Research Journal of Pure Algebra -2(9), 2012, Page: 286-290
Available online through www.rjpa.info ISSN 2248–9037

COMPATIBLE MAPPINGS OF TYPE (P) IN FUZZY METRIC SPACES FOR COMMON FIXED POINT THEOREMS

M. Rangamma[1] & A. Padma[2]*

[1]Department of Mathematics, Osmania University, Hyderabad, India
[2]304, Saptagiri Towers, Street No. 8, Habsiguda, Hyderabad-7, India

(Received on: 25-08-12; Revised & Accepted on: 26-09-12)

ABSTARACT

In this paper we prove compatible mappings of Type (P) In Fuzzy Metric Spaces For common Fixed point Theorems and Improve the results.

Mathematics subject classification: *54H25, 54E50.*

Keywords: *Compatible mappings of type (P), Common fixed point, Fuzzy metric space.*

1. INTRODUCTION

Zadeh [15] introduced the concept of fuzzy sets. The idea of fuzzy metric space was introduced by Kramosil and Michalek [10] which was modified by George and Veeramani [3, 4]. Gerald Jungck [7] in the theory of fixed point compatible mappings was obtained by as a generalization o f commuting mappings. Pathak, Chang and Cho [11] introduced the concept of compatible mappings of type (P).

Bijendra Singh and M. S. Chauhan [13] introduced the concept of compatibility in fuzzy metric space and proved some common fixed point theorems in fuzzy metric spaces in the sence of George and Veeramani with continuous t-norm * defined by $a*b = \min\{a, b\}$ for all $a, b \in [0,1]$.

This paper is to prove some common fixed point theorems of compatible mappings of type (P) by modifying the results of S.H. Cho.

2. PRELIMINARIES

Definition 2.1 [12] A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions

i * is associative and commutative.
ii * is continuous.
iii. $a*1 = a$ for all a $\in [0, 1]$.
iv $a*b \le c*d$ whenever $a \le c$ and $b \le d$, for each $a, b, c, d \in [0,1]$.

Definition 2.2: The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary (non-empty) set, * is continuous t-norm, and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(1) $M(x, y, t) > 0$,
(2) $M(x, y, t) = 1$ if and only if $x = y$,
(3) $M(x, y, t) = M(y, x, t)$,
(4) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$,
(5) $M(x, y, .) : (0,\infty) \to [0,1]$ is continuous for all $x, y, z \in X$ and $t, s > 0$.

Let (X, d) be a metric space, and let $a*b = ab$ or $a*b=\min\{a, b\}$. Let $M(x, y, t) = \frac{t}{t+d(x,y)}$ for all $x, y \in X$ and $t > 0$. Then $(X, M,*)$ is a fuzzy metric M induced by d is called the standard fuzzy metric space [3].

***Corresponding author:** *A. Padma*
304, Saptagiri Towers, Street No.8, Habsiguda, Hyderabad-7, India

Definition 2.3 : A sequence $\{x_n\}$ in a fuzzy metric space (*X, M,* *) is said to be convergent to a point $x \in X$ (denoted by llim$_{n\to\infty} x_n =$ x) , if for each ε >0 and each *t* >0, there exists $n_0 \in$N such that $M(x_n,, x,t) > 1$-ε for all $n_0 \geq$ n.

Definition 2.4 [3] A sequence $\{x_n\}$ *in* a fuzzy metric space (*X, M,* *) is called Cauchy sequence if for each ε >0 and each *t* >0, there exists $n_0 \in$ N such that M (x_n, x_m, t) > 1- ε for all n, m ≥ n_0.

Definition 2.5 [13]: Self mappings *A* and *S* of a fuzzy metric space (*X, M*,.∗) is said to be compatible if

$$\lim_{n\to\infty} M(AS\,SAx_n,, t) = 1$$

for all *t* > 0, whenever $\{x_n\}$ is a sequence in *X* such thatlim *Axn* = = *z* for some *z*∈ *X*.

Definition 2.6 [11] Self mappings *A* and *S* of a fuzzy metric space (*X, M,* *) is said to be compatible of type (P) if $\lim_{n\to\infty} M(AAx_n\ SAx_n,, t) = 1$ for all *t* > 0, whenever $\{x_n\}$ is a sequence in *X* such that lim *Axn* = $\lim_{n\to\infty} Sx_n$ = *z* for some *z* ∈ *X*.

