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COMMON FIXED POINT THEOREM ON M-FUZZY METRIC SPACE USING THE CONCEPT OF COMPATIBILITY OF TYPE (P)

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ABSTRACT

In this paper we prove a common fixed point theorem for compatible mappings of type (P) in M-fuzzy metric space modifying the result of M.Koireng and YumnamRohen [6] proved in fuzzy metric space.

Key words: *fixed point, compatible mapping of type (P), M-fuzzy metric space.*

Mathematics Subject Classification: *47H10, 54H25.*

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1. INTRODUCTION

In 1965, Zadeh [15] introduced the concept of fuzzy sets. In 1975, this basic tool impressed Kramosil and Michalek[7] to introduce the concept of fuzzy metric space, which was further extended by many authors as in 1994 George and Veeramani [4], in 1988 Grabiec [5], in 1995 Subrahmanyam[12] and in 1999Vasuki[13]. Following the track in 1992 Dhage [1, 2, 3] put forward the theory of D- metric space to prove fixed point theorems which were un fortunately not valid. Keeping in mind in beginning of the century Sedgiand Shobe [8] introduced D*-metric space by altering the tetrahedron inequality in D-metric and using D*-metric analogy, they defined M-fuzzy metric space and studied some fixed point theorems. Being the active field of research, researchers as S. S. Chauhan[9], S. S. Chauhan and Nidhi Joshi [10], S. S. Chauhan and Kiran Utreja [11]T. Veerapandi [14] proved their results in fixed point theorems using different contractive conditions. In this paper we are extending the result proved by M. Koireng and Yumnam Rohen [6] using the concept of compatible mappings in Fuzzy Metric Space to M-Fuzzy Metric Space.

2. PRELIMINARIES

Definition: 2.1 A binary operation *: [0, 1] × [0, 1]→[0, 1] is a continuous t- norm if it satisfies the following conditions

i. *is associative and commutative,
ii. * is continuous,
iii. a*1 = a for all a ∈ [0, 1],
iv. a*b≤ c *d whenever a≤c and b ≤ d ∀ a, b, c, d ∈ [0, 1].

Definition: 2.2 A 3-tuple(X, M, *) is called a M_ fuzzy metric space if X is an arbitrary (non- empty) set, * is a continuous t- norm, and M is a fuzzy set on X^3×(0,∞), satisfying the following conditions for eachx, y, z, a ∈X and t, s > 0,

i. M(x, y, z, t) > 0,
ii. M(x, y, z, t) = 1 if and only if x = y= z,
iii. M(x, y, z, t) = M(p{x, y, z}, t), (symmetry) where p is a permutation function,
iv. M(x, y, a, t) * M(a, z, z, s) ≤ M(x, y, z, t +s),
v. M(x, y, z, .): (0, ∞)→[0, 1] is a continuous,
vi. $\lim_{t\to\infty} M(x, y, z, t)$ = 1.

Lemma2.1. Let (X, M, *) be M- fuzzy metric space, then for each t > 0 and for every x, y ∈ X we have M(x, x, y, t) = M(x, y, y, t).

Lemma2.2 Let (X, M, *) be M- fuzzy metric space, then M(x, y, z, t) is non-decreasing w.r.t t ∀ x, y, z, a ∈ X.

Proof: For each x, y, z, a∈ X and s, t > 0, we have M(x, y, a, t) * M (a, z, z, s) ≤ M(x, y, z, t +s).

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If we set a = z, we get M(x, y, z, t) * M (z, z, z, s) ≤ M(x, y, z, t +s).

This implies M(x, y, z, t) * 1 ≤ M(x, y, z, t +s).

This implies M(x, y, z, t+s) ≥ M(x, y, z, t).

Thus M(x, y, z, t) is a non-decreasing with respect to t ∀ x, y, z, a ∈ X.

Lemma2.3. Let $\{x_n\}$ be a sequence in M-fuzzy metric space (X, M, *) with the vi condition .If ther exists a number q ∈ (0, 1) such that

M $(x_n, x_n, x_{n+1}, t) \geq M(x_{n-1}, x_{n-1}, x_n, {}^{t}/_{q})$∀ t > 0 and n = 1, 2, 3…... then $\{x_n\}$ is a Cauchy sequence.

