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A NEW GENERALISATION OF SAM-SOLAI'S MULTIVARIATE ADDITIVE NAGAKAMI-M DISTRIBUTION

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ABSTRACT

This paper proposed a new generalization of bounded Continuous multivariate symmetric probability distributions. In this paper, we visualize a new generalization of Sam-Solai's Multivariate additive Nagakami-m distribution from the univariate two parameter Nagakami-m distributions. Further, we find its Marginal, Multivariate Conditional distributions, Multivariate Generating functions, Multivariate survival, hazard functions and also discussed its special cases. The special cases includes the transformation of Sam-Solai's Multivariate additive Nagakami-m distribution into Multivariate additive half normal distribution, Multivariate additive chi-distribution, Multivariate additive Inverse Nagakami-m distribution, Multivariate additive log-Nagakami-m distribution, Multivariate additive Extreme value Nagakami-m distribution, Multivariate additive Gamma distribution, Multivariate additive Chi-square distribution and Multivariate additive Erlang-k distribution. Moreover, it is found that the bivariate correlation between two Nagakami random variables purely depends on the shape parameter and we simulated and established selected standard bivariate Nagakami correlation bounds from 2500 different combinations of values for shape parameter.

Keywords: *Multivariate additive half normal distribution, Multivariate additive chi-distribution, Multivariate additive Inverse Nagakami-m distribution, Multivariate additive log-Nagakami-m distribution, Multivariate additive Extreme value Nagakami-m distribution, Multivariate additive Gamma distribution, Multivariate additive Chi-square distribution and Multivariate additive Erlang-k distribution, correlation bounds.*

****Mathematics Subject Classification:*** *Primary 62H10; Secondary 62E15.*

INTRODUCTION

Cheriyan (1941) introduced a bi-variate Gamma type distribution function with assumption of the Gamma random variables are correlated and similarly Ramabhadran [1951] proposed a multivariate Gamma type distribution in the exponential family of functions. Moreover Krishnamoorthy et al. (1951) continued the work of cheriyan, Ramabhadran and proposed a similar type of multivariate Gamma distribution. On the other hand, Sarmanov (1968) proposed a generalized Gamma distribution with the assumption of symmetricity among random variables and Gaver (1970) established the mixture of multivariate Gamma distribution. Johnson et al (1972, 2000) highlighted the Multivariate system of Gamma distribution and Dussauchoy et al (1975) introduced a Multivariate Gamma distribution whose marginal are also followed a univariate Gamma laws. Becker et al(1981) studied the extension of gamma distribution for the bivariate case and similarly D'Este(1981) also described the Morgenstern type Generalization of bivariate Gamma distribution.Kowalczyk et al(1989) conducted a in-depth study about the properties of Multivariate Gamma distribution namely their shape, estimation of parameters and Mathai(1991,1992) studied a different form of multivariate Gamma distribution. Based on the past and present literatures, the authors proposed a new generalization of bounded Continuous multivariate symmetric probability distributions with special reference to the Gamma law and it is discussed in the next section. Thus the logical generalization of univariate probability distribution for a Multivariate case is an interesting task on the part of statisticians. The generalization of univariate two parameter Nagakami-m distribution to its Multivariate case based on the additive type distribution is discussed.

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SECTION 1: SAM-SOLAI'S MULTIVARIATE ADDITIVE NAGAKAMI-M DISTRIBUTION

Definition 1.1: Let $X_1, X_2, X_3 ..., X_p$ be the random variables followed Continuous univariate Nagakami-M distribution with shape parameter m_i and spread parameter Ω_i for all i (i=1 to p).Then the density function of the Multivariate Sam-Solai's additive Nagakami –M distribution is defined as

$$f(x_1, x_2, x_3 \dots, x_p) = 2^p \prod_{i=1}^{p} (m_i x_i / \Omega_i)(\sum_{i=1}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)) e^{-\sum_{i=1}^{p} \frac{m_i}{\Omega_i} x_i^2} \quad (1)$$

where $0 \le x_i < \infty$ $\Omega_i > 0$ $m_i \ge 0.5$

Theorem 1.2: The cumulative distribution function of the Sam-Solai's Multivariate additive Nagakami- distribution is defined by

$$F(x_1, x_2, x_3 \dots, x_p) = \int_0^{x_1}\int_0^{x_2}\int_0^{x_3} \dots \int_0^{x_p} 2^p \prod_{i=1}^{p} (m_i u_i / \Omega_i)(\sum_{i=1}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} u_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)) e^{-\sum_{i=1}^{p} \frac{m_i}{\Omega_i} u_i^2} du_1 du_2 du_3 \dots du_p \quad (2)$$

where $0 \le u_i < x_i$ $\Omega_i > 0$ $m_i \ge 0.5$

$$F(x_1, x_2, x_3 \dots, x_p) = \prod_{i=1}^{p} (1 - e^{-(m_i/\Omega_i) x_i^2})(\sum_{i=1}^{p} \frac{1}{1 - e^{-(m_i/\Omega_i) x_i^2}} \int_0^{x_i} (\frac{2(m_i / \Omega_i)^{m_i} u_i^{2m_i -1} e^{-\frac{m_i}{\Omega_i} u_i^2}}{\Gamma m_i} du_i) - (p-1))$$

$$F(x_1, x_2, x_3 \dots, x_p) = \prod_{i=1}^{p} (1 - e^{-(m_i/\Omega_i) x_i^2})(\sum_{i=1}^{p} (\frac{\phi_i(x_i, m_i, \Omega_i)}{1 - e^{-(m_i/\Omega_i) x_i^2}}) - (p-1))$$

where $\phi_i(x_i, m_i, \Omega_i) = \int_0^{x_i} \frac{2(m_i / \Omega_i)^{m_i} u_i^{2m_i -1} e^{-\frac{m_i}{\Omega_i} u_i^2}}{\Gamma m_i} du_i$ is the lower incomplete Nagakami integral of i^trandom variable.

