



Unsteady effect on free convective flow with heat, mass transfer and chemical Reaction

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ABSTRACT

The Objective of this paper is to study the unsteady effect on free convective flow with heat, mass transfer and chemical reaction. The dimensionless governing equations are solved using multi-parameter perturbation technique. The effects of velocity, temperature and concentration are studied for different parameters such as Grashof number, chemical reaction number, Prandtl number, Modified Grashof number and Radiative parameter, Schmidt number and presented by means of graphs and tables.

Keywords: Free convection, radiation, Heat and Mass transfer, chemical reaction, porous medium.

1. INTRODUCTION

Chemical reaction have numerous industrial applications such as several engineering, industrial, geophysical and astrophysical application, such as polymer production, manufacturing of ceramic, packed-bed catalytic reactors, food processing, cooling of nuclear reactors, enhanced oil recovery, underground energy transport, magnetized plasma flow, high-speed plasma wind, cosmic jets and stellar systems. A clear understanding of the nature of interaction between thermal and concentration buoyancies is necessary to control these processes. Muthucumaraswamy (2010) studied that the rate of diffusion is affected by chemical reaction This has led to extensive studies of unsteady MHD flow and improves in aerodynamics, nuclear energy system control, and improved designs in aerospace MHD energy systems, and manufacture of advanced aerospace materials. Mansour *et al.* (2009) studied the effect of chemical reactions and radiation on MHD free convective heat and mass transfer from a horizontal cylinder of elliptic cross section saturated in a porous media considering suction or injection. Mututhucumarasamy (2001) studied the effect of chemical reaction on a moving isothermal infinite long surface with suction. Devi and Kandersamy (2002) investigated the effects of chemical reaction and heat mass transfer on nonlinear MHD laminar boundary layer flow over a wedge with suction and injection. Murti *et al.* (2005) studied the effect of double dispersion on natural convection heat and mass transfer in non-Darcy, non-Newtonian fluid with radiation and chemical reaction along the vertical surface. El-Amin (2004) studied the effect of chemical reaction and double dissipation on non-Darcy free convective heat and mass transfer in porous medium. Also the study of Chamka (2003) on MHD flow over a uniform stretched vertical permeable surface used similar assumptions for the chemical reaction. Seddeek and Almushigeh (2010) investigated effects of radiation and variable viscosity on MHD free convective flow and mass transfer over a stretching sheet with chemical reaction. However Acharya *et al.* (1999) have studied heat and mass transfer on an accelerating surface subjected to both power law surface temperature and power law flux variation with a temperature dependent heat source in the presence of suction and injection. Okedoye and Ajala(2009) presented MHD Free convection flow past on oscillating plate in the presence of heat generation/absorption and chemical reaction. Beg and Ghosh (2010) have investigated analytical solution of MHD radiation convection with surface temperature oscillation and secondary flow effects. Uwanta *et.al*(2011) have studied radiative convective fluid flow with chemical reaction .Our present work concentrates the combined effects of unsteady heat, mass transfer and Radiative convection flow with chemical reaction , which is an extension work of Senapati *et.al*(2012), where they obtained Effects of Chemical reaction on free convection MHD flow through porous medium bounded by vertical surface with slip flow region. The effects of the parameters involved on the flow are discussed. The skin frictions at the wall, rate of heat transfer and the rate of mass transfer are also considered.

2. PROBLEM FORMULATION

We consider the region of unsteady MHD flow of a viscous, incompressible, electrically conducting fluid over a semi-infinite region perpendicular to an infinite vertical plate moving with constant velocity, in the Presence of a transverse magnetic field. The surface temperatures of the plate oscllates with small amplitude about a non-uniform mean temperature. The x' -axis is taken along the plate and the y' -axis is normal to it. The equations for the momentum, energy and concentration are respectively:

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$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) - \frac{\sigma B_0^2 u'}{\rho} - \frac{\nu u'}{k'} \quad (1)$$

$$\frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\rho C_p \partial y'^2} - \frac{\partial q_r}{\rho C_p \partial y'} - \frac{\theta_0}{\rho C_p} (T' - T'_\infty) \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y'^2} - k^*(C' - C'_\infty) \quad (3)$$

The boundary conditions are:

$$u' = U_0, T' = T'_\infty + \epsilon(T'_\omega - T'_\infty)e^{n't'}, C' = C'_\infty + \epsilon(C'_\omega - C'_\infty)e^{n't'}, \text{ at } y' = 0, \\ u' \rightarrow 0, T' \rightarrow T'_\infty, C' \rightarrow C'_\infty \text{ as } y' \rightarrow \infty \quad (4)$$

where ρ is the density, g acceleration due to gravity, β is the co-efficient of thermal expansion, k is the thermal conductivity, q_r the radiation flux, ν the kinematic viscosity, σ is electrical conductivity, $B_0 (= H_0 \mu e)$ is the electromagnetic induction. β^* is the co-efficient of expansion of mass and D is the diffusion constant and K^* is chemical reaction parameter.