Proof: By the simple induction, we have ∀ t > 0 and n = 1, 2, 3…..

$$\begin{aligned} M(x_n, x_n, x_{n+1}, t) &\geq M(x_{n-1}, x_{n-1}, x_n, t/q) \\ &\geq M(x_{n-2}, x_{n-2}, x_{n-1}, t/q^2) \\ &\ldots\ldots\ldots\ldots\ldots\ldots \\ &\geq M(x_0, x_0, x_1, t/q^n). \end{aligned}$$

$$M(x_n, x_n, x_{n+1}, t) \geq M(x_0, x_0, x_1, t/q^n).$$

Thus by axiom iv and i for any positive integer p and t > 0, we get

$$\begin{aligned} M(x_n, x_n, x_{n+p}, t) &\geq M(x_n, x_n, x_{n+1}, t/p) * \ldots\text{P–times}\ldots M(x_{n+p-1}, x_{n+p-1}, x_{n+p}, t/p) \\ &\geq M(x_0, x_0, x_1, t/pq^n)\ldots\text{p-times}\ldots M(x_0, x_0, x_1, t/pq^{n+p-1}) \end{aligned}$$

Thus by (vi) axiom, we get $\lim_{n\to\infty} M\left(x_n, x_n, x_{n+p}, t\right) \geq 1 * \ldots p - times \ldots * 1$.

Which implies that $\{x_n\}$ is a Cauchy sequence in M-fuzzy metric space X.

Lemma2.4.Let (X, M, *) be M-fuzzy metric space with the condition (vi). If for all x, y, z ∈X, t >0 with positive number q ∈ (0, 1) and M(x, y, z, qt) ≥ M(x, y, z, t), then x = y = z.

Proof: M(x, y, z, t) ≥ M(x, y, z, t/q)
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…...………..
≥ M(x, y, z, t/q^n)→1 as n → ∞.
Hence by ii, x = y= z.

Definition: 2.3 Let (X, M, *) be M-fuzzy metric space and $\{x_n\}$ be a sequence in X.
(a) $\{x_n\}$ is said to converges to a point x ∈ X if $\lim_{n\to\infty} M(x, x, x_n, t) = 1$∀ t > 0.
(b) $\{x_n\}$ is said to Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_{n+p}, x_n, t) = 1$∀ t > 0 and p > 0.
(c) An M-fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

3. MAIN THEOREM

Theorem 3.1: Let (X,M,*) be a complete M- fuzzy metric space and let A, B, P, Q, S and T be self–mappings of X satisfying the following conditions:

i. P(X)⊆ ST(X) ; Q(X) ⊆ AB(X);
ii. The pair {P, AB} and {Q, ST} are compatible mappings of type P.
iii. ST is continuous;
iv. M(Px, Qz, Qz, qt) ≥ min { M(ABx, Py, Qy, t), M(ABx, Py, STz, t), M(Qy, STz, Py, t), M(ABx, Qy, STz, t)}

then the mappings P, Q, AB and ST have a common fixed point in X.

Proof: Let $x_0$be any arbitrary point in X. Thus we construct a sequence $\{y_n\}$ in X such that
$y_{2n-1} = STx_{2n-1} = Px_{2n-2}$ andy_{2n}= ABx_{2n}= Qx_{2n-1}

Put x =x_{2n-1} , y = x_{2n-1}, , z $= x_{2n}$.

M(Px_{2n-1}, Qx_{2n}, Qx_{2n}, qt) ≥ min {M(ABx_{2n-1}, Px_{2n-1}, Qx_{2n-1}, t), M(ABx_{2n-1}, Px_{2n-1}, STx_{2n}, t),
M(Qx_{2n-1}, STx_{2n}, Px_{2n-1}, t), M(ABx_{2n-1}, Qx_{2n-1}, STx_{2n}, t)}.'

M($y_{2n}, y_{2n+1}, y_{2n+1}, qt$) ≥ min { M($y_{2n-1}$,$y_{2n}$, y_{2n}, t), M(y_{2n-1},y_{2n}, y_{2n}, t), M(y_{2n}, y_{2n}, y_{2n}, t),
M(y_{2n-1}, y_{2n}, y_{2n}, t)}.

M($y_{2n}, y_{2n+1}, y_{2n+1}, q\ t$) ≥ M($y_{2n-1}$, y_{2n}, y_{2n}, t).

This implies that M($y_{2n}, y_{2n+1}, y_{2n+1}, t$) is an increasing sequence of positive real numbers. Now to prove that M($y_n, y_{n+1}, y_{n+1}\ , t$) converges to 1 as n→ ∞.

By lemma (3) M($y_n, y_{n+1}, y_{n+1}\ , t$) ≥ M($y_{n-1}, y_n, y_n\ , t/q$)
≥ M ($y_{n-2}, y_{n-1}, y_{n-1}\ , t/q^2$)
………………………………
≥M($y_0, y_1,\ y_1\ , t/q^n$).

Thus M($y_n, y_{n+1}, y_{n+1}\ , t$) ≥ M($y_0, y_1, y_1\ , t/q^n$).

Then by axiom (iv) of M- fuzzy metric space.

