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COMPATIBLE MAPPINGS OF TYPE (P) IN FUZZY METRIC SPACES FOR COMMON FIXED POINT THEOREMS

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ABSTARACT

In this paper we prove compatible mappings of Type (P) In Fuzzy Metric Spaces For common Fixed point Theorems and Improve the results.

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Keywords: *Compatible mappings of type (P), Common fixed point, Fuzzy metric space.*

1. INTRODUCTION

Zadeh [15] introduced the concept of fuzzy sets. The idea of fuzzy metric space was introduced by Kramosil and Michalek [10] which was modified by George and Veeramani [3, 4]. Gerald Jungck [7] in the theory of fixed point compatible mappings was obtained by as a generalization o f commuting mappings. Pathak, Chang and Cho [11] introduced the concept of compatible mappings of type (P).

Bijendra Singh and M. S. Chauhan [13] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sence of George and Veeramani with continuous t-norm * defined by $a*b = \min\{a, b\}$ for all $a, b \in [0,1]$.

This paper is to prove some common fixed point theorems of compatible mappings of type (P) by modifying the results of S.H. Cho.

2. PRELIMINARIES

Definition 2.1 [12] A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if it satisfies the following conditions

i * is associative and commutative.
ii * is continuous.
iii. $a*1 = a$ for all $a \in [0, 1]$.
iv $a*b \le c*d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0,1]$.

Definition 2.2: The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, * is continuous t-norm, and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(5) $M(x, y, .) : (0,\infty) \to [0,1]$ is continuous for all $x, y, z \in X$ and $t, s > 0$.

Let (X, d) be a metric space, and let $a*b = ab$ or $a*b=\min\{a, b\}$. Let $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M,*)$ is a fuzzy metric M induced by d is called the standard fuzzy metric space [3].

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Definition 2.3 : A sequence $\{x_n\}$ in a fuzzy metric space (*X, M,* *) is said to be convergent to a point $x \in X$ (denoted by llim$_{n\to\infty} x_n = x$) , if for each ε >0 and each *t* >0, there exists $n_0 \in N$ such that $M(x_n, x, t) > 1-\varepsilon$ for all $n_0 \geq n$.

Definition 2.4 [3] A sequence $\{x_n\}$ *in* a fuzzy metric space (*X, M,* *) is called Cauchy sequence if for each ε >0 and each *t* >0, there exists $n_0 \in N$ such that $M(x_n, x_m, t) > 1-\varepsilon$ for all $n, m \geq n_0$.

Definition 2.5 [13]: Self mappings *A* and *S* of a fuzzy metric space (*X, M,.∗*) is said to be compatible if

$$\lim_{n\to\infty} M(AS\, SAx_n, t) = 1$$

for all *t* > 0, whenever $\{x_n\}$ is a sequence in *X* such thatlim *Axn* = = *z* for some $z \in X$.

Definition 2.6 [11] Self mappings *A* and *S* of a fuzzy metric space (*X, M,* *) is said to be compatible of type (P) if $\lim_{n\to\infty} M(AAx_n\ SAx_n, t) = 1$ for all *t* > 0, whenever $\{x_n\}$ is a sequence in *X* such that lim *Axn* = $\lim_{n\to\infty} Sx_n$ = *z* for some $z \in X$.

Lemma 2.7 [5] Let (*X, M,* *) be a fuzzy metric space. Then for all *x, y* in *X, M*(*x, y,* .),is non-decreasing.

Lemma 2.8 [14] Let (*X, M,* *) be a fuzzy metric space. If there exists $q\in(0, 1)$ such that $M(x, y, qt) \geq M(x, y, t/q\ n)$ for positive integer n. Taking limit as $n \to \infty$, $M(x, y, t) \geq 1$ and hence $x = y$.

Lemma 2.9 [9] The only *t*-norm * satisfying $r*r \geq r$ for all $r\in[0,1]$ is the minimum *t*-norm, that is, $a* b = \min\{a, b\}$ for all $a,b \in[0,1]$.

Proposition 2.10 [11] Let (*X, M,* *) be a fuzzy metric space and let *A* and *S* be continuous mappings of *X* then *A* and *S* are compatible if and only if they are compatible of type (P).

Proposition 2.11 [11] Let (*X, M,* *) be a fuzzy metric space and let *A* and *S* be compatible mappings of type(P) and *Az* = *Sz* for some $z \in X$, then AAz = ASz = SAz = SSz.

Proposition 2.12 [11] Let (X, M, *) be a fuzzy metric space and let A and S be compatible mappings of type(P) and let Axn, Sxn → z as n →∞ for some z ∈ X. Then)

$(i)\ \lim_{n\to\infty} SSx_n\ = AZ\ if\ A\ is\ continous\ at\ z$

$(ii)\ \lim_{n\to\infty} AAx_n\ = AZ\ if\ A\ is\ continous\ at\ z$

(iii) *ASz* = *Saz* and *Az* = *Sz* if *A* and *S* are continuous at *z*.

