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COMMON FIXED POINT THEOREM ON M-FUZZY METRIC SPACE USING THE CONCEPT OF COMPATIBILITY OF TYPE (P)

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ABSTRACT

In this paper we prove a common fixed point theorem for compatible mappings of type (P) in M-fuzzy metric space modifying the result of M.Koireng and YumnamRohen [6] proved in fuzzy metric space.

Key words: *fixed point, compatible mapping of type (P), M-fuzzy metric space.*

Mathematics Subject Classification: *47H10, 54H25.*

1. INTRODUCTION

In 1965, Zadeh [15] introduced the concept of fuzzy sets. In 1975, this basic tool impressed Kramosil and Michalek[7] to introduce the concept of fuzzy metric space, which was further extended by many authors as in 1994 George and Veeramani [4], in 1988 Grabiec [5], in 1995 Subrahmanyam[12] and in 1999Vasuki[13]. Following the track in 1992 Dhage [1, 2, 3] put forward the theory of D- metric space to prove fixed point theorems which were un fortunately not valid. Keeping in mind in beginning of the century Sedgiand Shobe [8] introduced D*-metric space by altering the tetrahedron inequality in D-metric and using D*-metric analogy, they defined M-fuzzy metric space and studied some fixed point theorems. Being the active field of research, researchers as S. S. Chauhan[9], S. S. Chauhan and Nidhi Joshi [10], S. S. Chauhan and Kiran Utreja [11]T. Veerapandi [14] proved their results in fixed point theorems using different contractive conditions. In this paper we are extending the result proved by M. Koireng and Yumnam Rohen [6] using the concept of compatible mappings in Fuzzy Metric Space to M-Fuzzy Metric Space.

2. PRELIMINARIES

Definition: 2.1 A binary operation *: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t- norm if it satisfies the following conditions

i. *is associative and commutative,
ii. * is continuous,
iii. $a*1 = a$ for all $a \in [0, 1]$,
iv. $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$ $\forall$ a, b, c, d $\in [0, 1]$.

Definition: 2.2 A 3-tuple(X, M, *) is called a M_ fuzzy metric space if X is an arbitrary (non- empty) set, * is a continuous t- norm, and M is a fuzzy set on $X^3 \times (0,\infty)$, satisfying the following conditions for eachx, y, z, a $\in$X and t, s > 0,

i. $M(x, y, z, t) > 0$,
ii. $M(x, y, z, t) = 1$ if and only if $x = y = z$,
iii. $M(x, y, z, t) = M(p\{x, y, z\}, t)$, (symmetry) where p is a permutation function,
iv. $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t+s)$,
v. $M(x, y, z, .): (0, \infty) \rightarrow [0, 1]$ is a continuous,
vi. $\lim_{t\to\infty} M(x, y, z, t) = 1$.

Lemma2.1. Let (X, M, *) be M- fuzzy metric space, then for each t > 0 and for every x, y $\in$ X we have $M(x, x, y, t) = M(x, y, y, t)$.

Lemma2.2 Let (X, M, *) be M- fuzzy metric space, then M(x, y, z, t) is non-decreasing w.r.t t $\forall$ x, y, z, a $\in$ X.

Proof: For each x, y, z, a$\in$ X and s, t > 0, we have $M(x, y, a, t) * M(a, z, z, s) \leq M(x, y, z, t+s)$.

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If we set a = z, we get M(x, y, z, t) * M (z, z, z, s) ≤ M(x, y, z, t +s).

This implies M(x, y, z, t) * 1 ≤ M(x, y, z, t +s).

This implies M(x, y, z, t+s) ≥ M(x, y, z, t).

Thus M(x, y, z, t) is a non-decreasing with respect to t ∀ x, y, z, a ∈ X.