M ($y_n, y_{n+p}, y_{n+p}\ , t$) ≥ M ($y_n, y_{n+1}, y_{n+1}\ , t/p$)*…p-times…M ($y_{n+p-1}, y_{n+p-1}, y_{n+p}\ , t/p$)
≥ M ($y_0, y_1, y_1\ , t/q^n$)…p-times…M($y_0, y_1, y_1\ , t/pq^{n+p-1}$)

Thus by vi axiom of M-fuzzy metric space,

$\lim_{n\to\infty}$ M ($y_n, y_{n+p}, y_{n+p}\ , t$) ≥1*…p-times…1.$\lim_{n\to\infty}$ M ($y_n, y_{n+p}, y_{n+p}\ , t) = 1$.

Thus $\{y_n\}$ is a Cauchy sequence in M- fuzzy metric space X. Since X is complete, there exist a point u∈ X such that $y_n \to u$.

Thus {ABx_{2n}}, {Qx_{2n-1}},$\{STx_{2n-1}\}$,$\{Px_{2n-2}\}$are Cauchy sequences converging to u.

Put x = ABx_{2n}, y = u, z = STx_{2n-1} in (iv), we get

M(PABx_{2n}, QSTx_{2n-1}, QSTx_{2n-1}, qt) ≥ min {M(ABABx_{2n}, Pu, Qu, t), M(ABABx_{2n}, Pu, STSTx_{2n-1}, t),
M(Qu, STSTx_{2n-1}, Pu, t), M(ABABx_{2n}, Qu, STSTx_{2n-1}, t)}.

Now take the limit as n→ ∞ and using (ii) we get

M(Pu, Qu, Qu, qt) ≥ min{M(Pu, Pu, Qu, t), M(Pu, Pu, Qu, t), M(Qu, Qu, Pu, t),M(Pu, Qu, Qu, t)}.

Then by lemma (i) we get

M(Pu, Qu, Qu, qt) ≥ M(Pu, Qu, Qu, t).

Therefore Pu = Qu.

Now put x = ABx_{2n}, y =x_{2n-1}, z =x_{2n-1}in (iv), we get

M(P ABx_{2n}, Qx_{2n-1}, Qx_{2n-1}, qt) ≥ min {M(ABABx_{2n}, Px_{2n-1}, Qx_{2n-1}, t), M(ABABx_{2n}, Px_{2n-1}, STx_{2n-1}, t),
M(Qx_{2n-1}, STx_{2n-1}, Px_{2n-1}, t), M(ABABx_{2n}, Qx_{2n-1}, STx_{2n-1}, t)}.

Now on taking the limit as n→ ∞and on using (ii) we get

M(Pu, u, u, qt) ≥ min { M(Pu, u, u, t), M(Pu, u, u, t), M(u, u, u, t), M(Pu, u, u, t)}.

Thus we have M(Pu, u, u, qt) ≥ M(Pu, u, u, t).

Therefore Pu = u.

This implies Pu = Qu = u.

Now put x = Px_{2n-2}, y = Px_{2n-2}, z = u in (iv) we get

M(P Px_{2n-2}, Qu, Qu, qt) ≥ min {M(ABPx_{2n-2}, PPx_{2n-2}, QPx_{2n-2}, t), M(ABPx_{2n-2}, PPx_{2n-2}, STu, t),
M(QPx_{2n-2}, STu, PPx_{2n-2}, t), M(ABPx_{2n-2}, QPx_{2n-2}, STu, t)}.

Now take the limit as n→ ∞and on using (ii) & (iii) we get

M(ABu, u, u, qt) ≥ min { M(ABu, ABu, u, t), M(ABu, ABu, u, t), M(Qu, u, ABu, t), M(ABu, Qu, u, t)}.

This implies M(ABu, u, u, qt) ≥ min { M(ABu, ABu,u, t), M(ABu, ABu, u, t), M(u, u, ABu, t), M(ABu, u, u, t)}.

Therefore by lemma 1 we have ABu = u.

Thus Pu = Qu = ABu= u.

Put x = u, y = u, z = Qx_{2n-1} in (iv) we get

M(Pu, QQx_{2n-1}, QQx_{2n-1}, qt) ≥ min {M(ABu, Pu, Qu, t), M(ABu, Pu, STQx_{2n-1}, t),
M (Qu, ST Qx_{2n-1}, Pu, t), M(ABu, Qu, ST Qx_{2n-1}, t)}.

Take the limit as limit as n→ ∞and on using (ii) & (iii) we get

M(u, STu, STu, qt) ≥ min { M(u, u, u, t), M(u, u, STu, t), M(u, STu, u, t), M(u, u, STu, t)}.

On using lemma 1 we have M (STu, STu, u qt) ≥ M(STu, STu, u, t).

Thus STu= u.

We get Pu = Qu= ABu= STu= u.

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