The radiation flux on the basis of the Rossel and diffusion model for radiation heat transfer is expressed as:

$$q_r = -\frac{4\sigma^* \partial T'^4}{3k^* \partial y'} \quad (5)$$

in which σ^* and k^* are Stefan-Boltzmann constant and the spectral mean absorption coefficient of the medium. It is assumed that the temperature differences within the flow are sufficiently small such that T'^4 may be expressed as linear function of the temperature. It can be established by expanding T'^4 in a Taylor series about T'_∞ and neglecting higher order term, that T'^4 can be expressed in the following way:

$$T'^4 = 4T_\infty^3 T' - 3T_\infty^4 \quad (6)$$

We introduce the following non-dimensional quantities:

$$u = \frac{u'}{U_0}, y = \frac{V_0 y'}{\nu}, t = \frac{t' V_0^2}{\nu}, \theta = \frac{T' - T'_\infty}{T'_\omega - T'_\infty}, C = \frac{C' - C'_\infty}{C'_\omega - C'_\infty}, Pr = \frac{\nu \rho C_p}{k}, M = \frac{\sigma \nu B_0^2}{\rho V_0^2} \\ N_r = \frac{16\sigma^* T_\infty^3}{3k^* k}, S = \frac{\nu \theta_0}{\rho c_p V_0^2}, K^* = \frac{K V_0^2}{\nu}, Gr = \frac{g \beta \nu (T'_\omega - T'_\infty)}{U_0 V_0^2}, Gm = \frac{g \beta^* \nu (C'_\omega - C'_\infty)}{U_0 V_0^2} \quad (7) \\ v = \frac{v'}{V_0}, K = \frac{k' V_0^2}{\nu^2}, Sc = \frac{\nu}{D}$$

where $u, \kappa, C, Pr, Gr, Gm, S, Sc, M, K, N_r, \theta$, and ω are dimensionless velocity function, thermal conductivity, non-dimensional mass concentration, Prandtl number, Grashoff number, modified Grashoff number, Sink strength, Schmidt number, Hartmann number, Chemical reaction parameter, Radiation parameter, non-dimensional temperature and Oscillating frequency respectively.

Substituting eqn. (7) into eqn. (1), (2), (3), (4) we have

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + Gr\theta + GmC - (M + \frac{1}{K})u \quad (8)$$

$$\frac{\partial \theta}{\partial t} = \left(\frac{1+N_r}{Pr}\right) \frac{\partial^2 \theta}{\partial y^2} - S\theta \quad (9)$$

$$\frac{\partial c}{\partial t} = \frac{1}{Sc} \frac{\partial^2 c}{\partial y^2} - KC \quad (10)$$

With the following boundary conditions

$$u = 1, \theta = 1 + \epsilon e^{nt}, C = 1 + \epsilon e^{nt} \text{ at } y = 0, \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \quad (11)$$

METHOD OF SOLUTION

In view of boundary condition (11), we assume the solutions of equation (8) to (10) as

$$u(y, t) = u_0(y)e^{i\omega t} \quad (12)$$

$$\theta(y, t) = \theta_0(y)e^{i\omega t} \quad (13)$$

$$c(y, t) = c_0(y)e^{i\omega t} \quad (14)$$

Where ω the frequency of the oscillation and t is the time

The boundary conditions (11) changes to

$$u_0 = e^{-i\omega t}, \theta_0 = (1 + \varepsilon e^{nt})e^{-i\omega t}, C_0 = (1 + \varepsilon e^{nt})e^{-i\omega t} \text{ at } y=0, \\ u \rightarrow 0, \theta \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \quad (15)$$

