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$\tau_i \tau_j$ - Q^{**} CLOSED SETS IN BITOPOLOGICAL SPACES

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ABSTRACT

In the present paper, we introduced τ_i τ_j-Q^{**} closed sets in bitopological spaces and studied its some of their bitopological properties. Also some relations are established with known generalized closed sets.

Keywords: *$\tau_i \tau_j$ - Q^{**} closed, $\tau_i \tau_j$ - Q^{**}open sets.*

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1. INTRODUCTION

A triple (X, τ_1, τ_2) where X is a non - empty set and τ_1, τ_2 are topologies on X is called a bitopological space and Kelly initiated the study of such spaces. Maheswari and prasad [11] introduced semi open sets in bitopological spaces in 1977.

Closed sets are fundamental objects in a topological space. For example one can define the topology on a set by using either the axioms for the closed sets or the Kuratowski closure axioms. In 1971, Levine [10] introduced the concept of generalized closed sets in topological spaces. Also he introduced the notion of semi open sets in topological spaces. Bhattacharyya and Lahiri [3] introduced a class of sets called semi generalized closed sets by means of semi open sets of Levine and obtained various topological properties.

In 1985, Fukutake [7] introduced the concepts of g - closed sets in bitopological spaces and after that several authors turned their attention towards generalizations of various concepts of topology by considering bitopological spaces.

In 1991, Chattopadhyay and Bandyopadhyay [2] introduced δ set. A subset A of a topological space is called δ set if int (cl (A)) $\subset$ cl (int (A)) .

In 2004 [23], Sheik john. M and Sundaram. P introduced g* closed sets in bitopological spaces .The notion of Q^* - closed sets in a topological space was introduced by Murugalingam and Lalitha [12] in 2010.

Recently, P. Padma and S .Udayakumar [12] introduced the concept of (τ_1, τ_2) * - Q^* closed sets in bitopological spaces .
In the present paper, we introduced τ_i τ_j-Q^{**} closed sets in bitopological spaces and studied its some of their bitopological properties. Also some relations are established with Known generalized closed sets.

2.1 PRELIMINARIES

Throughout this paper X and Y always represent nonempty bitopological spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2). For a subset A of X , τ_i - cl (A) , τ_i - Q^*cl (A) (resp. τ_i - int (A) .τ_i - Q^*int (A)) represents closure of A and Q^*closure of A (resp. interior of A, Q^* - interior of A) with respect to the topology τ_i. We shall now require the following known definitions.

Definition 2.2 - A set A of a bitopological space (X, τ_1, τ_2) is called

a) $\tau_i \tau_j$ - semi open if there exists an τ_i - open set U such that U$\subseteq$A $\subseteq$ τ_j - cl (A). Equivalently, a set A is $\tau_i \tau_j$ - semi open if A $\subset$ τ_j - cl (τ_i - int (A))
b) $\tau_i \tau_j$ - semi closed if X – A is $\tau_i \tau_j$ - semi open.
c) $\tau_i \tau_j$ - generalized open ($\tau_i \tau_j$ - g open) if X – A is $\tau_i \tau_j$ - generalized closed.
d) $\tau_i \tau_j$ - generalized closed ($\tau_i \tau_j$ - g closed) if τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - open in X.
e) $\tau_i \tau_j$ - generalized open ($\tau_i \tau_j$ - g open) if X – A is $\tau_1 \tau_2$ - g closed.
f) $\tau_i \tau_j$ - semi generalized closed ($\tau_i \tau_j$ - sg closed) if τ_j - scl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - semi open in X.

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g) $\tau_i\tau_j$ - semi generalized open ($\tau_i\tau_j$ - sg open) if X – A is $\tau_i\tau_j$ - sg closed.
h) $\tau_i\tau_j$ - generalized semi closed ($\tau_i\tau_j$ - gs closed) if τ_2 - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - open in X.
i) $\tau_i\tau_j$ - generalized semi open ($\tau_i\tau_j$ - gs open) if X – A is $\tau_i\tau_j$ - gs closed.
j) $\tau_i\tau_j$ - regular open if A = τ_i - int [τ_j - cl (A)].
k) $\tau_i\tau_j$ - regular closed if A = τ_i - cl [τ_j - int (A)].
l) $\tau_i\tau_j$ - g* closed sets if τ_j - cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is τ_i - g open in X.
m) $\tau_i\tau_j$ - g* open ($\tau_i\tau_j$- g* open) if X – A is $\tau_i\tau_j$ - g* closed.