Lemma2.3. Let $\{x_n\}$ be a sequence in M-fuzzy metric space (X, M, *) with the vi condition .If ther exists a number q ∈ (0, 1) such that

M $(x_n, x_n, x_{n+1}, t) \geq M(x_{n-1}, x_{n-1}, x_n, {}^{t}/_{q})$∀ t > 0 and n = 1, 2, 3…... then $\{x_n\}$ is a Cauchy sequence.

Proof: By the simple induction, we have ∀ t > 0 and n = 1, 2, 3…..

$$
\begin{aligned}
M(x_n, x_n, x_{n+1}, t) &\geq M(x_{n-1}, x_{n-1}, x_n, t/q)\\
&\geq M(x_{n-2}, x_{n-2}, x_{n-1}, t/q^2)\\
&\quad \ldots\ldots\ldots\ldots\ldots\ldots\\
&\geq M(x_0, x_0, x_1, t/q^n).
\end{aligned}
$$

$$M(x_n, x_n, x_{n+1}, t) \geq M(x_0, x_0, x_1, t/q^n).$$

Thus by axiom iv and i for any positive integer p and t > 0, we get

$$
\begin{aligned}
M(x_n, x_n, x_{n+p}, t) &\geq M(x_n, x_n, x_{n+1}, t/p) * \ldots\text{P–times}\ldots M(x_{n+p-1}, x_{n+p-1}, x_{n+p}, t/p)\\
&\geq M(x_0, x_0, x_1, t/pq^n)\ldots\text{p-times}\ldots M(x_0, x_0, x_1, t/pq^{n+p-1})
\end{aligned}
$$

Thus by (vi) axiom, we get $\lim_{n\to\infty} M\left(x_n, x_n, x_{n+p}, t\right) \geq 1 * \ldots p - times \ldots * 1$.

Which implies that $\{x_n\}$ is a Cauchy sequence in M-fuzzy metric space X.

Lemma2.4.Let (X, M, *) be M-fuzzy metric space with the condition (vi). If for all x, y, z ∈X, t >0 with positive number q ∈ (0, 1) and M(x, y, z, qt) ≥ M(x, y, z, t), then x = y = z.

Proof: M(x, y, z, t) ≥ M(x, y, z, t/q)
…………….
…...………..
≥ M(x, y, z, t/q^n)→1 as n → ∞.
Hence by ii, x = y= z.

Definition: 2.3 Let (X, M, *) be M-fuzzy metric space and $\{x_n\}$ be a sequence in X.
(a) $\{x_n\}$ is said to converges to a point x ∈ X if $\lim_{n\to\infty} M(x, x, x_n, t) = 1$∀ t > 0.
(b) $\{x_n\}$ is said to Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x_{n+p}, x_n, t) = 1$∀ t > 0 and p > 0.
(c) An M-fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

3. MAIN THEOREM

Theorem 3.1: Let (X,M,*) be a complete M- fuzzy metric space and let A, B, P, Q, S and T be self–mappings of X satisfying the following conditions:

i. P(X)⊆ ST(X) ; Q(X) ⊆ AB(X);
ii. The pair {P, AB} and {Q, ST} are compatible mappings of type P.
iii. ST is continuous;
iv. M(Px, Qz, Qz, qt) ≥ min { M(ABx, Py, Qy, t), M(ABx, Py, STz, t), M(Qy, STz, Py, t), M(ABx, Qy, STz, t)}

then the mappings P, Q, AB and ST have a common fixed point in X.

Proof: Let $x_0$be any arbitrary point in X. Thus we construct a sequence $\{y_n\}$ in X such that
$y_{2n-1} = STx_{2n-1} = Px_{2n-2}$ andy_{2n}= ABx_{2n}= Qx_{2n-1}

Put x =x_{2n-1} , y = x_{2n-1}, , z $= x_{2n}$.

M(Px_{2n-1}, Qx_{2n}, Qx_{2n}, qt) ≥ min {M(ABx_{2n-1}, Px_{2n-1}, Qx_{2n-1}, t), M(ABx_{2n-1}, Px_{2n-1}, STx_{2n}, t),
M(Qx_{2n-1}, STx_{2n}, Px_{2n-1}, t), M(ABx_{2n-1}, Qx_{2n-1}, STx_{2n}, t)}.'