Theorem 1.3: The Probability density function of Sam-Solai's Multivariate additive Conditional Nagakami-M distribution of X_1 on $X_2, X_3, \dots X_P$ is

$$f(x_1 / x_2, x_3 \dots, x_p) = \frac{2(m_1 x_1 / \Omega_1)(\sum_{i=1}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)) e^{-\frac{m_1}{\Omega_1} x_1^2}}{(\sum_{i=2}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))} \quad (3)$$

where $0 \le x_1 < \infty$ $\Omega_1 > 0$ $m_1 \ge 0.5$

Proof: It is obtained from

$$f(x_1 / x_2, x_3 \dots, x_p) = \frac{f(x_1, x_2, x_3 \dots, x_p)}{f(x_2, x_3 \dots, x_p)}$$

Theorem 1.4: Mean and Variance of Sam - Solai's Multivariate additive Conditional Nagakami-M distribution are

$$E(x_1 / x_2, x_3 \dots, x_p) = \frac{\frac{1}{(m_1 / \Omega_1)^{1/2}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\sqrt{\pi}}{2} (\sum_{i=2}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)))}{(\sum_{i=2}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))} \quad (4)$$

$$V(x_1 / x_2, x_3 \dots, x_p) = E(x_1^2 / x_2, x_3 \dots, x_p) - (E(x_1 / x_2, x_3 \dots, x_p))^2 \quad (5)$$

where $E(x_1^2 / x_2, x_3 \dots, x_p) = \frac{\frac{1}{(m_1 / \Omega_1)} (\frac{\Gamma(m_1 + 1)}{\Gamma m_1} + (\sum_{i=2}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)))}{(\sum_{i=2}^{p} (\frac{(m_i / \Omega_i)^{m_i -1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))}$

Proof: The n^{th} order moment of the distribution is

$$E(x_1^n / x_2, x_3 \ldots, x_p) = \int_0^\infty x_1^n f(x_1 / x_2, x_3 \ldots, x_p) dx_1$$

$$E(x_1^n / x_2, x_3 \ldots, x_p) = \int_0^\infty x_1^n \frac{2(m_1 x_1 / \Omega_1)(\sum_{i=1}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)) e^{-\frac{m_1}{\Omega_1}x_1^2}}{(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))} dx_1$$

$$E(x_1^n / x_2, x_3 \ldots, x_p) = \frac{\frac{1}{(m_1/\Omega_1)^{n/2}}(\frac{\Gamma(m_1 + (n/2))}{\Gamma m_1} + \Gamma(\frac{n+2}{2})(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)))}{(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))}$$

If **n=1**, then the Conditional expectation is

$$E(x_1 / x_2, x_3 \ldots, x_p) = \frac{\frac{1}{(m_1/\Omega_1)^{1/2}}(\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\sqrt{\pi}}{2}(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)))}{(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))}$$

If **n=2**, then the second order moment is

$$E(x_1^2 / x_2, x_3 \ldots, x_p) = \frac{\frac{1}{(m_1/\Omega_1)}(\frac{\Gamma(m_1 + 1)}{\Gamma m_1} + (\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)))}{(\sum_{i=2}^{p}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-2))}$$

The conditional variance of the distribution is obtained by Substituting the first and second moments in (5).

Theorem 1.5: If there are **p = (q + k)** random variables, such that **q** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depends on the **k** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$, then the density function of Sam-Solai's multivariate additive conditional Nagakami-M distribution is

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{2^q \prod_{i=1}^{q}(m_i x_i / \Omega_i)(\sum_{i=1}^{q+k}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (q+k-1)) e^{-\sum_{i=1}^{q}\frac{m_i}{\Omega_i}x_i^2}}{(\sum_{i=q+1}^{q+k}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (k-1))} \quad (6)$$

where $0 \le x_i < \infty \quad \Omega_i > 0 \quad m_i \ge 0.5$

Proof: Let the multivariate conditional law for **q** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depending on the **k** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$ is given as

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{f(x_1, x_2, x_3, \ldots x_q, x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}{f(x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{2^{q+k} \prod_{i=1}^{q+k}(m_i x_i / \Omega_i)(\sum_{i=1}^{q+k}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (q+k-1)) e^{-\sum_{i=1}^{q+k}\frac{m_i}{\Omega_i}x_i^2}}{\int_0^\infty \int_0^\infty \ldots \int_0^\infty 2^{q+k} \prod_{i=1}^{q+k}(m_i x_i / \Omega_i)(\sum_{i=1}^{q+k}(\frac{(m_i / \Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (q+k-1)) e^{-\sum_{i=1}^{q+k}\frac{m_i}{\Omega_i}x_i^2} \prod_{i=1}^{q} dx_i}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{2^q \prod_{i=1}^{q} (m_i x_i / \Omega_i)(\sum_{i=1}^{q+k} (\frac{(m_i / \Omega_i)^{m_i - 1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (q+k-1)) e^{-\sum_{i=1}^{q} \frac{m_i}{\Omega_i} x_i^2}}{(\sum_{i=q+1}^{q+k} (\frac{(m_i / \Omega_i)^{m_i - 1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (k-1))}$$

where $0 \le x_i < \infty$ $\Omega_i > 0$ $m_i \ge 0.5$

Section 2: Constants of Sam-Solai's multivariate additive Nagakami-M distribution