3. $\tau_i\tau_j$ - Q^{**} CLOSED SETS

In this section, the concepts of $\tau_i\tau_j$ - Q^{**} closed sets is introduced and their basic properties in bitopological spaces are discussed. Recall that a set A os a bitopological space (X, τ_1 , τ_2) is called $\tau_i\tau_j$ - Q* closed if τ_i - int (A) = ϕ and A is τ_j - closed . The family of all $\tau_i\tau_j$ - Q^{**} closed subsets of a bitopological space (X, τ_1, τ_2) is denoted by (τ_i, τ_j) - Q^{**}.

Definition 3.1- A subset A of a bitopological space (X, τ_1, τ_2) is called
i) $\tau_i\tau_j$ - Q^{**} closed if τ_i - int (A) = ϕ and A is τ_j - Q* closed , where i, j = 1, 2 and i $\neq$ j .
ii) $\tau_i\tau_j$ - Q^{**} open if X – A is $\tau_i\tau_j$ - Q^{**} closed in X, where i, j = 1, 2 and i $\neq$ j.

Example 3.1. Let X = {a, b, c},τ_1 = {ϕ, X, {c}}, τ_2 = {ϕ, X, {b, c}, {c}}. Then τ_j - Q* closed sets are {a}, {a, b}, ϕ . Clearly ϕ, {a, b} and {a} are $\tau_i\tau_j$ - Q^{**} closed.

Definition 3.2. Let (X,τ_1, τ_2) be a bitopological spaces. Let A $\subset$ X. The intersection of all $\tau_i\tau_j$ - Q^{**} closed sets of X containing a subset A of X is called $\tau_i\tau_j$ - Q^{**} **closure of A** and is denoted by $\tau_i\tau_j$ - Q^{**} cl (A).

Definition 3.3. Let (X,τ_1, τ_2) be a bitopological spaces. Let A $\subset$ X. The union of all $\tau_i\tau_j$ - Q^{**} open sets contained in a subset A of X is called $\tau_i\tau_j$ - Q^{**} **interior of A** and is denoted by $\tau_i\tau_j$ - Q^{**} int (A).

Remark 3.1. Since every $\tau_i\tau_j$ - Q^{**} closed is τ_j - closed and τ_j - closed set is $\tau_i\tau_j$ - g closed, $\tau_i\tau_j$ - sg closed, we have $\tau_i\tau_j$ - Q^{**} closed is $\tau_i\tau_j$ - g closed, $\tau_i\tau_j$ - sg closed. But the converse is not true in general. The following example supports our claim.

Example 3.4. In example 3.1, {c} $\tau_i\tau_j$ - g closed, $\tau_i\tau_j$ - sg closed and $\tau_i\tau_j$ - gs closed but not $\tau_i\tau_j$ - Q^{**} closed.

Remark 3.3. Since every $\tau_i\tau_j$ - Q^{**} closed set is $\tau_i\tau_j$ - Q* closed. But the converse is not true in general. The following example supports our claim.

Example 3.5. Let X = {a, b, c} ,τ_1 = {ϕ, X, {a}}, τ_2= {ϕ, X, {a}, {b, c}}. Then τ_j - Q* closed sets are ϕ. Clearly {b, c} is $\tau_i\tau_j$ - Q* closed but it is not $\tau_i\tau_j$ - Q^{**} closed.

Proposition 3.1. If A B$\in$(τ_i, τ_j) - Q^{**} then A$\cup$B $\in$(τ_i , τ_j) - Q^{**}.

Proof: Let A and B be (τ_i, τ_j) - Q** closed sets in (X, τ_1, τ_2).

Claim: A $\cup$ B be a (τ_i , τ_j) - Q** closed sets in (X, τ_1,τ_2).
i.e) to prove τ_i - int (A$\cup$B) = ϕ and A is τ_j - Q* closed.

Since, A and B be (τ_i , τ_j) - Q* closed sets in (X, τ_1,τ_2) we have τ_i - int (A) = ϕ and A is τ_j - Q* closed and τ_i - int (B) = ϕ and B is τ_j - Q* closed.

Since (X, τ_1, τ_2) be a bitopological space, we have finite union of τ_j - Q* closed sets are τ_j - Q* closed.
$\Rightarrow$ τ_i - int (A $\cup$ B) = ϕ and A is τ_j - Q* closed.
$\Rightarrow$ A $\cup$ B is (τ_i, τ_j) - Q** closed sets in (X, τ_1, τ_2).
$\Rightarrow$ A $\cup$ B $\in$ (τ_i, τ_j) - Q*.