Using eqns (12), (13), (14) & (15), the solution of the eqns (8), (9) & (10) are expressed as velocity, temperature and concentration as follows

$$u(y) = e^{-y\sqrt{a_1}} - [A_1 e^{-y\sqrt{a_2}}(e^{-y\sqrt{a_1}} - 1) + A_2 e^{-y\sqrt{a_3}}(e^{-y\sqrt{a_1}} - 1)] \quad (16)$$

$$\theta(y) = b_1 e^{-y\sqrt{a_2}} \quad (17)$$

$$C(y) = b_1 e^{-y\sqrt{a_3}} \quad (18)$$

$$\text{where } a_1 = M + \frac{1}{K} + i\omega, a_2 = \frac{Pr(S+i\omega)}{1+Nr}, a_3 = (i\omega + K)Sc, A_1 = \frac{Gr b_1}{a_1}, A_2 = \frac{Gm b_1}{a_1}, b_1 = 1 + \varepsilon e^{nt}$$

Skin friction:

$$u'(y)]_{y=0} = (A_1 + A_2 - 1)\sqrt{a_1} \quad (19)$$

Nusselt number:

$$\theta'(y)]_{y=0} = -b_1\sqrt{a_2}$$

Sherwood number:

$$C'(y)]_{y=0} = -b_1\sqrt{a_3}$$

RESULT AND DISCUSSION

In this paper we have studied the unsteady effect on free convection fluid flow with heat, mass transfer and chemical reaction. The effect of the parameters Gr , Gm , Pr , Sc , M , K , Nr , S , ω on flow characteristics have been studied and shown by means of graphs and tables. In order to have physical correlations, we choose suitable values of flow parameters. To obtain the graphs the velocity, mass concentration and temperature w.r.t parameters(y) and nusselt number, shearwood number and skin friction are shown in tables.

Velocity profiles: The velocity profiles are depicted in Figs 1-4. Figure-(1) shows the effects of Pr , Nr and S on velocity at any point of the fluid, when $Sc=0.24$, $Gr=5$, $Gm=6$, $K=0.5$, $M=0.5$, $\omega=0.1$, $n=0.5$, $t=1$ and $\varepsilon=0.02$. It is noticed that the velocity decreases with the increase of Prandtl number (Pr) where as increases with the increase of Radiation parameter (Nr) and Sink strength (S).

Figure-(2) shows the effects of Sc , K and ω on velocity at any point of the fluid, when $Pr=0.71$, $Gr=5$, $Gm=6$, $S=1$, $M=0.5$, $Nr=0.5$, $n=0.5$, $t=1$ and $\varepsilon=0.02$. It is noticed that the velocity decreases with the increase Schmidt number (Sc) where as increases with the increase of Chemical reaction parameter (K) and Oscillating frequency (ω).

Figure-(3) shows the effects of Pr , Nr , Sc , K and ω on velocity at any point of the fluid, when $Gr=5$, $Gm=6$, $S=1$, $M=0.5$, $\omega=0.1$, $n=0.5$, $t=1$ and $\varepsilon=0.02$. It is noticed that the velocity decreases with the increase of Prandtl number (Pr) and Schmidt number (Sc), where as increases with the increase of Radiation parameter (Nr) and Chemical reaction parameter (K).

Figure-(4) shows the effects of Gr, Gm, M on velocity at any point of the fluid, when Pr=0.71, Sc=0.24, S=1, Nr=0.5, K=0.5, $\omega=0.1$, n=0.5, t=1 and $\epsilon=0.02$. It is noticed that the velocity increases with the increase of Grashoff number (Gr), modified Grashoff number (Gm) where as decreases with the increase Hartmann number (M).

Temperature profile: The temperature profile is depicted in Figs-5 only. Figure-(5) shows the effect of the parameters Pr, Nr, S, ω on temperature profile at any point of the fluid, when n=0.5, t=1 and $\epsilon=0.02$. It is noticed that the temperature rises in the increase of Radiation parameter (Nr), but falls for the increase of Prandtl number (Pr), Sink strength (S) and Oscillating frequency (ω).

Mass concentration profile: The mass concentration profile is depicted in Fig-6 only. Figure-(6) shows the effect of the parameters Sc, K and ω on mass concentration profile at any point of the fluid, when n=0.5, t=1 and $\epsilon=0.02$. It is noticed that the mass concentration decreases with the increase of Schmidt number (Sc), Chemical reaction parameter (K) and Oscillating frequency (ω).

