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APPLICATION OF REDUCE DIGITS ALGORITHM IN DIVISIBILITY OF NUMBERS

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ABSTRACT

We give a new algorithm entitled reduce digits algorithm (RDA) for divisibility of numbers. In the other words, we study divisibility of numbers with a new algorithm about decreasing the numbers of digits. In this paper, we introduce some new methods for high speed of divisibility in RDA.

AMS subject classification: *13AXX, 13F15*

Keywords: *Congruence, Divisibility, Partition of numbers, Reduce Digits Algorithm.*

1- INTRODUCTION

In number theory, divisibility methods of whole numbers are very useful because they help us to quickly determine if a number can be divided by n. There are several different methods for divisibility of numbers with many variants, and some of them can be found in [4, 5, 6, 7].For example in [6] presented that numbers which are dividable to 11 should have the (sum of the odd numbered digits) - (sum of the even numbered digits) is divisible by 11. Similarly some studies are presented for special numbers such as 15, 17, 19, etc ([1, 2, 3]). In this paper, we suppose that z = $a_n a_{n-1} \ldots a_1$ and w = $b_m b_{m-1} \ldots b_1$ are dividend and odd divisor respectively. Also, we show z = $a_n a_{n-1} \ldots a_1$ and w = $b_m b_{m-1} \ldots b_1$ are dividend and prime divisor respectively, then:

1) If $w|(b_m b_{m-1} \ldots b_2)a_1 - (a_n a_{n-1} \ldots a_2)$ and $b_1 = 1$ then $w|z$.
2) If $w|(w - (7w-1)/10)a_1 + (a_n a_{n-1} \ldots a_2)$ and $b_1 = 3$ then $w|z$.
3) $w|z$ if $w|\ w - (3w-1)/10)\ a_1 + (a_n a_{n-1} \ldots a_2)$and $b_1 = 7$ then $w|z$.
4) $w|z$ if $w|w\ - ((9w\ -1)/10)\ a_1 + (a_n a_{n-1} \ldots a_2)$ and $b_1 = 9$ then $w|z$.
5) If $w = 5$ is prime divisor then the proof of $w|z$ is clear.
6) If w=$b_m b_{m-1} \ldots b_1$ is composite divisor, then with using of fundamental theorem of arithmetic, the proof of $w|z$ is obvious.

Also, we show that in [3], if z = $a_n a_{n-1} \ldots a_1$ and w = $b_m b_{m-1} \ldots b_1$ are dividend and odd divisor respectively, then:

1) If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=1, b_2=0, then $w|z$ if $w||(b_m b_{m-1} \ldots b_3)a_2 a_1 - a_n a_{n-1} \ldots a_3|$.
2) If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=3, b_2=4, then $w|z$ if $w||((7w-1)/100)a_2 a_1 - (a_n a_{n-1} \ldots a_3)|$.
3) If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=7, b_2=6, then $w|z$ if $w||\ ((3w-1)/100)\ a_2 a_1$-$(a_n a_{n-1} \ldots a_3)\ |$.
4) If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=9, b_2=8, then $w|z$ if $w||\ ((9w\text{-}1)/100)a_2 a_1$-$(a_n a_{n-1} \ldots a_3)|$.

Now, in this paper we study divisibility of numbers with application of RDA.

2- APPLICATION OF REDUCE DIGITS ALGORITHM IN DIVISIBILITY OF NUMBERS AND RESULTS

Theorem 2.1. If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=1, b_2=0 , b_3=0, then $w|z$ if $w||(b_m b_{m-1} \ldots b_4)a_3 a_2 a_1 - a_n a_{n-1} \ldots a_4|$.

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Proof. If $w||(b_m b_{m-1} \ldots b_4)a_3a_2a_1 - a_n a_{n-1} \ldots a_4|$, then there exists an integer k such that $kw= (b_m b_{m-1} \ldots b_4)a_3a_2a_1 - a_n a_{n-1} \ldots a_4$. Therefore,$1000kw= (b_m b_{m-1} \ldots b_4) * 1000a_3a_2a_1 - (a_n a_{n-1} \ldots a_4) * 1000$,hence we have $(1000kw\text{-}a_3a_2a_1)w = -z$, so w|z.