3. COMMON FIXED POINT THEOREMS

Theorem 3.1: *Let* (*X, M,**) *be a complete fuzzy metric space and let A, B, S and T be a self mappings of X satisfying the following conditions:*

(*i*) *A*(*X*) ⊂ *T*(*X*), *B*(*X*) ⊂ *S*(*X*),

(*ii*) *S and T are continuous*

(*iii*) *The pairs* {*A, S*} *and* {*B, T*} *are compatible mapping of type* (*P*) *on X.*

(iv) There exists k∈(0,1) such that for every u, v∈X and t > 0,

$$M(Au, Bv, Kt) \geq min\{M(Su, Tv(x)^2)*M(S_U,A_U(x))*M(T_V.B_V(x\ M(S_U.B_V(2x))*M(T_V.A_u(x))*\ M(T_u.A_v(x)*$$
$$M(Su.\ Bv(2x))M(Tv.\ Bv(x))$$

Then A, B, S and T have a unique common fixed point in X.

Proof: Since *A*(*X*) ⊂ *T*(*X*) and *B*(*X*) ⊂ *S*(*X*), for any $x_0 \in X$, there there exists a point $x_1 \varepsilon$ X such that $Axo = Tx_1$. Since $B(X) \subset S(X)$, for this point X_l we can choose a point $x_2 \varepsilon X$ such that $BX_l = SX_2$ and so on. Inductively, we can define a sequence $\{Y_n\}$ in X such that

$$\begin{cases} y_{2n} = Tx_{2n+1} = Ax_{2n} \\ y_{2n+1} = Sx_{2n+2=Bx_{2n+1}} \end{cases}$$

for $n = 0,1,2,\cdots$. Now, we prove $M(Y_{2n}, Y_{2n+l}(kx)) \geq M(Y_{2n-1}, Y_{2n}(x)$ for all $x > 0$, where $k\ \varepsilon\ (0,1)$.

Suppose that $M(Y_{2n}, Y_{2n+l}(kx)) < M(Y_{2n-11}, Y_{2n}(X))$ for some: x>0.

Then by using and $M(Y_{2n}, Y_{2n+1}(kx)) \geq M(Y_{2n}, Y_{2n}, y_{2n+1}(x))$, We have

$$M\big(y_{2n}, y_{2n+1}(kx)\big)^2 \geq M\big(Ax_{2n}, Bx_{2n+1}, (kx)\big)^2$$

$$> \min\{M\big(Sx_{2n}, Tx_{2n+1}, (kx)\big)^2 * M(Sx_{2n,} Ax_{2n}(x)) * M(Tx_{2n+1,} Bx_{2n+1}(x))$$

$$* M(Sx_{2n,} Bx_{2n+1}(2x) * M(Tx_{2n+1,} Ax_{2n}(x)) * M(Sx_{2n,} Ax_{2n}(x))$$

$$* M(Tx_{2n+1,} Ax_{2n}(x)) * M(Sx_{2n,} Bx_{2n+1}(\mathrm{x})) * M(Tx_{2n+1,} Bx_{2n+1}(\mathrm{x}))$$

$$= \min\{M(y_{2n-1} y_{2n}\ (\mathrm{x}))^2 * M(y_{2n-1ss,} y_{2n}(x)) * M(y_{2n-1,} y_{2n+1}(x))$$

$$* M(y_{2n-1,} y_{2n+1}(2x)) * M(y_{2n,} y_{2n}(x)) * M(y_{2n-1,} y_{2n}(x))$$

$$* M(y_{2n,} y_{2n}(x)) * M(y_{2n,-1} y_{2n+1}(2x)) * M(y_{2n,} y_{2n+1}(\mathrm{x}))$$

$$> min\{M(y_{2n-1} y_{2n}\ (\mathrm{x}))^2 * M(y_{2n-1ss,} y_{2n}(x)) * M(y_{2n} y_{2n+1}(x))$$

$$* \mathrm{t}(M(y_{2n-1,} y_{2n}(x)) * M(y_{2n,} y_{2n+1}(x)) * M(y_{2n-1,} y_{2n}(x))$$

$$* \mathrm{t}(M(y_{2n,-1,} y_{2n}(x)) * M(y_{2n,} y_{2n+1}(x)) * M(y_{2n,} y_{2n+1}(\mathrm{x}))$$

$$> \min\{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^{2*}\{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^2$$

$$* \{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},} y_{2n+1}(\mathrm{kx}))^2$$

Which is a contradiction. Thus, we have

$$M(y_{2n}, y_{2n+1,}(kx)) \geq M(y_{2n-1,} y_{2n}\ (x)$$

Similarly, we have also.

$$M(y_{2n+1}, y_{2n+2,}(kx) \geq M(y_{2n,} y_{2n+1}\ (x)$$

Therefore, for every $n \varepsilon N$,

$$M(y_n, y_{n+1,}(kx) \geq M(y_{n-1,} y_n\ (x)$$

Therefore, $\{_{\mathrm{Yn}}\}$ is a Cauchy sequence in *X*. Since the *(X, M, t)*is complete, $\{y_u\}$ converges to a point *z* in *X* and the subsequences $\{AX_{2n}\}$, $\{BX_{2n+1}\}$, $\{Sx_{2n}\}$, $\{TX_{2n+1}$ *}of { $\{_{Yn}\}$* also converge to *z* . *X*.