Theorem 2.1: The marginal product moments, Co-variance and Correlation Co-efficient between the Nagakami variables x_1 and x_2 are given as

$$E(x_1 x_2) = \frac{\sqrt{\pi}}{2\sqrt{(m_1/\Omega_1)}\sqrt{(m_2/\Omega_2)}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\Gamma(m_2 + (1/2))}{\Gamma m_2} - \frac{\sqrt{\pi}}{2}) \tag{7}$$

$$COV(x_1, x_2) = \frac{\sqrt{\pi}}{2\sqrt{(m_1/\Omega_1)}\sqrt{(m_2/\Omega_2)}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\Gamma(m_2 + (1/2))}{\Gamma m_2} - \frac{\sqrt{\pi}}{2} - \frac{2\Gamma(m_1 + (1/2))\Gamma(m_2 + (1/2)}{\sqrt{\pi}\Gamma m_1 \Gamma m_2}) \tag{8}$$

$$\rho(x_1, x_2) = \frac{\sqrt{\pi}}{2\sqrt{m_1(1 - \frac{1}{m_1}(\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1})^2)}\sqrt{m_2(1 - \frac{1}{m_2}(\frac{\Gamma(m_2 + (1/2))}{\Gamma m_2})^2)}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\Gamma(m_2 + (1/2))}{\Gamma m_2} - \frac{\sqrt{\pi}}{2} - \frac{2\Gamma(m_1 + (1/2))\Gamma(m_2 + (1/2)}{\sqrt{\pi}\Gamma m_1 \Gamma m_2}) \tag{9}$$

where $-1 \le \rho(x_1, x_2) \le +1$ for certain values of shape parameters(see result 3.4)

Proof: Assume that x_1 and x_2 are random variables from Sam-Solai's multivariate additive Nagakami-M distribution. Let the product moment of the distribution is

$$E(x_1 x_2) = \int_0^\infty \int_0^\infty \ldots \int_0^\infty x_1 x_2 f(x_1, x_2, x_3, \ldots x_p) \prod_{i=1}^{p} d x_i$$

Its Co-variance is $COV(x_1, x_2) = E(x_1 x_2) - E(x_1)E(x_2)$ (10)

Then

$$E(x_1 x_2) = \int_0^\infty \int_0^\infty \ldots \int_0^\infty x_1 x_2 2^p \prod_{i=1}^{p} (m_i x_i / \Omega_i)(\sum_{i=1}^{p} (\frac{(m_i / \Omega_i)^{m_i - 1} x_i^{2(m_i-1)}}{\Gamma m_i}) - (p-1)) e^{-\sum_{i=1}^{p} \frac{m_i}{\Omega_i} x_i^2} \prod_{i=1}^{p} d x_i$$

By evaluation, it follows that $E(x_1 x_2) = \frac{\sqrt{\pi}}{2\sqrt{(m_1/\Omega_1)}\sqrt{(m_2/\Omega_2)}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\Gamma(m_2 + (1/2))}{\Gamma m_2} - \frac{\sqrt{\pi}}{2})$ .The marginal expectation of Nagakami variables x_1 and x_2 are $E(x_1) = \frac{\Gamma(m_1 + (1/2))}{(m_1/\Omega_1)^{1/2} \Gamma m_1}$ and $E(x_2) = \frac{\Gamma(m_2 + (1/2))}{(m_2/\Omega_2)^{1/2} \Gamma m_2}$ respectively.

The Marginal Product moment for $E(x_1 x_2)$ is obtained by substituting the above Marginal expectations for x_1 and x_2 in (10).

Thus

$$COV(x_1, x_2) = \frac{\sqrt{\pi}}{2\sqrt{(m_1/\Omega_1)}\sqrt{(m_2/\Omega_2)}} (\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1} + \frac{\Gamma(m_2 + (1/2))}{\Gamma m_2} - \frac{\sqrt{\pi}}{2} - \frac{2\Gamma(m_1 + (1/2))\Gamma(m_2 + (1/2)}{\sqrt{\pi}\Gamma m_1 \Gamma m_2}) \tag{11}$$

Correlation coefficient of a distribution is (12a)

$$\rho(x_1, x_2) = \frac{COV(x_1, x_2)}{\sigma_1 \sigma_2}$$

It observes that $\sigma_{x_1} = \sqrt{\Omega_1 (1 - \frac{1}{m_1}(\frac{\Gamma(m_1 + (1/2))}{\Gamma m_1})^2)}$ and $\sigma_{x_2} = \sqrt{\Omega_2 (1 - \frac{1}{m_2}(\frac{\Gamma(m_2 + (1/2))}{\Gamma m_2})^2)}$ (12b)

From (11), (12a) and (12b), it follows that

$$\rho(x_1,x_2)=\frac{\sqrt{\pi}}{2\sqrt{m_1(1-\frac{1}{m_1}(\frac{\Gamma(m_1+(1/2))}{\Gamma m_1})^2)}\sqrt{m_2(1-\frac{1}{m_2}(\frac{\Gamma(m_2+(1/2))}{\Gamma m_2})^2)}}(\frac{\Gamma(m_1+(1/2))}{\Gamma m_1}+\frac{\Gamma(m_2+(1/2))}{\Gamma m_2}-\frac{\sqrt{\pi}}{2}-\frac{2\Gamma(m_1+(1/2))\Gamma(m_2+(1/2)}{\sqrt{\pi}\Gamma m_1\Gamma m_2}) \quad (13)$$

where $-1\le\rho(x_1,x_2)\le+1$

Remark 2.1: The Product moments, Co-variance and population Correlation Co-efficient between the i^{th} and j^{th} of Nagakami variables are given as