M($y_{2n}, y_{2n+1}, y_{2n+1}, qt$) ≥ min { M($y_{2n-1}$,$y_{2n}$, y_{2n}, t), M(y_{2n-1},y_{2n}, y_{2n}, t), M(y_{2n}, y_{2n}, y_{2n}, t),
M(y_{2n-1}, y_{2n}, y_{2n}, t)}.

M($y_{2n}, y_{2n+1}, y_{2n+1}, q\ t$) ≥ M($y_{2n-1}$, y_{2n}, y_{2n}, t).

This implies that M($y_{2n}, y_{2n+1}, y_{2n+1}, t$) is an increasing sequence of positive real numbers. Now to prove that M($y_n, y_{n+1}, y_{n+1}\ , t$) converges to 1 as n→ ∞.

By lemma (3) M($y_n, y_{n+1}, y_{n+1}\ , t$) ≥ M($y_{n-1}, y_n, y_n\ , t/q$)
≥ M ($y_{n-2}, y_{n-1}, y_{n-1}\ , t/q^2$)
………………………………
≥M($y_0, y_1,\ y_1\ , t/q^n$).

Thus M($y_n, y_{n+1}, y_{n+1}\ , t$) ≥ M($y_0, y_1, y_1\ , t/q^n$).

Then by axiom (iv) of M- fuzzy metric space.

M ($y_n, y_{n+p}, y_{n+p}\ , t$) ≥ M ($y_n, y_{n+1}, y_{n+1}\ , t/p$)*…p-times…M ($y_{n+p-1}, y_{n+p-1}, y_{n+p}\ , t/p$)
≥ M ($y_0, y_1, y_1\ , t/q^n$)…p-times…M($y_0, y_1, y_1\ , t/pq^{n+p-1}$)

Thus by vi axiom of M-fuzzy metric space,

$\lim_{n\to\infty}$ M ($y_n, y_{n+p}, y_{n+p}\ , t$) ≥1*…p-times…1.$\lim_{n\to\infty}$ M ($y_n, y_{n+p}, y_{n+p}\ , t$) $= 1$.

Thus $\{y_n\}$ is a Cauchy sequence in M- fuzzy metric space X. Since X is complete, there exist a point u∈ X such that $y_n \to u$.

Thus {ABx_{2n}}, {Qx_{2n-1}},$\{STx_{2n-1}\}$,$\{Px_{2n-2}\}$are Cauchy sequences converging to u.

Put x = ABx_{2n}, y = u, z = STx_{2n-1} in (iv), we get

M(PABx_{2n}, QSTx_{2n-1}, QSTx_{2n-1}, qt) ≥ min {M(ABABx_{2n}, Pu, Qu, t), M(ABABx_{2n}, Pu, STSTx_{2n-1}, t),
M(Qu, STSTx_{2n-1}, Pu, t), M(ABABx_{2n}, Qu, STSTx_{2n-1}, t)}.

Now take the limit as n→ ∞ and using (ii) we get

M(Pu, Qu, Qu, qt) ≥ min{M(Pu, Pu, Qu, t), M(Pu, Pu, Qu, t), M(Qu, Qu, Pu, t),M(Pu, Qu, Qu, t)}.

Then by lemma (i) we get

M(Pu, Qu, Qu, qt) ≥ M(Pu, Qu, Qu, t).

Therefore Pu = Qu.

Now put x = ABx_{2n}, y =x_{2n-1}, z =x_{2n-1}in (iv), we get

M(P ABx_{2n}, Qx_{2n-1}, Qx_{2n-1}, qt) ≥ min {M(ABABx_{2n}, Px_{2n-1}, Qx_{2n-1}, t), M(ABABx_{2n}, Px_{2n-1}, STx_{2n-1}, t),
M(Qx_{2n-1}, STx_{2n-1}, Px_{2n-1}, t), M(ABABx_{2n}, Qx_{2n-1}, STx_{2n-1}, t)}.