Lemma 2.7 [5] Let (*X, M,* *) be a fuzzy metric space. Then for all *x, y* in *X, M*(*x, y,* .),is non-decreasing.

Lemma 2.8 [14] Let (*X, M,* *) be a fuzzy metric space. If there exists *q*∈(0, 1) such that *M*(*x, y, qt*) ≥ *M*(*x, y, t/q n*) for positive integer n. Taking limit as *n* → ∞, *M*(*x, y, t*) ≥ 1 and hence *x* = *y*.

Lemma 2.9 [9] The only *t*-norm * satisfying *r*r* ≥ *r* for all *r*∈[0,1] is the minimum *t*-norm, that is, *a* b* = min {*a, b*} for all *a,b* ∈[0,1].

Proposition 2.10 [11] Let (*X, M,* *) be a fuzzy metric space and let *A* and *S* be continuous mappings of *X* then *A* and *S* are compatible if and only if they are compatible of type (P).

Proposition 2.11 [11] Let (*X, M,* *) be a fuzzy metric space and let *A* and *S* be compatible mappings of type(P) and *Az* = *Sz* for some *z* ∈ *X*, then AAz = ASz = SAz = SSz.

Proposition 2.12 [11] Let (X, M, *) be a fuzzy metric space and let A and S be compatible mappings of type(P) and let Axn, Sxn → z as n →∞ for some z ∈ X. Then)

$(i)\ \lim_{n\to\infty} SSx_n\ = AZ\ if\ A\ is\ continous\ at\ z$

$(ii)\ \lim_{n\to\infty} AAx_n\ = AZ\ if\ A\ is\ continous\ at\ z$

(iii) *ASz* = *Saz* and *Az* = *Sz* if *A* and *S* are continuous at *z*.

3. COMMON FIXED POINT THEOREMS

Theorem 3.1: *Let* (*X, M,**) *be a complete fuzzy metric space and let A, B, S and T be a self mappings of X satisfying the following conditions:*
(*i*) *A*(*X*) ⊂ *T*(*X*), *B*(*X*) ⊂ *S*(*X*),
(*ii*) *S and T are continuous*
(*iii*) *The pairs* {*A, S*} *and* {*B, T*} *are compatible mapping of type* (*P*) *on X.*
(iv) There exists k∈*(0,1) such that for every u, v*∈*X and t > 0,*

$$M(Au, Bv, Kt) \geq min\{M(Su, Tv(x)^2)*M(S_U,A_U(x))*M(T_V.B_V(x\ M(S_U.B_V(2x))*M(T_V.A_u(x))*M(T_u.A_v(x))* M(Su.\ Bv(2x))M(Tv.\ Bv(x))$$

Then A, B, S and T have a unique common fixed point in X.

Proof: Since *A*(*X*) ⊂ *T*(*X*) and *B*(*X*) ⊂ *S*(*X*), for any $x_0 \in X$, there there exists a point x $_1$ε X such that $Axo = Tx_1$. Since *B(X)* ⊂ *S(X)*, for this point X_l we can choose a point $x_2 ε$ *X* such that $BX_l = SX_2$ and so on. Inductively, we can define a sequence $\{Y_n\}$ in X such that

$$\begin{cases} y_{2n} = Tx_{2n+1} = Ax_{2n} \\ y_{2n+1} = Sx_{2n+2=Bx_{2n+1}} \end{cases}$$

for $n = 0,1,2,\cdots$. Now, we prove $M(Y_{2n}, Y_{2n+l}(kx)) \geq M(Y_{2n-1}, Y_{2n}(x)$ for all *x*> 0, where *k ε* (0,1).

Suppose that $M(Y_{2n}, Y_{2n+l}(kx)) < M(Y_{2n-11}, Y_{2n}(X))$ for some: x>0.

Then by using and $M(Y_{2n}, Y_{2n+1}(kx)) \geq M(Y_{2n}, Y_{2n}, y_{2n+1}(x)),$

We have

$$M\big(y_{2n}, y_{2n+1}(kx)\big)^2 \ \geq M\big(Ax_{2n}, Bx_{2n+1}, (kx)\big)^2$$

$$> \ \min\{M\big(Sx_{2n}, Tx_{2n+1}, (kx)\big)^2 * M(Sx_{2n,}Ax_{2n}(x)) * M(Tx_{2n+1,}Bx_{2n+1}(x))$$