Shearing stress of mean velocity: The shearing stresses of velocity are depicted in Table 1-2. Table-(1) shows the effect of the parameters Gr, M, K, ω on shearing stress of velocity at the plate of the fluid when Pr=0.71, n=0.5 and $\epsilon=0.02$. It is noticed that shearing stress at plate decreases with the increase of Grashoff number (Gr) and Chemical reaction parameter (K), whereas increases with the increase of Hartmann number (M) and Oscillating frequency (ω).

Table-(2) shows the effect of the parameters Gm, M, K, ω on shearing stress of velocity at the plate of the fluid when Sc=0.71, n=0.5 and $\epsilon=0.02$. It is noticed that shearing stress at plate decreases with the increase of modified Grashoff number (Gm) and Chemical reaction parameter (K) whereas increases with the increase of Hartmann number (M) and Oscillating frequency (ω).

Nusselt number: The Nusselt number of temperature is depicted in Table-3. it shows the effect of the parameters Pr, S, Nr and ω rate of heat transfer at the upper and lower plates, when n=0.5 and $\epsilon=0.02$. It is observed that the rate of heat transfer decreases with the increase in Prandtl number (Pr), Sink strength (S) and Oscillating frequency (ω) whereas it increases with the increase in Radiation parameter (Nr).

Sherwood Number: The Sherwood number of mass concentration is depicted in Table-4. it shows the effect of the parameters Sc, K, ω rate of mass transfer at the upper and lower plates, when n=0.5 and $\epsilon=0.02$. It is observed that the rate of mass transfer decreases with the increase of Schmidt number (Sc), Chemical reaction parameter (K) and Oscillating frequency (ω).

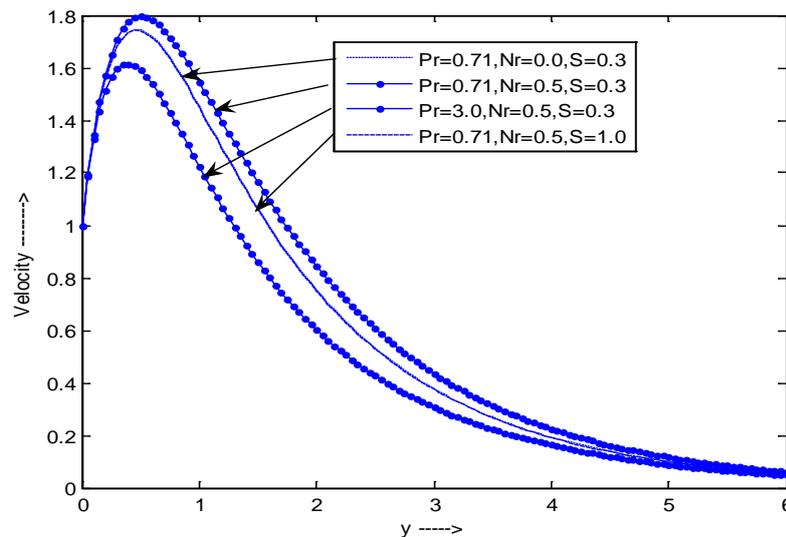


Fig-(1): Effects of Pr, Nr, S on Velocity profile, when Sc=0.24, Gr=5, Gm=6, K=0.5, M=0.5, $\omega=0.1$, n=0.5, t=1, $\epsilon=0.02$.

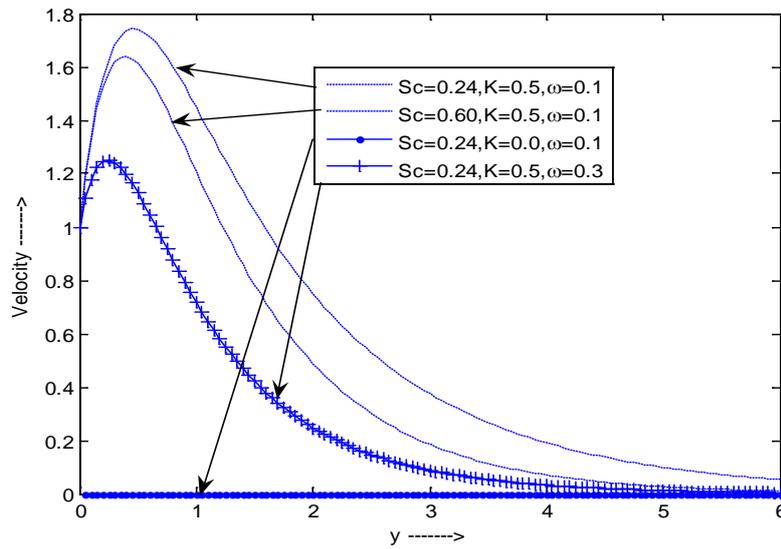


Fig-2: Effects of Sc, K, ω on Velocity profile, when $Pr=0.71$, $Gr=5$, $Gm=6$, $S=1$, $M=0.5$, $Nr=0.5$, $n=0.5$, $t=1$, $\epsilon=0.02$.