Proposition 3.2 - Every $\tau_i\tau_j$ - Q^{**} closed set is τ_j - closed.

Proof: Let A be a $\tau_i\tau_j$ - Q^{*} closed set in X.
Then X – A is $\tau_i\tau_j$ - Q^{*} open.

We have to show that

A is $\tau_i\tau_j$ - Q^{**}closed.

Since every $\tau_i \tau_j$ - Q^{**} open set is τ_j - open, we have X – A is τ_j - open.

Thus, A is τ_j - closed.

Remark 3.5. The converse of the above proposition is not true in general ie) τ_j - closed is not $\tau_i \tau_j$ - Q^{**} closed.

Remark 3.6. $\tau_i \tau_j$ - regular closed sets and $\tau_i \tau_j$ - Q^{**} closed sets are independent of each other in general. It is proved in the following example.

Example 3.6. X = {a, b, c},τ_1 = {ϕ, X, {b, c}}, τ_2= {ϕ, X, {a}, {b, c}}. Then {a} is $\tau_i \tau_j$ - regular closed but not $\tau_i \tau_j$ - Q^{**} closed set.

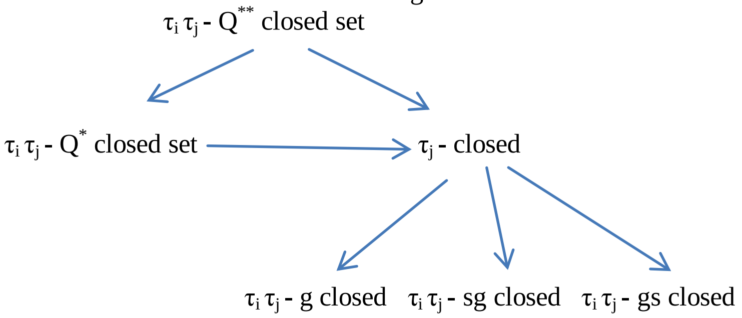
Remark 3.7. $\tau_i \tau_j$ - g* closed sets and $\tau_i \tau_j$ - Q^{**} closed sets are independent of each other in general. It is proved in the following example.

Example 3.6. X = {a, b, c},τ_1 = {ϕ, X, {c}, {a, c}}, τ_2= {ϕ, X, {a}}. Then {b} is $\tau_i \tau_j$ - g*closed but not $\tau_i \tau_j$ - Q^{**} closed set.

Remark 3.8. $\tau_i \tau_j$ - g* closed sets and $\tau_i \tau_j$ - Q^{*} closed sets are independent of each other in general. It is proved in the following example.

Example 3.7. X = {a, b, c},τ_1 = {ϕ, X, {c}, {a, c}} , τ_2 = {ϕ, X, {a}}. Then {b, c} is $\tau_i \tau_j$ - g* closed but not $\tau_i \tau_j$ - Q^{*} closed set.

Result 3.1. From the above results we conclude the following



Theorem 3.1. If A is $\tau_i \tau_j$ - Q^{**} closed then A is nowhere dense.

Proof: Since A is $\tau_i \tau_j$ - Q^{**} closed, we have τ_i - int (A) = ϕ and A is τ_j - Q* closed.

Therefore,

$$\tau_j - cl\,[\tau_i - int\,(A)] = \phi.$$

Hence A is nowhere dense.

Theorem 3.2. Every $\tau_i \tau_j$ - Q^{**} closed set is $\tau_i \tau_j$ - δ set.

Proof: Let A be $\tau_i \tau_j$ - Q^{**} closed.

Then τ_i - int (A) = ϕ and A is τ_j - Q* closed.

Consequently,

$$\begin{aligned}\tau_i - int\,\{\tau_j - cl\,[\tau_i - int\,(A)]\} &= \tau_i - int\,\{\tau_j - cl\,(\phi)\}\\ &= \tau_i - int\,(\phi)\\ &= \phi\,.\end{aligned}$$

Therefore, A is $\tau_i \tau_j$ - δ set.

Theorem 3.3.
a) Every $\tau_i \tau_j$ - Q* closed set is $\tau_i \tau_j$ - semi closed set.
b) Every $\tau_i \tau_j$ - Q** closed set is $\tau_i \tau_j$ - semi closed set.