$$E(x_ix_j)=\frac{\sqrt{\pi}}{2\sqrt{(m_i/\Omega_i)}\sqrt{(m_j/\Omega_j)}}(\frac{\Gamma(m_i+(1/2))}{\Gamma m_i}+\frac{\Gamma(m_j+(1/2))}{\Gamma m_j}-\frac{\sqrt{\pi}}{2}) \quad (14)$$

$$COV(x_i,x_j)=\frac{\sqrt{\pi}}{2\sqrt{(m_i/\Omega_i)}\sqrt{(m_j/\Omega_j)}}(\frac{\Gamma(m_i+(1/2))}{\Gamma m_i}+\frac{\Gamma(m_j+(1/2))}{\Gamma m_j}-\frac{\sqrt{\pi}}{2}-\frac{2\Gamma(m_i+(1/2))\Gamma(m_j+(1/2)}{\sqrt{\pi}\Gamma m_i\Gamma m_j}) \quad (15)$$

$$\rho(x_i,x_j)=\frac{\sqrt{\pi}}{2\sqrt{m_i(1-\frac{1}{m_i}(\frac{\Gamma(m_i+(1/2))}{\Gamma m_i})^2)}\sqrt{m_j(1-\frac{1}{m_j}(\frac{\Gamma(m_j+(1/2))}{\Gamma m_j})^2)}}(\frac{\Gamma(m_i+(1/2))}{\Gamma m_i}+\frac{\Gamma(m_j+(1/2))}{\Gamma m_j}-\frac{\sqrt{\pi}}{2}-\frac{2\Gamma(m_i+(1/2))\Gamma(m_j+(1/2)}{\sqrt{\pi}\Gamma m_i\Gamma m_j}) \quad (16)$$

where $i\neq j \quad -1\le\rho(x_1,x_2)\le+1$

Theorem 2.2: The Moment generating function of Sam-Solai's Multivariate additive Nagakami-M distribution is

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_{3,\dots}t_p)=\prod_{i=1}^{p}\phi_1(t_i)(\sum_{i=1}^{p}(\frac{(\pi^{3/2}/\Gamma(m_i))(\phi_2(t_i)+\phi_3(t_i))}{\phi_1(t_i)})-(p-1)) \quad (17)$$

where $\phi_1(t_i)=1+((t_ie^{t_i^2/(4m_i/\Omega_i)}\sqrt{\pi/(m_i/\Omega)})/2)(1+erf(t_i/2\sqrt{m_i/\Omega_i}))$

$$\phi_2(t_i)=\frac{2(t_i^2/(4m_i/\Omega_i)+1)L(-m_i,1/2,(t_i^2/(4m_i/\Omega_i))-(t_i^2/(4m_i/\Omega_i))L(-m_i,3/2,t_i^2/(4m_i/\Omega_i))}{2\sin(m_i\pi)\Gamma((3/2)-m_i)}$$

$$\phi_3(t_i)=(\frac{t_i/(2\sqrt{m_i/\Omega_i})(2(t_i^2/(4m_i/\Omega_i)+2m_i-1))((1/2)-m_i)L((1/2)-m_i,1/2,t_i^2/(4m_i/\Omega_i))-2(t_i/2\sqrt{m_i/\Omega_i})^3)((1/2)-m_i)L((1/2)-m_i,3/2,t_i^2/(4m_i/\Omega_i))}{(2m_i-1)\cos(m_i\pi)\Gamma(2-m_i)}$$

$L(-m_i,1/2,(t_i^2/(4m_i/\Omega_i))$, $L(-m_i,3/2,t_i^2/(4m_i/\Omega_i))$, $L((1/2)-m_i,1/2,t_i^2/(4m_i/\Omega_i))$, $L((1/2)-m_i,3/2,t_i^2/(4m_i/\Omega_i))$ are the Laguerre-L-functions and $erf(t_i/2\sqrt{m_i/\Omega_i})$ is the error function.

Proof: Let the moment generating function of a Multivariate distribution is given as

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_{3,\dots}t_p)=\int_0^\infty\int_0^\infty\dots\int_0^\infty e^{\sum_{i=1}^{p}t_ix_i}f(x_1,x_2,x_3,\dots x_p)\prod_{i=1}^{p}dx_i$$

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_{3,\dots}t_p)=\int_0^\infty\int_0^\infty\dots\int_0^\infty e^{\sum_{i=1}^{p}t_ix_i}2^p\prod_{i=1}^{p}(m_ix_i/\Omega_i)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}x_i^{2(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}x_i^2}\prod_{i=1}^{p}dx_i$$

By integrating the above equation

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_{3,\dots}t_p)=\prod_{i=1}^{p}\phi_1(t_i)(\sum_{i=1}^{p}(\frac{(\pi^{3/2}/\Gamma(m_i))(\phi_2(t_i)+\phi_3(t_i))}{\phi_1(t_i)})-(p-1))$$

Theorem 2.3: The Cumulant of the Moment generating function of the Sam-Solai's Multivariate additive Nagakami-M distribution is

$$C_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\sum_{i=1}^{p}\log(\phi_1(t_i))+\log(\sum_{i=1}^{p}(\frac{(\pi^{3/2}/\Gamma(m_i))(\phi_2(t_i)+\phi_3(t_i))}{\phi_1(t_i)})-(p-1)) \quad (18)$$

Proof: It is found from

$C_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\log(M_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p))$.