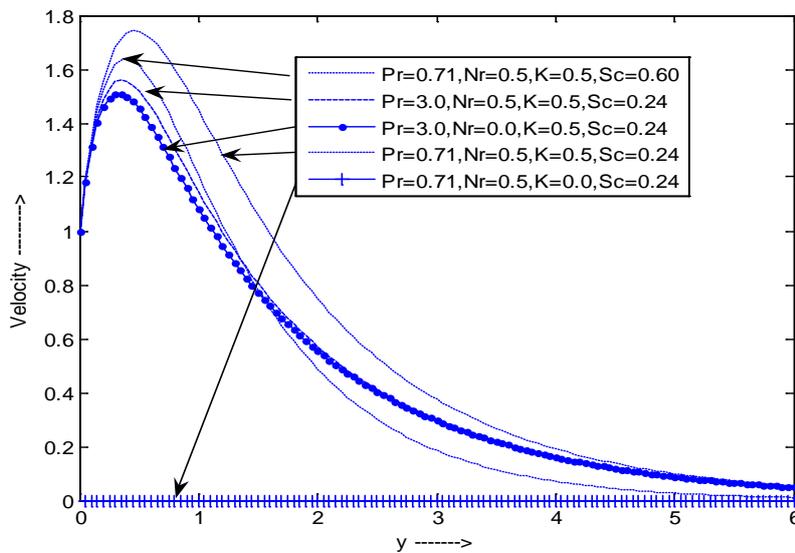


Fig-3: Effects of Pr, Nr, Sc, K on Velocity profile, when $Gr=5$, $Gm=6$, $S=1$, $M=0.5$, $\omega=0.1$, $n=0.5$, $t=1$, $\epsilon=0.02$.

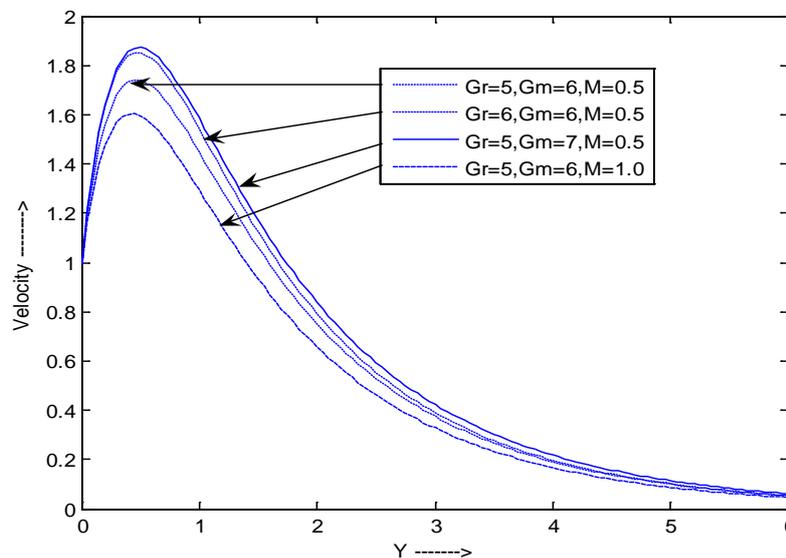


Fig-4: Effects of Gr, Gm, M on Velocity profile, when $Pr=0.71$, $Sc=0.24$, $S=1$, $Nr=0.5$, $K=0.5$, $\omega=0.1$, $n=0.5$, $t=1$, $\epsilon=0.02$.