Remark 3.7. But the converse of the assertions of above theorem are not true in general as can be seen in the following example.

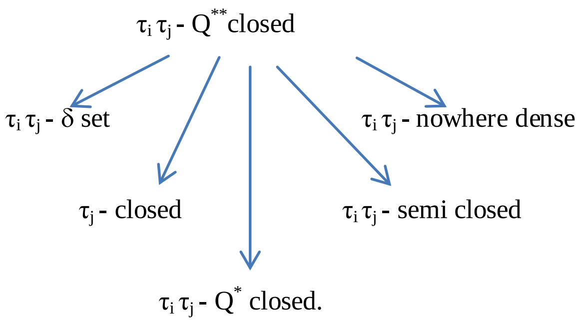
Example 3.7. Let X = {a, b, c}, τ_1 = {ϕ, X, {a}, {b, c}} and τ_2 = {ϕ, X, {a, c}}. Then {a} is a $\tau_i \tau_j$ - semi closed set, but not $\tau_i \tau_j$ - Q* closed and $\tau_i \tau_j$ - Q** closed.

Theorem 3.4. $(\tau_i \tau_j - Q^{**} \text{ int } A)^c = \tau_i \tau_j - Q^{**} \text{ cl } (A^c)$.

Proof: $\tau_i\tau_j$ - Q** int A = $\cup$ {B/B is $\tau_i \tau_j$ - Q** open and B$\subset$A}.

$$\begin{aligned}(\tau_i \tau_j - Q^{**} \text{ int } A)^c &= \cup\{B/B \text{ is } \tau_i \tau_j - Q^{**} \text{ open and } B \subset A\}^c.\\ &= \cap\{ B^c/B^c \text{ is } \tau_i \tau_j - Q^{**} \text{ open and } B \subset A\}\\ &= \cap\{B^c/B^c \text{ is } \tau_i \tau_j - Q^{**} \text{ closed and } A^c \subset B^c\}\\ &= \cap \{F/F \text{ is } \tau_i \tau_j - Q^{**} \text{ closed and } A^c \subset F\}\\ &= \tau_i \tau_j - Q^{**} \text{ cl } (A^c).\end{aligned}$$

Result 3.2. The relationship between $\tau_i \tau_j$ - Q^{**} closed sets and other generalizations is given by the following figure



4. $\tau_i \tau_j$ - Q^{**} OPEN SETS

Definition 4.1. A subset A of a bitopological spaces (X, τ_1, τ_2) is called $\tau_i \tau_j$ - Q^{**} open if X – A is $\tau_i \tau_j$ - Q^{**} closed in X .

Example 4.1. In example 3.1, X, {c}, {a, b} are $\tau_i \tau_j$ - Q^{**} open.

Remark 4.1. Since every $\tau_i \tau_j$ - Q^{**}open set is τ_j - open and every τ_j - open set is $\tau_i \tau_j$ - g open, $\tau_i \tau_j$ - sg open, $\tau_i \tau_j$ - gs we have every $\tau_i \tau_j$ - Q^{**}open set is $\tau_i \tau_j$ - g open, $\tau_i \tau_j$ - sg open and $\tau_i \tau_j$ - gs open . But the converse need not be true in general. The following example supports our claim.

Example 4.2. In example 3.1,{a, b} is $\tau_i \tau_j$ - g open , $\tau_i \tau_j$ - sg open and $\tau_i \tau_j$ - gs open but not $\tau_i \tau_j$ - Q^{**}open.

Theorem 4.1. A set A of a bitopological space (X, τ_1,τ_2) is $\tau_i \tau_j$ - Q^{**}open if and only if τ_i - cl (A) = X and A is τ_j - Q* open.

Proof: Necessity: Suppose that A is $\tau_i \tau_j$ - Q^{**}open.

Then A^c is $\tau_i \tau_j$ - Q^{**}closed.

Therefore,

$$\tau_i - \text{int } (A^c) = [\tau_i - \text{cl } (A)]^c = \phi \text{ and } A^c \text{ is } \tau_j - Q^* \text{ closed.}$$

Consequently,

$$\tau_i - \text{cl } (A) = X \text{ and } A \text{ is } \tau_j - Q^* \text{ open.}$$

Sufficiency: Suppose that τ_i - cl (A) = X and A is τ_j - Q* open.

Then $[\tau_i - \text{cl } (A)]^c = \tau_i - \text{int } (A^c) = \phi$ and A^c is τ_j - Q* closed.

