $\rho(a, b) = 0.4, \rho(b, c) = \sigma(b, c) = 0.7, \mu(a) = 0.6$  and  $\mu(b) = 0.8$

Now  $\rho(b, c) \leq \sigma(b, c)$

$$(\mu(a) \bar{L} \rho(a, b)) = \text{MaxMax}[\mu(a) + \rho(a, b) - 1, 0] = 0 \tag{3}$$

$$(\mu(a) \bar{L} \sigma(a, b)) = \text{MaxMax}[\mu(a) + \sigma(a, b) - 1, 0] = 0.5 \tag{4}$$

From (3) and (4) we have  $(\mu(a) \bar{L} \rho(a, b)) \leq (\mu(b) \bar{L} \sigma(a, b))$

iv. If  $\mu(a) = 0.6, \rho(a, e) = 0.3, \rho(a, d) = \sigma(a, d) = 0.5$

$$v. (\mu(a) \bar{L} \rho(a, e)) = \text{MaxMax}[\mu(a) + \rho(a, e) - 1, 0] = 0.1$$

$$(\rho(a, e) \bar{L} \sigma(a, d)) = \text{MaxMax}[\rho(a, e) + \sigma(a, d) - 1, 0] = 0$$

$$(\mu(a) \bar{L} (\rho(a, e) \bar{L} \sigma(a, d))) = 0,$$

$$((\mu(a) \bar{L} \rho(a, e)) \bar{L} \sigma(a, d)) = 0$$

Therefore, we get  $(\mu(a) \bar{L} (\rho(a, e) \bar{L} \sigma(a, d))) = ((\mu(a) \bar{L} \rho(a, e)) \bar{L} \sigma(a, d))$

v. If  $\mu(b) = 0.8, \mu(c) = 0.7, \rho(c, d) = 0.7$

$$(\mu(b) \bar{L} \mu(c)) = \text{MaxMax}[\mu(b) + \mu(c) - 1, 0] = 0.5$$

$$((\mu(b) \bar{L} \mu(c)) \bar{L} \rho(c, d)) = 0.2 \tag{5}$$

$$(\mu(b) \bar{L} \rho(c, b)) = 0.5, (\mu(c) \bar{L} \rho(c, b)) = 0.4$$

$$((\mu(b) \bar{L} \rho(c, b)) \bar{L} (\mu(c) \bar{L} \rho(c, b))) = \text{MaxMax}[0.5 + 0.4 - 1, 0] = 0 \tag{6}$$

From (5) and (6)

We have  $\left(\left(\mu(b) \bar{L} \mu(c)\right) \underline{L} \rho(c, d)\right) \geq \left(\left(\mu(b) \underline{L} \rho(c, b)\right) \bar{L} \left(\mu(c) \underline{L} \rho(c, b)\right)\right)$ .

vi. If  $\mu(d) = 0.9, \rho(b, d) = \sigma(b, d) = 0.6, \rho(e, d) = 0.5$

$$\left(\mu(d) \underline{L} \left(\rho(e, b) \bar{L} \sigma(d, b)\right)\right) = 0$$

$$\left(\mu(d) \underline{L} \rho(e, d)\right) = 0.4, \left(\mu(d) \underline{L} \sigma(d, b)\right) = 0.5,$$

$$\left(\mu(d) \underline{L} \rho(e, d)\right) \bar{L} \left(\mu(d) \underline{L} \sigma(d, b)\right) = 0,$$

Therefore, we have  $\left(\mu(d) \underline{L} \left(\rho(e, d) \bar{L} \sigma(d, b)\right)\right) = \left(\mu(d) \underline{L} \rho(e, d)\right) \bar{L} \left(\mu(d) \underline{L} \sigma(d, b)\right)$ .

From the above example we consider  $\mu(b) = 0.8, \rho(b, d) = 0.6, \rho(b, c) = \sigma(b, c) = 0.7$

$$\left(\rho(b, d) \bar{L} \sigma(b, c)\right) = 0.3$$

$$\left(\mu(b) \underline{L} \rho(b, d) \bar{L} \sigma(b, c)\right) = 0.1 \tag{7}$$

$$\left(\mu(b) \underline{L} \rho(b, d)\right) = 0.4, \left(\mu(b) \underline{L} \sigma(b, c)\right) = 0.5$$

$$\left(\mu(b) \underline{L} \rho(b, d)\right) \bar{L} \left(\mu(b) \underline{L} \sigma(b, c)\right) = 0 \tag{8}$$

From (7) and (8) we have  $\left(\mu(b) \underline{L} \left(\rho(b, d) \bar{L} \sigma(b, c)\right)\right) \geq \left(\mu(b) \underline{L} \rho(b, d)\right) \bar{L} \left(\mu(b) \underline{L} \sigma(b, c)\right)$ .