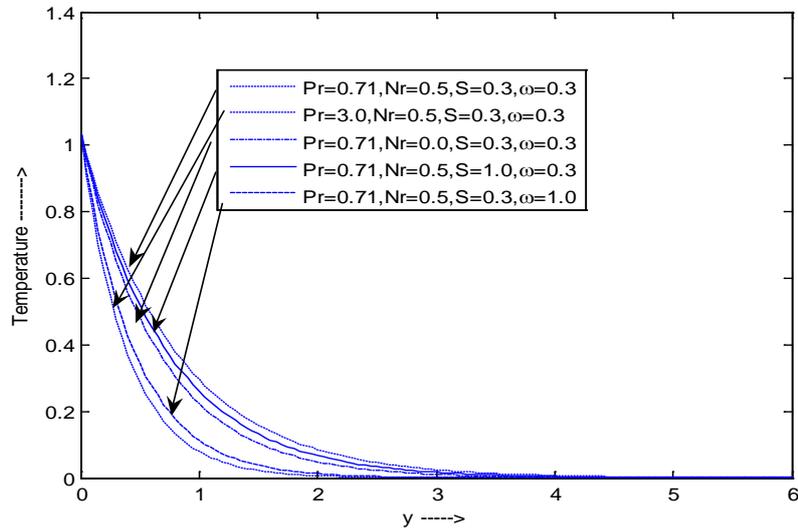


Fig-5: Effects of Pr, Nr, S, ω on temperature profile, when $n=0.5$, $t=1$, $\epsilon=0.02$.

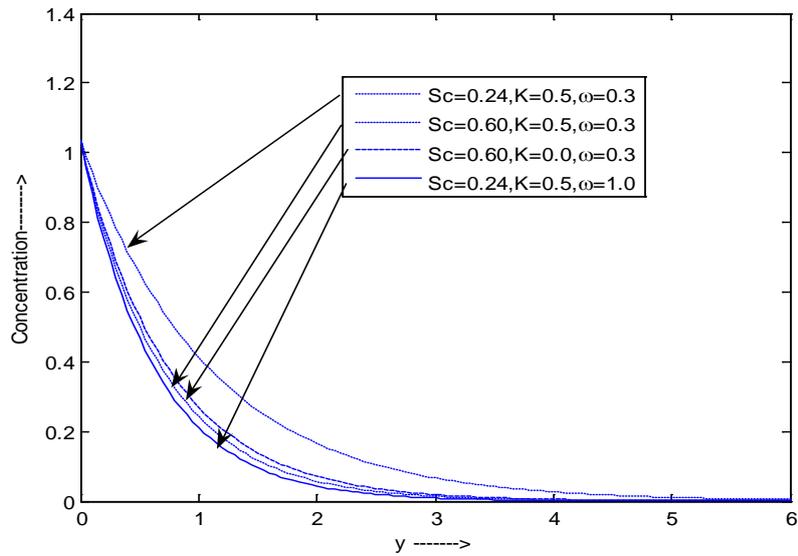


Fig-6: Effects of Sc, K, ω on mass concentration profile, when $n=0.5$, $t=1$, $\epsilon=0.02$.

Table-1: Effect of K, M, Gr, ω on Skin friction

| S/NO | Gr | M | K | ω | τ |
|------|----|-----|-----|----------|---------|
| 1 | 2 | 0.5 | 0.5 | 0.1 | -0.3378 |
| 2 | 3 | 0.5 | 0.5 | 0.1 | -0.8899 |
| 3 | 3 | 1.0 | 0.5 | 0.1 | -0.5824 |
| 4 | 2 | 0.5 | 0.5 | 0.3 | 0.1429 |
| 5 | 2 | 0.5 | 0.2 | 0.1 | 0.5237 |

Table-2: Effect of K, M, Gm, ω on Skin friction

| S/NO | Gm | M | K | ω | τ |
|------|----|-----|-----|----------|---------|
| 1 | 2 | 0.5 | 0.5 | 0.1 | -0.8899 |
| 2 | 3 | 0.5 | 0.5 | 0.1 | -1.4421 |
| 3 | 3 | 1.0 | 0.5 | 0.1 | -1.0989 |
| 4 | 2 | 0.5 | 0.5 | 0.3 | 0.1429 |
| 5 | 2 | 0.5 | 0.2 | 0.1 | 0.5237 |

Table-3: Effect of Pr, S, Nr, ω on Nusselt number

| S/NO | Pr | S | Nr | ω | Nu |
|------|------|-----|-----|----------|---------|
| 1 | 0.71 | 0.3 | 0.5 | 0.1 | -0.8103 |
| 2 | 3 | 0.3 | 0.5 | 0.1 | -1.6656 |
| 3 | 0.71 | 1.0 | 0.5 | 0.1 | -1.0051 |
| 4 | 0.71 | 0.3 | 0.0 | 0.1 | -0.9924 |
| 5 | 0.71 | 0.3 | 0.5 | 0.3 | -1.2910 |