Consequently,

$$A^c \text{ is } \tau_i \tau_j - Q^{**} \text{ closed.}$$

This completes the proof.

Corollary 4.1. A set A of a bitopological space (X, τ_1 ,τ_2) is $\tau_i \tau_j$ - Q^{**}open if and only if A is τ_i - dense and τ_j - open.

Theorem 4.2. If A and B are $\tau_i \tau_j$ - Q^{**}open sets then so is A $\cap$ B.

Proof: Suppose that A and B are $\tau_i \tau_j$ - Q^{**}open sets.

Then A^c and B^c are $\tau_i\tau_j$ - Q^{**}closed sets.

Therefore,

$$A^c \cup B^c \text{ is } \tau_i\tau_j \text{ - } Q^{**}\text{closed sets.}$$

But $A^c \cup B^c = (A \cap B)^c$.

Hence $A \cap B$ $\tau_i\tau_j$ - Q^{**}open.

Theorem 4.3.

i) X is not $\tau_i\tau_j$ - Q^{**}closed.
ii) ϕ is $\tau_i\tau_j$ - Q^{**}closed.
iii) X is $\tau_i\tau_j$ - Q^{**}open
iv) X is not $\tau_i\tau_j$ - Q^{**}open.

Remark 4.3. It is obvious that every $\tau_i\tau_j$ - Q^{**}open set is τ_j - open, but the converse is not true in general.

Remark 4.4. Every $\tau_i\tau_j$ - Q^{**}open is $\tau_i\tau_j$ -semi open. But the converse need not be true. The following example supports our claim.

Example 4.5. In example 3.2, {b.c} is $(\tau_1,\tau_2)^*$ - semi open but not $\tau_i\tau_j$ - Q^{**}open.

Remark 4.5. $\tau_i\tau_j$ - g* open sets and $\tau_i\tau_j$ - Q^{**}open sets are independent of each other in general. It is proved in the following example.

Example 3.6. X = {a, b, c},τ_1 ={ϕ, X, {c}, {a, c}}, τ_2= {ϕ, X, {a}}. Then {a, c} is $\tau_i\tau_j$ - g*open but not $\tau_i\tau_j$ - Q^{**}open set.

Remark 3.8. $\tau_i\tau_j$ - g* open sets and $\tau_i\tau_j$ - Q^{*}open sets are independent of each other in general. It is proved in the following example.

Example 3.7. X = {a, b, c},τ_1 = {ϕ, X, {c}, {a, c}}, τ_2 = {ϕ, X, {a}}. Then {a} is $\tau_i\tau_j$ - g*open but not $\tau_i\tau_j$ - Q^{*} open set.

Theorem 4.4. If $B \subset A \subset X$, where A is $\tau_i\tau_j$ - Q^{**}open and B is $\tau_i\tau_j$ - Q^{**}open in A then B is $\tau_i\tau_j$ - Q^{**}open in X.

Proof: Since B is τ_j - open in A, A is τ_j - open in X and B is τ_j - open in X.

We claim that τ_i - cl (B) = X.

Let U be any τ_j - Q** open set.

Since τ_1 - cl (B) is A, $(U \cap A) \cap B \neq \phi$.

Then

$$(U \cap A) \cap B \neq \phi.$$

Hence

$$\tau_1 \text{ - cl (B)} = X.$$

Therefore,

B is $\tau_i\tau_j$ - Q^{**}open in X.

Theorem 4.5. If A and B are τ_2 - open sets with $A \cap B = \phi$ then A and B are not $\tau_i\tau_j$ - Q^{**}open.

Proof: Since $A \cap B = \phi$, the points of B cannot be limit points of A.

Then τ_1 - cl (A) $\neq$ X.

Hence A is not $\tau_i\tau_j$ - Q^{**}open.

Similarly, B is not $\tau_i\tau_j$ - Q^{**}open.

Theorem 4.6. Let (X, τ_1,τ_2) be a hyper connected bitopological space. Let $A \subset X$. If A is τ_j - open then A is τ_i τ_j - Q^{**}open in X.

Proof: It is enough to prove that A is τ_i - dense.

Suppose that τ_i - cl (A) $\neq$ X.

Then $[\tau_i$ - cl $(A)]^c \neq \phi$.

Consequently,

$$A \cap [\tau_i - cl\ (A)]^c \neq \phi.$$

This is a contradiction to the fact that (X, τ_1, τ_2) is a hyper connected bitopological space.

Hence A is τ_i - dense.

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