Remark 2.2. In this paper, with using theorems for dividend and odd divisor, we can introduce the new numbers (as same as 0 in follow Example). If we don't know whether a new number is divisible by odd divisor, we should apply the theorems again. In this case, the new numbers are dividend.

Example 2.3. Is 8265015 divisible by 551001?

Solution: With using above theorem | (551*15)-8265 | = 0. Therefore, 8265015 is divisible by 551001.

Example 2.4. Is 8542396580235184251305 divisible by 5872311825742001?

Solution: With using above theorem | (5872311825742*305)- 8542396580235184251 | = 854060552512833294. But the divisibility 854060552512833294 by 5872311825742001is not clear. Therefore, with using above theorem for 854060552512833294 to 5872311825742001, we have | (5872311825742*294) - 854060552512833 |=3014760097105110. Therefore, 8542396580235184251305 is not divisible by 5872311825742001. (3014760097105110<5872311825742001).

Theorem 2.5. If $z = a_n a_{n-1} \ldots a_1$ is dividend and $w = b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=3, b_2=4, b_3=1, then w|z if $w||((7w-1)/1000)a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)|$.

Proof. If $w||((7w-1)/1000)a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)|$, then there exists an integer k such that $kw=((7w-1)/1000)a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)$. Therefore, $(1000k\text{-}7a_3a_2a_1)w = -z$, so w|z.

Example 2.6. Is 28522195 divisible by 78143?

Solution: With using above theorem | (547*195)-28522 | = 78143. Therefore, 28522195 is divisible by 78143.

Example 2.7. Is 16496634249311499 divisible by 2181375143?

Solution: With using above theorem | (15269626*499)- 16496634249311 | = 16489014705937. But the divisibility 16489014705937 by 2181375143 is not clear. Therefore, with using above theorem for 16489014705937 to 2181375143, we have | (15269626*937) - 16489014705 |=2181375143. Therefore, 16496634249311499 is divisible by 2181375143.

Theorem 2.8. If $z = a_n a_{n-1} \ldots a_1$ is dividend and $w = b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=7, b_2=6, b_3=6, then w|z if $w||((3w-1)/1000)\, a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)|$.

Proof. If $w||((3w-1)/1000)\, a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)|$,then there exists an integer k such that

$kw= ((3w-1)/1000)\, a_3a_2a_1 - (a_n a_{n-1} \ldots a_4)$. Therefore, $(1000k\text{-}3a_3a_2a_1)w = -z$, so w|z.

Example 2.9. Is 156481996 divisible by 15667?

Solution: With using above theorem | (47*996)-156481 | = 109669. But the divisibility 109669 by 15667 is not clear. Therefore, with using above theorem for 109669 to 15667, we have | (47*669)-109 |=31334. Therefore, with using above theorem for 31334 to 15667, we have | (47*334)-31 |=15667. Therefore, 156481996 is divisible by 15667.

Example 2.10. Is 635238182481180927181235820 divisible by 325667?

Solution: With using above theorem | (977*820)- 635238182481180927181235 | = 635238182481180926380095. But the divisibility 635238182481180926380095 by 325667 is not clear. Therefore, with using above theorem for 635238182481180926380095 to 325667,we have | (977*095)- 635238182481180926380 |=635238182481180833565 But the divisibility 635238182481180833565 by 325667 is not clear . Therefore, with using above theorem for

635238182481180833565 to 325667, we have | (977*565) - 635238182481180833 |=635238182480628828. But the divisibility 635238182480628828 by 325667 is not clear. Therefore, with using above theorem for 635238182480628828 to 325667, we have | (977*828)- 635238182480628 |=635238181671672. But the divisibility 635238181671672 by 325667 is not clear. Therefore, with using above theorem for 635238181671672 to 325667, we have | (977*672) - 635238181671|=635237525127. But the divisibility 635237525127 by 325667 is not clear.