Theorem 2.4: The Characteristic function of the Sam-Solai's Multivariate additive Nagakami-M distribution is

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\prod_{i=1}^{p}\omega_1(t_i)(\sum_{i=1}^{p}(\frac{(\pi^{3/2}/\Gamma(m_i))(\omega_2(t_i)+\omega_3(t_i))}{\omega_1(t_i)})-(p-1)) \quad (19)$$

Where $\omega_1(t_i)=1+((it_i e^{-t_i^2/(4m_i/\Omega_i)}\sqrt{\pi/(m_i/\Omega)})/2)(1+erf(it_i/2\sqrt{m_i/\Omega_i}))$

$$\omega_2(t_i)=\frac{2(1-t_i^2/(4m_i/\Omega_i))L(-m_i,1/2,(-t_i^2/(4m_i/\Omega_i))+(t_i^2/(4m_i/\Omega_i))L(-m_i,3/2,-t_i^2/(4m_i/\Omega_i))}{2\sin(m_i\pi)\Gamma((3/2)-m_i)}$$

$$\omega_3(t_i)=(\frac{it_i/(2\sqrt{m_i/\Omega_i})(2(-t_i^2/(4m_i/\Omega_i)+2m_i-1))((1/2)-m_i)L((1/2)-m_i,1/2,-t_i^2/(4m_i/\Omega_i))-2(it_i/2\sqrt{m_i/\Omega_i})^3)((1/2)-m_i)L((1/2)-m_i,3/2,-t_i^2/(4m_i/\Omega_i))}{(2m_i-1)\cos(m_i\pi)\Gamma(2-m_i)}$$

$L(-m_i,1/2,(-t_i^2/(4m_i/\Omega_i))$, $L(-m_i,3/2,-t_i^2/(4m_i/\Omega_i))$, $L((1/2)-m_i,1/2,-t_i^2/(4m_i/\Omega_i))$ $L((1/2)-m_i,3/2,-t_i^2/(4m_i/\Omega_i))$ are the Laguerre-L-functions and $erf(it_i/2\sqrt{m_i/\Omega_i})$ is the complex error function.

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{j=1}^{p}t_jx_j} f(x_1,x_2,x_3,\ldots x_p)\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\int_0^\infty\int_0^\infty\ldots\int_0^\infty e^{i\sum_{i=1}^{p}t_ix_i}2^p\prod_{i=1}^{p}(m_ix_i/\Omega_i)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}x_i^{2(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}x_i^2}\prod_{i=1}^{p}dx_i$$

$$\phi_{x_1,x_2,x_3,\ldots x_p}(t_1,t_2,t_{3,}\ldots t_p)=\prod_{i=1}^{p}\omega_1(t_i)(\sum_{i=1}^{p}(\frac{(\pi^{3/2}/\Gamma(m_i))(\omega_2(t_i)+\omega_3(t_i))}{\omega_1(t_i)})-(p-1))$$ By integrating the above equation.

Theorem 2.5: The survival function of the Sam-Solai's Multivariate additive Nagakami-M distribution is

$$S(x_1,x_2,x_3\ldots,x_p)=1-\prod_{i=1}^{p}(1-e^{-(m_i/\Omega_i)x_i^2})(\sum_{i=1}^{p}(\frac{\phi_i(x_i,m_i,\Omega_i)}{1-e^{-(m_i/\Omega_i)x_i^2}})-(p-1)) \quad (20)$$

Proof: Let the survival function of a multivariate distribution is given as

$$S(x_1,x_2,x_3\ldots,x_p)=1-F(x_1,x_2,x_3\ldots,x_p)$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-\int_0^{x_1}\int_0^{x_2}\int_0^{x_3}\ldots\int_0^{x_p}2^p\prod_{i=1}^{p}(m_iu_i/\Omega_i)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}u_i^{2(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}u_i^2}du_1du_2du_3\ldots du_p$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-\prod_{i=1}^{p}(1-e^{-(m_i/\Omega_i)x_i^2})(\sum_{i=1}^{p}\frac{1}{1-e^{-(m_i/\Omega_i)x_i^2}}\int_0^{x_i}(\frac{2(m_i/\Omega_i)^{m_i}u_i^{2m_i-1}e^{-\frac{m_i}{\Omega_i}u_i^2}}{\Gamma m_i}du_i)-(p-1))$$

$$S(x_1,x_2,x_3\ldots,x_p)=1-\prod_{i=1}^{p}(1-e^{-(m_i/\Omega_i)x_i^2})(\sum_{i=1}^{p}(\frac{\phi_i(x_i,m_i,\Omega_i)}{1-e^{-(m_i/\Omega_i)x_i^2}})-(p-1))$$

where $\phi_i(x_i, m_i, \Omega_i) = \int_0^{x_i} \frac{2(m_i/\Omega_i)^{m_i} u_i^{2m_i-1} e^{-\frac{m_i}{\Omega_i}u_i^2}}{\Gamma m_i} du_i$ is the lower incomplete Nagakami integral of i[th] random variable.