Now, suppose that T is continuous. Since B and T are compatible of type (A), $BTx_{2n+l,}$ $TTx_{2n+I} \rightarrow Tz$ as $n \rightarrow \infty$.

Putting U= X_{2n} and v = Tx_{2n+l} we Compatible mappings of type (A) and

$(M(Ax_{2n}, BTx_{2n+l}(kx))^2 \geq \min\{(M(sx_{2n}, TTx_{2n+1}(x))^2$,

$M(Sx_{2n}, Ax_{2n}(X)*M(TTx_{2n+l}, BTx_{2n+1}(x), M(Sx_{2n}, BTx_{2n+11}(2x)*M(TTx_{2n+l}, Ax_{2n}(x)$,

$M(Sx_{2n}), Ax_{2n}(x)*M(TTx_{2n}, Ax_{2n}(x), M(Sx_{2n}, BTx_{2n+11}(2x)*M(TTx_{2n+l}, BTx_{2n+1}(x)\}$.

Taking $n \rightarrow \infty$, we have

$(Mz, Tz(kx))^2 \geq \min\{(M(z, Tz(x))^2 * M(Tz, Tz(x) * Mz, Tz(2) * M(Tz, z(x) * Mz, z(x) *$

$M(_{Tz}, z(x) * Mz, Tz(2x) * M(Tz, Tz(x)\}$

$= (Mz, Tz(x))^2$,

which implies that Tz = z. Again, replacing 'u by x_{2n} and v by z in we have

$(M(Ax_{2n}, B_z(kx))^2 \geq \min\{(Mx_{2n}, Tz(x))^2 * M(Sx_{2n}, Ax_{2n}(x) * M_T z, Bz(x)$,

$M(SX_{2n}, Bz(2x) * M(Tz, Ax_{2n}(x) * M(SX_{2n}, Ax_{2n}(X) * M(Tz, Ax_{2n}(x), M(SX_{2n}, Bz(2x) * M(Tz, Bz(X)\}$

Taking n $\rightarrow\infty$, we have

$(M(z, Bz(kx))^2 \geq \min\{(M(z, Tz(X))^2,*M(z, Bz(X)* M(z, Bz(2x)*M(Tz, z(X),$

$*M(z, z(x)*M(Tz, z(X)* M(z, Bz(2x)*M(z, Bz(X)\}$

$= (M (z, Bz(x))^2,$

which implies that Bz = z. Since B(X) ⊂ S(X), there exists a point w in X such that B z = Sw = z. again, we have

$(M(Atu,z(kx))^2 = (M(Aw, Bz(kx))^2$

$\geq \min\{(M(sw, Tz(x))^2* M(Sw, Aw(x) *M(Tz, Bz(X)$

$*M(Sw, BA2x)*M(Tz, Aw(x)* M(Sw, Aw(X)*M(Tz, Aw(x), M(S_w , B_z(2x)* M(Tz, Bz(x)\}$

$= (M_{Aw},z(X))^2,$

which means that Aw = z. Since A and S are compatible of ' type(A) and Aw = Sw = z, we have, for every ε > 0, M(ASw, SSw(ε) = 1 and so Az = ASw = SSw = Sz. again, we have A.z = z. Therefore, Az = Bz = Sz = Tz = z, that is, z is a common fixed point of the given mappings. The uniqueness of the common fixed point z. This completes the proof.

Theorem 3.3. Let A, B, Sand T be mappings from a complete metric space (X, d) into itself such that
(i) A(X) C T(X) and B(X) c SeX),
(ii) one of A,B,S and T is continuous,
(iii) the pairs .4, Sand B, T are compatible of type (A),
(iv) there exists a constant k ∈ (0,1) SUcll that

$$M(Ax,By)^2 \leq k\max\{M(Sx,Ty)^2 * M(Sx,Ax) * M(Ty,By) * \frac{1}{2}M(Sx,By) * M(Ty,Ax) * M(Sx,Ax) * M(Ty,Ax)\frac{1}{2}M(Sx,By) * M(Ty,$$

For all x, y in X. Then A, B, S and T llave a unique common fixed point in X.

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