Therefore, with using above theorem for 635237525127 to 325667, we have | (977*127)- 635237525|=635113464. But the divisibility 635113464 by 325667 is not clear. Therefore, with using above theorem for 635113464 to 325667, we have | (977*464)- 635113|=199371. Therefore, 635238182481180927181235820 is not divisible by 325667. (199371<325667).

Remark 2.11. $3*\underbrace{66\ldots6}_{n-th}-1 \overset{10^{n+1}}{\equiv} 0$.[6]

Corollary 2.12. If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=7, $b_2 = b_3$=…=b_m=6, then w|z if w|| $((3w-1)/10^m)\, a_m \ldots a_2 a_1 - (a_n a_{n-1} \ldots a_{m+1})$ |.

Theorem 2.13. If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=9, b_2=8, b_3=8, then w|z if w|| ((9w-1)/1000) $a_3 a_2 a_1 - (a_n a_{n-1} \ldots a_4)$| .

Proof. If w|| ((9w-1)/1000) $a_3 a_2 a_1 - (a_n a_{n-1} \ldots a_4)|$, then there exists an integer k such that

kw= ((9w-1)/1000) $a_3 a_2 a_1 - (a_n a_{n-1} \ldots a_4)$. Therefore, (1000k-9$a_3 a_2 a_1$)$w = -z$, so w|z.

Example 2.14. Is 90292806 divisible by 2889?

Solution: With using above theorem | (26*806)-90292 | = 69336. But the divisibility 69336 by 2889 is not clear. Therefore, with using above theorem for 69633 to 2889, we have | (26*336)-69 |=8667=3*2889. Therefore, 90292806 is divisible by 2889.

Example 2.15. Is 321480154879362731940753142 divisible by 841889?

Solution: With using above theorem | (7577*142)-321480154879362731940753|= 321480154879362730864819. But the divisibility 321480154879362730864819 by 841889 is not clear. Therefore, with using above theorem for 321480154879362730864819 to 841889, we have |(7577*819)-321480154879362730864| =321480154879356525301. But the divisibility 321480154879356525301 by 841889 is not clear. Therefore, with using above theorem for 321480154879356525301 to 841889, we have | (7577*301)- 321480154879356525 |=321480154877075848. But the divisibility 321480154877075848 by 841889 is not clear. Therefore, with using above theorem for 321480154877075848 to 841889, we have | (7577*848) - 321480154877075 |=321480148451779. But the divisibility 321480148451779 by 841889 is not clear. Therefore, with using above theorem for 321480148451779 to 841889, we have | (7577*779) - 321480148451|=321474145968. But the divisibility 321474145968 by 841889 is not clear. Therefore, with using above theorem for 321474145968 to 841889, we have | (7577*968) - 321474145|=314139709. But the divisibility 314139709 by 841889 is not clear. Therefore, with using above theorem for 314139709 to 841889, we have | (7577*709) - 314139|=5057954. But the divisibility 5057954. by 841889 is not clear. Therefore, with using above theorem for 5057954 to 841889, we have | (7577*954) - 5057|=7223401. But the divisibility 7223401 by 841889 is not clear. Therefore, with using above theorem for 7223401 to 841889, we have | (7577*401) - 7223|=3031154. But the divisibility 3031154 by 841889 is not clear. Therefore, with using above theorem for 3031154 to 841889, we have | (7577*154) - 3031|=1163827. Therefore, 321480154879362731940753142 is not divisible by 841889.

Remark 2.16. $9*\underbrace{88\ldots8}_{n-th}9-1 \overset{10^{n+1}}{\equiv} 0$. [6]

Corollary2.17. If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=9,$b_2 = b_3$=…=b_m=8 , then w|z if w|| $((9w-1)/10^m)\, a_m \ldots a_2 a_1 - (a_n a_{n-1} \ldots a_{m+1})$ |.

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