Theorem 2.6: The hazard function of the Sam-Solai's Multivariate additive Nagakami-M distribution is

$$h(x_1, x_2, x_3 \ldots, x_p) = \frac{2^p \prod_{i=1}^{p}(m_i x_i/\Omega_i)\left(\sum_{i=1}^{p}\left(\frac{(m_i/\Omega_i)^{m_i-1} x_i^{2(m_i-1)}}{\Gamma m_i}\right) - (p-1)\right) e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}x_i^2}}{1 - \prod_{i=1}^{p}(1 - e^{-(m_i/\Omega_i)x_i^2})\left(\sum_{i=1}^{p}\left(\frac{\phi_i(x_i, m_i, \Omega_i)}{1 - e^{-(m_i/\Omega_i)x_i^2}}\right) - (p-1)\right)} \quad (21)$$

Proof: It is obtained from

$$h(x_1, x_2, x_3 \ldots, x_p) = \frac{f(x_1, x_2, x_3 \ldots, x_p)}{S(x_1, x_2, x_3 \ldots, x_p)} \text{ and}$$

$$S(x_1, x_2, x_3 \ldots, x_p) = 1 - F(x_1, x_2, x_3 \ldots, x_p)$$

Theorem 2.7: The Cumulative hazard function of the Sam-Solai's Multivariate additive Nagakami-M distribution is

$$H(x_1, x_2, x_3 \ldots, x_p) = -\log(1 - \prod_{i=1}^{p}(1 - e^{-(m_i/\Omega_i)x_i^2})(\sum_{i=1}^{p}(\frac{\phi_i(x_i, m_i, \Omega_i)}{1 - e^{-(m_i/\Omega_i)x_i^2}}) - (p-1))) \quad (22)$$

Proof: Let the Cumulative hazard function of a multivariate distribution is given as

$$H(x_1, x_2, x_3 \ldots, x_p) = -\log(1 - F(x_1, x_2, x_3 \ldots, x_p))$$

$$H(x_1, x_2, x_3 \ldots, x_p) = -\log(S(x_1, x_2, x_3 \ldots, x_p))$$

$$H(x_1, x_2, x_3 \ldots, x_p) = -\log(1 - \prod_{i=1}^{p}(1 - e^{-(m_i/\Omega_i)x_i^2})(\sum_{i=1}^{p}(\frac{\phi_i(x_i, m_i, \Omega_i)}{1 - e^{-(m_i/\Omega_i)x_i^2}}) - (p-1)))$$

SECTION 3: SOME SPECIAL CASES

Result 3.1: The uni-variate marginal of the Sam-Solai's multivariate additive Nagakami-M distribution is the uni-variate two parameter Nagakami distributions.

Result 3.2: From (1) and if **P=1,** the Sam-Solai's multivariate additive Nagakami-M density is reduced to density of univariate two parameter Nagakami-M distribution.

Result 3.3: From (1) and if **P=2**, then the density of Sam-Solai's Multivariate Nagakami-M distribution was reduced into

$$f(x_1, x_2) = \left(\frac{(m_1/\Omega_1)^{m_1-1} x_i^{2(m_1-1)}}{\Gamma m_1} + \frac{(m_2/\Omega_2)^{m_2-1} x_i^{2(m_2-1)}}{\Gamma m_2} - 1\right) 4(m_1 x_1/\Omega_1)(m_2 x_2/\Omega_2) e^{-(\frac{m_1}{\Omega_1}x_1^2 + \frac{m_2}{\Omega_2}x_2^2)} \quad (23)$$

where $0 \le x_1, x_2 < \infty, \Omega_1, \Omega_2 > 0, m_1, m_2 \ge 0.5$

This is called the density of Sam-Solai's Bi-variate additive Nagakami-M distribution.

Result 3.4: The tables 1, table 2 and Bi-variate probability surface for (23) show the selected simulated standard Bi-variate correlations between two Nagakami variables which are bounded between -1 and +1 calculated from 2500 different combinations of shape parameters (m_1, m_2).

Table 1: Simulation runs for selected values of shape parameter with correlation bounds when $\rho(x_1,x_2)=0$

Runs	**477**	**1081**	**1327**	**80**	**2365**	**444**	**1597**	**54**	**2442**	**1832**	**1040**	**934**
m_1	5.1	3.8	1.3	5.3	4.4	3.5	5.3	5.1	5.2	3.1	4.5	4.8
m_2	1.2	3.8	0.8	2.4	0.7	1.5	2	4.9	3.6	0.9	2.9	1.1

Table 2: Simulation runs and combination of shape parameters with Bi-variate correlation bounds

Runs	**m_1**	**m_2**	$\rho(x_1,x_2)$
94	**0.5**	**4.3**	**-0.969**
839	2.3	4.9	-0.798
1312	0.6	0.9	-0.690
1268	1.0	4.4	-0.602
2024	**1.5**	**4.3**	**-0.508**
2341	4.0	5.3	-0.402
800	3.4	3.4	-0.307
461	5.0	3.5	-0.201
770	**2.7**	**1.9**	**-0.100**
1044	**3.5**	**2.2**	**+0.100**
74	4.7	3.1	+0.200
638	5.4	2.0	+0.300
1542	4.0	2.5	+0.408
2281	**2.1**	**2.2**	**+0.500**
2066	4.0	1.5	+0.602
414	4.1	2.4	+0.713
1356	4.7	3.3	+0.815
1192	0.7	4.9	+0.894
1358	**4.6**	**5.2**	**+0.981**