Hence Property (vi). Does not hold

**Remarks:** the following axioms

$$i. \left(\left(\mu_1 \underline{L} \mu_2\right) \underline{L} \rho\right) = \left(\mu_1 \underline{L} \rho\right) \underline{L} \left(\mu_2 \underline{L} \rho\right)$$

$$ii. \left(\mu_1 \underline{L} \left(\rho \underline{L} \sigma\right)\right) = \left(\mu_1 \underline{L} \rho\right) \underline{L} \left(\mu_1 \underline{L} \sigma\right)$$

But we get  $\left(\left(\mu_1 \underline{L} \mu_2\right) \underline{L} \rho\right) \geq \left(\mu_1 \underline{L} \rho\right) \underline{L} \left(\mu_2 \underline{L} \rho\right)$  and

$$\left(\mu_1 \underline{L} \left(\rho \underline{L} \sigma\right)\right) \geq \left(\mu_1 \underline{L} \rho\right) \underline{L} \left(\mu_1 \underline{L} \sigma\right).$$

**Definition:** a weak lattice is an algebraic system  $(W, \bar{L}, \underline{L})$  with two binary logical operators  $\bar{L}$  and  $\underline{L}$  on non empty set  $W$  which satisfies both commutative and associative laws.

i. Commutative laws:  $(a \bar{L} b) = (b \bar{L} a)$  **and**  $(a \underline{L} b) = (b \underline{L} a)$

ii. Associative law:  $(a \bar{L} b) \bar{L} c = a \bar{L} (b \bar{L} c)$  **and**  $(a \underline{L} b) \underline{L} c = a \underline{L} (b \underline{L} c) \forall a, b, \text{ and } c \in W$

**Remarks:** in this paper we consider an algebraic system  $(\wp(X), \bar{L}, \underline{L})$  is a weak lattice under inclusion  $\mu_1 \subseteq \mu_2 \Leftrightarrow (\forall X(\mu_1(x) \leq \mu_2(x)))$ . let  $G = (\mu, \rho)$  be a graph, then  $(\mu(x), \bar{L}, \underline{L})$  is a weak lattice under condition for each  $a, b \in X \Rightarrow \mu(a) \leq \mu(b)$  and the composition  $\underline{L}$ .

**Example:** let  $\mu_1$  and  $\mu_2$  be any sets of  $(\wp(X))$  where  $\mu_1(x) = 0.4, \mu_2(x) = 0.7$  and  $\mu_3(x) = 0.6$  for each  $x \in X$ , then we have

1) Idempotent Laws:

$$i. \left(\mu_1 \bar{L} \mu_2\right) = 0 \neq \mu_1$$

ii.  $\left(\mu_1 \underline{L} \mu_2\right) = 0.8 \neq \mu_1$ , Therefore, idempotent Laws are not satisfied.

2) Commutative Laws:

i.  $(\mu_1 \bar{L} \mu_2) = 0.1 = (\mu_2 \bar{L} \mu_1)$

ii.  $(\mu_1 \underline{L} \mu_2) = 1 = (\mu_2 \underline{L} \mu_1)$ , Therefore, Commutative laws are satisfied.

3) Associative Laws:

i.  $(\mu_1 \bar{L} \mu_2) \bar{L} \mu_3 = 0 = \mu_1 \bar{L} (\mu_2 \bar{L} \mu_3)$

ii.  $(\mu_1 \underline{L} \mu_2) \underline{L} \mu_3 = 1 = \mu_1 \underline{L} (\mu_2 \underline{L} \mu_3)$ , Therefore Associative laws satisfied.

4) Absorption Laws:

i.  $\mu_1 \bar{L} (\mu_1 \underline{L} \mu_2) = 0.4 = \mu_1$

ii.  $\mu_1 \underline{L} (\mu_1 \bar{L} \mu_2) = 0.5 \neq \mu_1$

Suppose if  $\mu_1(x) = 0.2, \mu_2(x) = 0.6$  then we have  $\mu_1 \bar{L} (\mu_1 \underline{L} \mu_2) = 0 \neq \mu_1$  therefore absorption laws are not satisfied. Hence  $(\wp(X), \bar{L}, \underline{L})$  is a weak lattice.

**Definition:** Let  $(W, \bar{L}, \underline{L})$  is a weak lattice then the two non empty sets  $S$  of the set  $W$  is said to be sub – weak lattice if it is closed under the operations  $\bar{L}$ , and  $\underline{L}$  that is if  $(a \bar{L} b) \in S$  and  $(a \underline{L} b) \in S, \forall a, b \in S$ .