$$* M(Sx_{2n,}Bx_{2n+1}(2x) * M(Tx_{2n+1,}Ax_{2n}(x)) * \ M(Sx_{2n,}Ax_{2n}(x))$$

$$* M(Tx_{2n+1,}Ax_{2n}(x)) * M(Sx_{2n,}Bx_{2n+1}(\mathrm{x})) * M(Tx_{2n+1,}Bx_{2n+1}(\mathrm{x}))$$

$$= \min\{M(y_{2n-1}y_{2n}\ (\mathrm{x}))^2 * M(y_{2n-1ss,}y_{2n}(x)) * M(y_{2n-1,}y_{2n+1}(x))$$

$$* M(y_{2n-1,}y_{2n+1}(2x)) * M(y_{2n,}y_{2n}(x)) * \ M(y_{2n-1,}y_{2n}(x))$$

$$* M(y_{2n,}y_{2n}(x)) * M(y_{2n,-1}y_{2n+1}(2x)) * M(y_{2n,}y_{2n+1}(\mathrm{x}))$$

$$> min\{M(y_{2n-1}y_{2n}\ (\mathrm{x}))^2 * M(y_{2n-1ss,}y_{2n}(x)) * M(y_{2n}y_{2n+1}(x))$$

$$* \mathrm{t}(M(y_{2n-1,}y_{2n}(x)) * M(y_{2n,}y_{2n+1}(x)) * \ M(y_{2n-1,}y_{2n}(x))$$

$$* \mathrm{t}(M(y_{2n,-1,}y_{2n}(x)) * M(y_{2n,}y_{2n+1}(x)) * M(y_{2n,}y_{2n+1}(\mathrm{x}))$$

$$> \min\{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^{2*}\{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^2$$

$$* \{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^2 * \{M(\mathrm{y}_{2\mathrm{n},}y_{2n+1}(\mathrm{kx}))^2$$

Which is a contradiction. Thus, we have

$$M(y_{2n}, y_{2n+1,}(kx)) \geq M(y_{2n-1,}y_{2n}\ (x)$$

Similarly, we have also.

$$M(y_{2n+1}, y_{2n+2,}(kx) \geq M(y_{2n,}y_{2n+1}\ (x)$$

Therefore, for every n ε N,

$$M(y_n, y_{n+1,}(kx) \geq M(y_{n-1,}y_n\ (x)$$

Therefore, $\{_{Y_n}\}$ is a Cauchy sequence in *X*. Since the *(X, M, t)*is complete, $\{y_u\}$ converges to a point *z* in *X* and the subsequences $\{AX_{2n}\}$, $\{BX_{2n+1}\}$, $\{Sx_{2n}\}$, $\{TX_{2n+1}$ *}of { $\{_{Y_n}\}$* also converge to *z . X*.

Now, suppose that T is continuous. Since B and T are compatible of type (A), $BTx_{2n+l,}$ $TTx_{2n+I} \rightarrow Tz$ as n → ∞.

Putting U= X_{2n} and v = Tx_{2n+l} we Compatible mappings of type (A) and

$(M(Ax_{2n}, BTx_{2n+l}(kx))^2 \geq \min\{(M(sx_{2n}, TTx_{2n+1}(x))^2$,

M $(Sx_{2n}, Ax_{2n}(X)*M(TTx_{2n+l}, BTx_{2n+1}(x), M(Sx_{2n}, BTx_{2n+11}(2x)*M(TTx_{2n+l}, Ax_{2n}(x)$,

M $(Sx_{2n}), Ax_{2n}(x)*M(TTx_{2n}, Ax_{2n}(x), M(Sx_{2n}, BTx_{2n+11}(2x)*M(TTx_{2n+l}, BTx_{2n+1}(x)\}$.

Taking $n \to \infty$, we have

$(Mz, Tz(kx))^2 \geq \min\{(M(z, Tz(x))^2 * M(Tz, Tz(x) * Mz, Tz(2\ ) * M(Tz, z(x) * \ Mz, z(x) *$

$M(_{Tz}, z(x) * Mz, Tz(2x) * M(Tz, Tz(x)\}$

$= (Mz, Tz(x))^2$,

which implies that Tz = z. Again, replacing 'u by x_{2n} and v by z in we have

$(M(Ax_{2n}, B_z(kx))^2 \geq \ \min\{(Mx_{2n}, Tz(x))^2 * M(Sx_{2n}, Ax_{2n}(x) * M_Tz, Bz(x)$,