$m_1 = 0.5\ m_2 = 4.3\ \rho(x_1, x_2) = -0.969$

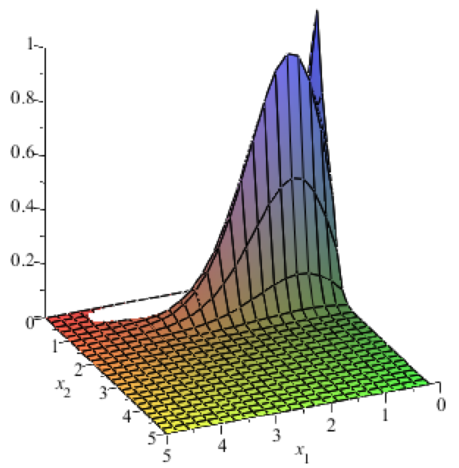

Fig.1

$m_1 = 1.5\ m_2 = 4.3\ \rho(x_1, x_2) = -0.508$

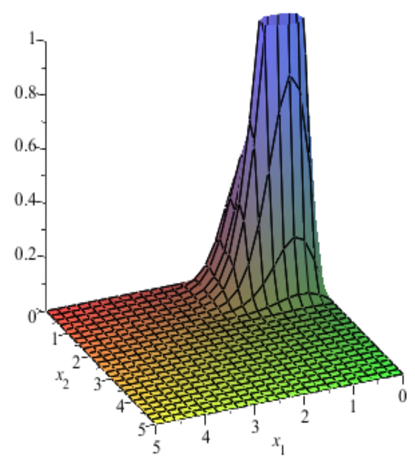

Fig.2

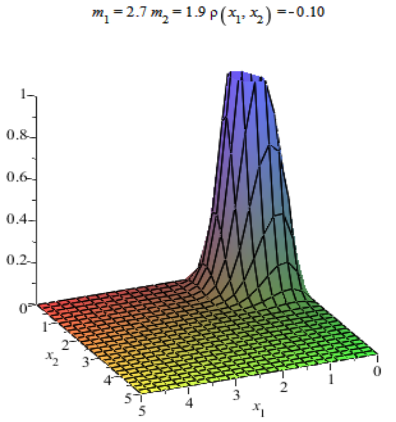

Fig.3

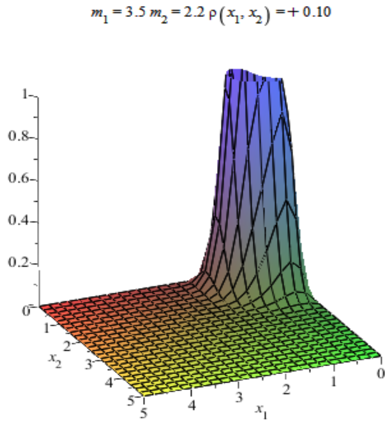

Fig.4

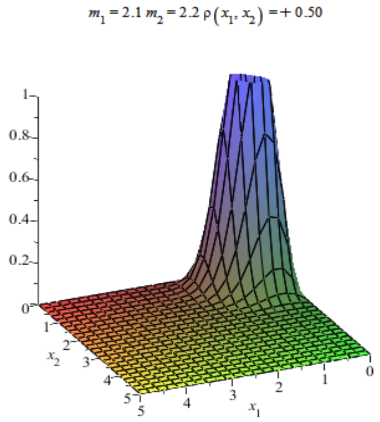

Fig.5

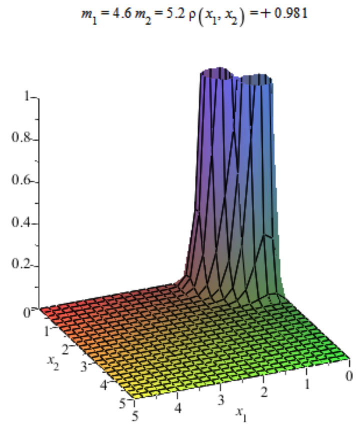

Fig.6

Result 3.5: From (1) and if $m_i = 1, \Omega_i = 2\lambda_i^2$, then the correlation between the i^{th} and j^{th} Nagakami variable is given as $\rho(x_i, x_j) = 0$ and the density of Sam-Solai's Multivariate additive Nagakami-M distribution is reduced as product of univariate one parameter Rayleigh distribution, and its density function is

$$f(x_1, x_2, x_3 \ldots, x_p) = (\prod_{i=1}^{p} x_i / \lambda_i^2) e^{-\frac{1}{2}\sum_{i=1}^{p}(\frac{x_i^2}{\lambda_i})}$$

$$f(x_1, x_2, x_3 \ldots, x_p) = \prod_{i=1}^{p} (x_i / \lambda_i^2 e^{-\frac{x_i^2}{2\lambda_i}}) \tag{24}$$

where $0 \le x_i < \infty \quad \lambda_i > 0$

Result 3.6: From (1) and if $m_i = 1$, $m_i / \Omega_i = \lambda_i$ then the correlation between the i^{th} and j^{th} Nagakami variable is given as $\rho(x_i, x_j) = 0$ and the density of Sam-Solai's Multivariate additive Nagakami-M distribution is reduced as product of univariate exponential distribution, and its density function is

$$f(x_1, x_2, x_3 \ldots, x_p) = (\prod_{i=1}^{p} \lambda_i) e^{-\sum_{i=1}^{p} \lambda_i x_i}$$

$$f(x_1, x_2, x_3 \ldots, x_p) = \prod_{i=1}^{p} \lambda_i e^{-\lambda_i x_i} \tag{25}$$

where $0 \le x_i < \infty \quad \lambda_i > 0$

Result 3.7: From (1) and if $m_i = m$ and $\Omega_i = \Omega$, then the Sam-solai's Multivariate additive Nagakami-M distribution is reduced into Sam-solai's Multivariate two parameter additive Nagakami-M distribution with parameters$(m, \Omega)$and its density function is given as

$$f(x_1,x_2,x_3\dots,x_p)=(2m/\Omega)^p(\prod_{i=1}^{p}x_i)(\frac{(m/\Omega)^{m-1}}{\Gamma m}\sum_{i=1}^{p}(x_i^{2(m-1)})-(p-1))e^{-\frac{m}{\Omega}\sum_{i=1}^{p}x_i^2} \quad (26)$$

where $0\le x_i<\infty \quad m\ge 0.5,\Omega>0$

Result 3.8: From (1) and if $m_i=1/2$,$\Omega_i=1$,then the Sam-solai's Multivariate additive Nagakami-M distribution is modified into Sam-solai's Multivariate additive Half normal distribution and its density function is given as