**Definition:** let  $G = (\mu, \rho)$  be a graph without loops and with underlying set  $X$  where  $\mu: X \rightarrow [0,1], \rho: X \times X \rightarrow [0,1]$  and  $a, b \in X$  we shall say that in  $G$  a is composition  $\bar{L} \text{ Ned} \Leftrightarrow \left( \overline{(\mu(a) \underline{L} \rho(a, b))} \leq \overline{\mu(a)} \right)$  and  $(\mu(a) \underline{L} \rho^{-1}(a, b) \leq \mu a$ . We denote it by  $\text{Ned } \rho, \underline{L}$ , the set of all sets satisfying the equivalent condition.

**Proposition of the set Ned  $(\rho, \underline{L})$**

**Proposition:** let  $G = (\mu, \rho)$  be a graph without loops and with underlying set  $X$  where  $\mu: X \rightarrow [0,1], \rho: X \times X \rightarrow [0,1]$  and  $a \in X$  the set  $\text{Ned } (\rho, \underline{L})$

i. Is a sub – weak lattice of  $(\mu(X), \bar{L}, \underline{L})$

ii. Contains any constant k.1 of set  $(\wp(X))$

**Proof:**

i. Let  $a, b \in \text{Ned } (\rho, \underline{L})$  then we have,

$$(\mu(a) \underline{L} \rho^{-1}(a, b) \leq \mu(a)) \text{ and } (\mu(b) \underline{L} \rho^{-1}(a, b) \leq \mu(b)) \tag{9}$$

$$\left( \overline{(\mu(a) \underline{L} \rho(a, b))} \leq \overline{\mu(a)} \right) \text{ and } \left( \overline{(\mu(b) \underline{L} \rho(b, a))} \leq \overline{\mu(b)} \right) \tag{10}$$

To prove  $((\mu(a) \bar{L} \mu(b)) \underline{L} \rho^{-1}(a, b) \leq (\mu(a) \bar{L} \mu(b)))$

Now  $((\mu(a) \bar{L} \mu(b)) \underline{L} \rho^{-1}(a, b) \leq (\mu(a) \underline{L} \rho^{-1}(a, b)) \bar{L} (\mu(b) \underline{L} \rho^{-1}(a, b)))$

$$\leq (\mu(a) \bar{L} \mu(b)) \quad \text{from (9)}$$

Similarly, we to prove that  $(\overline{(\mu(a) \bar{L} \mu(b))} \underline{L} \rho(a, b) \leq \overline{(\mu(a) \bar{L} \mu(b))})$  from (10)

Same method for the operator  $\underline{L}$ , we have  $(\overline{(\mu(a) \underline{L} \mu(b))} \underline{L} \rho(a, b) \leq \overline{(\mu(a) \underline{L} \mu(b))})$  and

$$\left( (\mu(a) \underline{L} \mu(b)) \underline{L} \rho(a, b) \leq (\mu(a) \underline{L} \mu(b)) \right).$$

Hence  $\text{Ned}(\rho, \underline{L})$  is a sub-weak lattice of  $(\mu(X), \bar{L}, \underline{L})$

ii. Let  $k \in [0,1] \forall x \in X$  then  $\exists y \in X$ ,

$$\text{iii. } ((k.1)\underline{L})\rho^{-1}(x,y) = \text{Max Max } [k + \rho^{-1}(x,y) - 1,0] \leq k \text{ since } 0 \leq \rho(x,y) \leq \quad (11)$$

$$\text{And } ((\bar{k}.1)\underline{L})\rho(x,y) = \text{Max Max } [\bar{k} + \rho(x,y) - 1,0] \leq \bar{k} \quad (12)$$

From (11) and (12), we get condition for an element in  $\text{Ned}(\rho, \underline{L})$

Hence (ii) proved

Example: if  $k \in [0,1]$  and any relation  $\rho$  of a non empty set  $X$  such that  $0 \leq \rho(x,y) \leq 1$  for all  $x, y \in X$  consider  $k = 0.6$  and  $\rho(x,y) = 0.6$  Then

$$\begin{aligned} ((k.1)\underline{L})\rho^{-1}(x,y) &= \text{Max Max } \{0.6 + 0.6 - 1,0\} = 0.2 \\ &= 0.2 \leq 0.6 = k \end{aligned} \quad (13)$$

$$\begin{aligned} \text{And } ((k.1)\underline{L})\rho(x,y) &= \text{Max Max } \{\bar{k} + \rho(x,y) - 1,0\} \\ &= \text{Max Max } \{0.4 + 0.6 - 1,0\} = 0 \leq \bar{k} = 0.4 \end{aligned} \quad (14)$$

From (13) and (14) we have the conditions (11) and (12) respectively

Hence the condition (ii) of proposition 3.2 is proved.

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