$M(SX_{2n}, Bz(2x) * M(Tz, Ax_{2n}(x) * M(SX_{2n}, Ax_{2n}(X) * M(Tz, Ax_{2n}(x), M(SX_{2n}, Bz(2x) * M(Tz, Bz(X)\}$

Taking n →∞, we have

$(M(z, Bz(kx))^2 \geq \min\{(M(z, Tz(X))^2, *M(z, Bz(X)* M(z, Bz(2x)*M(Tz, z(X),$

$*M(z, z(x)*M(Tz, z(X)* M(z, Bz(2x)*M(z, Bz(X)\}$

$= (M (z, Bz(x))^2,$

which implies that Bz = z. Since B(X) ⊂ S(X), there exists a point w in X such that B z = Sw = z. again, we have

$(M(Atu,z(kx))^2 = (M(Aw, Bz(kx))^2$

$\geq \min\{(M(sw, Tz(x))^2 * M(Sw, Aw(x) *M(Tz, Bz(X)$

$*M(Sw, BA2x)*M(Tz, Aw(x)* M(Sw, Aw(X)*M(Tz, Aw(x), M(S_w , B_z(2x)* M(Tz, Bz(x)\}$

$= (M_{Aw},z(X))^2,$

which means that Aw = z. Since A and S are compatible of ' type(A) and Aw = Sw = z, we have, for every ε > 0, M(ASw, SSw(ε) = 1 and so Az = ASw = SSw = Sz. again, we have A.z = z. Therefore, Az = Bz = Sz = Tz = z, that is, z is a common fixed point of the given mappings. The uniqueness of the common fixed point z. This completes the proof.

Theorem 3.3. Let A, B, Sand T be mappings from a complete metric space (X, d) into itself such that
(i) A(X) C T(X) and B(X) c SeX),
(ii) one of A,B,S and T is continuous,
(iii) the pairs .4, Sand B, T are compatible of type (A),
(iv) there exists a constant k ∈ (0,1) SUcll that

$$M(Ax,By)^2 \leq k \max\{M(Sx,Ty)^2 * M(Sx,Ax) * M(Ty,By) * \frac{1}{2}M(Sx,By) * M(Ty,Ax) * M(Sx,Ax) * M(Ty,Ax)\frac{1}{2}M(Sx,By) * M(Ty,$$

For all x, y in X. Then A, B, S and T llave a unique common fixed point in X.

REFERENCES

[1] Deng Zi- Ke, Fuzzy pseudo metric spaces, J. Math. Anal. Appl. **86**(1982), 74-95.

[2] M. A.Erceg., Metric spaces in Fuzzy set theory, J. Math. Anal. Appl., **69**(1979), 205-230.

[3] A. George and P. Veeramani, On some results in fuzzy metric spaces, Fuzzy Sets and Systems, **64**(1994), 395-399.

[4] A. George and P. Veeramani, On some results of analysis for fuzzy metric spaces, Fuzzy sets and Systems, **90**(1997), 365-368.

[5] Grabiec M., Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, **27**(1988), 385-389.

[6] Valentin Gregori and Almanzor Sapena, On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems, **125** (2002), 245-252.

[7] G. Jungck , Compatible mappings and common fixed points. Internat. J. Math. and Math. Sci., **9(4)**, (1986), 771-779.

[8] O. Kaleva and S. Seikkala, On fuzzy metric spaces, Fuzzy Sets and Systems, **12** (1984), 215-229.

[9] E.P. Klement, R. Mesiar and E. Pap, Triangular Norms, Kluwer Academic Publishers.

[10] Kramosil and J. Machalek, Fuzzy metric and statistical metric spaces, Kybernetika **11** (1975), 336-344.

[11] Pathak, Chang, Cho, Fixed point theorems for compatible mappings of type (P), Indian journal of Math. **36(2)** (1994), 151-166.

[12] B. Schweizer and A. Sklar, Statistical metric spaces, Pacific J. Math., **10**(1960), 314- 334.

[13] Bijedra Singh and M. S. Chauhan, Common fixed points of compatible maps in fuzzy spaces, Fuzzy sets and Systems, **115**(2000), 471-475.

[14] Seong Hoon Cho, On common fixed points in fuzzy metric spaces, Inter. Math. Forum, **1 (10)** (2006), 471-479.

[15] Zadeh L.A., Fuzzy sets, Inform and Control, **8** (1965), 338-353.

Source of support: Nil, Conflict of interest: None Declared