$$f(x_1,x_2,x_3\dots,x_p)=\prod_{i=1}^{p}x_i(\frac{1}{\sqrt{2\pi}}\sum_{i=1}^{p}(\frac{1}{x_i})-(p-1))e^{-\frac{1}{2}\sum_{i=1}^{p}x_i^2} \quad (27)$$

where $0\le x_i<\infty$

Result 3.9: From (1) and if $m_i=k_i, m_i/\Omega_i=1/2$, then the Sam-solai's Multivariate additive Nagakami-M distribution is modified into Sam-solai's Multivariate additive Chi-distribution and its density function is given as

$$f(x_1,x_2,x_3\dots,x_p)=\prod_{i=1}^{p}(x_i)(\sum_{i=1}^{p}(\frac{(1/2)^{k_i-1}x_i^{2(k_i-1)}}{\Gamma k_i})-(p-1))e^{-\frac{1}{2}\sum_{i=1}^{p}x_i^2} \quad (28)$$

where $0\le y_i<\infty$, $k_i>0$

Result 4.0: From (1) and if $y_i=1/x_i$, then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive Inverse Nagakami-M distribution and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=2^p\prod_{i=1}^{p}(m_i/\Omega_i y_i^3)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}(1/y_i)^{2(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}(\frac{1}{y_i})^2} \quad (29)$$

where $0\le y_i<\infty \quad m\ge 0.5,\Omega>0$

Result 4.1: From (1) and if $y_i=e^{x_i}$, then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive log-Nagakami-M distribution and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=2^p\prod_{i=1}^{p}(m_i\log y_i/\Omega_i y_i)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}(\log y_i)^{2(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i}{\Omega_i}(\log y_i)^2} \quad (30)$$

where $1\le y_i<\infty \quad m\ge 0.5,\Omega>0$

Result 4.2: From (1) and if $y_i=\log x_i$,then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive Extreme value Nagakami-M distribution and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=2^p\prod_{i=1}^{p}(m_i e^{2y_i}/\Omega_i)(\sum_{i=1}^{p}(\frac{(m_i/\Omega_i)^{m_i-1}e^{2y_i(m_i-1)}}{\Gamma m_i})-(p-1))e^{-\sum_{i=1}^{p}\frac{m_i e^{2y_i}}{\Omega_i}} \quad (31)$$

where $-\infty<y_i<+\infty \quad m\ge 0.5,\Omega>0$

Result 4.3: From (1) and if $m_i=k_i$, $m_i/\Omega_i=\lambda_i$ and $y_i=x_i^2$, then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive Gamma distribution and its density function is given as

$$f(y_1,y_2,y_3\dots,y_p)=\prod_{i=1}^{p}\lambda_i(\sum_{i=1}^{p}(\frac{y_i^{k_i-1}}{\Gamma k_i})-(p-1))e^{-\sum_{i=1}^{p}\lambda_i y_i} \quad (32)$$

where $0\le y_i<\infty$, $\lambda_i,k_i>0$

Result 4.4: From (1) and if $m_i = k_i / 2$, $m_i / \Omega_i = 1/2$ and $y_i = x_i^2$, then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive Chi-square distribution with k_i degrees of freedom and its density function is given as

$$f(y_1, y_2, y_3 \ldots, y_p) = (1/2)^p (\sum_{i=1}^{p} (\frac{(1/2)^{(k_i/2)-1} y_i^{(k_i/2)-1}}{\Gamma(k_i/2)}) - (p-1)) e^{-\frac{1}{2}\sum_{i=1}^{p} y_i} \quad (33)$$

where $0 \le y_i < \infty$, $k_i > 0$

Result 4.5: From (1) and if $m_i = k_i$, $m_i / \Omega_i = \mu_i k_i$ and $y_i = x_i^2$, then the Sam-solai's Multivariate additive Nagakami-M distribution is transformed into Sam-solai's Multivariate additive Erlang-k distribution with parameters (μ_i, k_i) and its density function is given as

$$f(y_1, y_2, y_3 \ldots, y_p) = \prod_{i=1}^{p} (\mu_i k_i) (\sum_{i=1}^{p} (\frac{(\mu_i k_i)^{k_i - 1} y_i^{k_i - 1}}{\Gamma k_i}) - (p-1)) e^{-\sum_{i=1}^{p} \mu_i k_i y_i} \quad (34)$$

where $0 \le y_i < \infty$, $\mu_i, k_i > 0$

CONCLUSION

The multivariate generalization of two parameter Nagakami-M-distribution in an additive form of Sam-Solai's generalization having some interesting features. At first, the marginal uni-variate distributions of the Sam-Solai's Multivariate additive Nagakami-M distribution are uni-variate and enjoyed the symmetric property. Secondly, the Population Correlation co-efficient of the proposed distribution is bounded between -1 and +1 for certain values of shape parameter and the authors established the simulated standard bivariate correlations. Thirdly, the Conditional variance of Sam-Solai's Multivariate additive conditional Nagakami-M distribution is heteroscedastic in nature and this feature is a unique for the proposed distribution. Finally, the multivariate generalization of two parameter Nagakami-M distribution in an additive form open the way for the same additive form of the Multivariate additive half normal distribution, Multivariate additive chi-distribution, Multivariate additive Inverse Nagakami-m distribution, Multivariate additive log-Nagakami-m distribution, Multivariate additive Extreme value Nagakami-m distribution, Multivariate additive Gamma distribution, Multivariate additive Chi-square distribution and Multivariate additive Erlang-k distribution

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