Table-4: Effect of Sc, K, ω on Sherwood number

| S/NO | Sc | K | ω | Sh |
|------|------|-----|----------|---------|
| 1 | 0.24 | 0.5 | 0.1 | -0.6198 |
| 2 | 0.60 | 0.5 | 0.1 | -0.9800 |
| 3 | 0.24 | 0.0 | 0.1 | -0.5061 |
| 4 | 0.24 | 0.5 | 0.3 | -0.9467 |

REFERENCES:

- [1] Anjalidevi, S. P. and Kandarsamy, R. (2002). "Effects of Chemical Reaction, Heat or and mass Transfer for on Non-Linear MHD Laminar Boundary Layer Flow over a wedge with suction and injection". Intl. Comm. of Heat and mass Transfer Vol. 29, Pp. 707-716.
- [2] Acharya, M., Singh, L. P. and Dash G.G. (1999). "Heat and Mass Transfer on an Accelerating surface subjected to both power Law surface temperature and power law heat flux variations with Temperature dependent heat source in the presence of suction and injection". Int. J. Engr. Sc. Vol.37.Pp.189 – 211.
- [3] Beg Anwar, O. and Ghosh, S.K. (2010). "Analytical study of MHD radiation-convection with surface temperature oscillation and secondary flow effects". Int. J. of Applied Mathematical and Mech., Pp. 1-22.
- [4] Chamka, A. J. (2003), "MHD Flow of a Uniformly Stretched Vertical Permeable surface in the presence of heat generation/absorption and a chemical reaction". Int. Comm. Heat Mass Transf. Vol. 30, Pp. 413 – 422.
- [5] El-Amin, M. F. (2004). "Double Dispersion Effects on Natural Convection Heat and Mass Transfer in Non- Darcy Porous Medium". Applied Mathematics and Computer. Vol. 156, Pp. 1-7.
- [6] Mansour, M.A and Aly, A. A. (2009). "Effect of Chemical Reaction and Radiation on MHD Free Convection Heat and Mass Transfer from a Horizontal Cylinder of Elliptic Cross section saturated in a porous media with considering suction or injection". International Journal of Applied Mathematics and Mech., Vol. 5(2) Pp.
- [7] Muthucumaraswamy, R. (2001). "Effect of a Chemical Reaction on moving Isothermal Vertical surface with suction". Acta Mechanica, Vol. 147, Pp. 45-55.
- [8] Murti, A.S.N, Kemaswara, P.K and Porna, T. K. (2005). "Radiation, chemical reaction, Double dispersion effects on Heat and mass transfer in non-Newtonian fluids". International journal of Engineering (IJE), Volume 4: Issue (1) Pp.243-254.
- [9] Muthucumaraswamy, R. (2010). "Chemical Reaction Effects on Vertical Oscillating Plate with Variable Temperature". Chemical Industry and Engineering Quarterly, Vol. 16(2), Pp. 167-173.
- [10] Okedoye, A. M. and Ajala A. O. (2009). "MHD Free convection flow past on oscillating plate in the presence of heat generation/absorption and chemical reaction". Journal of the Nigerian Association of Mathematical Physics Vol.15, Pp. 491-500. [18] Pop, I. and Ingham D. B. (2001) "Convection Heat Transfer Mathematical and Computational Modeling of Viscous Fluids and Porous media". Pergomon, Oxford, Vol. 2, Pp. 127-139.
- [11] Seddeek, M. A., and Almushigh A. A. (2010). "Effects of Radiation and Variable Viscosity on MHD Free convective flow and mass transfer over a stretching sheet with chemical reaction". International Journal of Applications and Applied Mathematics, Vol. 5(1), Pp. 181 – 197.
- [12] Uwanta,I.J.,Isah,B.Y. and Ibrahim,M.O.(2011).''Radiative convection flow with chemical reaction''. International journal of Computer Application,Vol.36(2),pp.25-32.
- [13] Senapati,N., Dhal,R.K.,and Das,T.K.(2012).''Effects of Chemical reaction on free convection MHD flow through porous medium bounded by vertical surface with slip flow region''. American journal of computational and applied mathematics,2(3),pp.124-135.

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