Now on taking the limit as n→ ∞and on using (ii) we get

M(Pu, u, u, qt) ≥ min { M(Pu, u, u, t), M(Pu, u, u, t), M(u, u, u, t), M(Pu, u, u, t)}.

Thus we have M(Pu, u, u, qt) ≥ M(Pu, u, u, t).

Therefore Pu = u.

This implies Pu = Qu = u.

Now put x = Px_{2n-2}, y = Px_{2n-2}, z = u in (iv) we get

M(P Px_{2n-2}, Qu, Qu, qt) ≥ min {M(ABPx_{2n-2}, PPx_{2n-2}, QPx_{2n-2}, t), M(ABPx_{2n-2}, PPx_{2n-2}, STu, t),
M(QPx_{2n-2}, STu, PPx_{2n-2}, t), M(ABPx_{2n-2}, QPx_{2n-2}, STu, t)}.

Now take the limit as n→ ∞and on using (ii) & (iii) we get

M(ABu, u, u, qt) ≥ min { M(ABu, ABu, u, t), M(ABu, ABu, u, t), M(Qu, u, ABu, t), M(ABu, Qu, u, t)}.

This implies M(ABu, u, u, qt) ≥ min { M(ABu, ABu,u, t), M(ABu, ABu, u, t), M(u, u, ABu, t), M(ABu, u, u, t)}.

Therefore by lemma 1 we have ABu = u.

Thus Pu = Qu = ABu= u.

Put x = u, y = u, z = Qx_{2n-1} in (iv) we get

M(Pu, QQx_{2n-1}, QQx_{2n-1}, qt) ≥ min {M(ABu, Pu, Qu, t), M(ABu, Pu, STQx_{2n-1}, t),
M (Qu, ST Qx_{2n-1}, Pu, t), M(ABu, Qu, ST Qx_{2n-1}, t)}.

Take the limit as limit as n→ ∞and on using (ii) & (iii) we get

M(u, STu, STu, qt) ≥ min { M(u, u, u, t), M(u, u, STu, t), M(u, STu, u, t), M(u, u, STu, t)}.

On using lemma 1 we have M (STu, STu, u qt) ≥ M(STu, STu, u, t).

Thus STu= u.

We get Pu = Qu= ABu= STu= u.

REFERENCES

1. B. C. Dhage, A common fixed point principe in D-metric spaces, Bull. Calcutta Math. Soc.91 (1999), 475-480.

2. B. C. Dhage, A. M. Pathan and B. E. Rhoades, A general existence priciple for fixed point theorem in D-metric spaces, Int. J. Math. Math. Sci., 23 (2000), 441-448.

3. B. C. Dhage, Generalised metric spaces and mappings with fixed point, Bull. Calcutta Math. Soc., 84(4) (1992), 329-336.

4. George, A., Veeramani, P., On some results in fuzzy metric spaces, Fuzzy Sets and Systems 46 (1992), 107–113.

5. Grabiec, M., Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems 27 (1988), 385–389.

6. M.Koireng and Yumnam Rohen, Common Fixed Point Theorems OF Compatible Mappings OF Type (P) in Fuzzy Metric Space, Int. Journal OF Math. Analysis, Vol.6, 2012, no. 4, 181-188.

7. O. Kramosil, J. Michelak, Fuzzy metric and statistical metric space, Kybernetika, 11 (1975), 326-334.

8. S. Sedghi and N. Shobe, Fixed point theorem in M-fuzzy metric spaces with property (E), Advances in Fuzzy Mathematics, 1(1) (2006), 55-65.

9. S. S. Chauhan,Common Fixed Point Theorem for Two Pairs of Weakly Compatible Mappings in M-Fuzzy Metric Spaces Int. Journal of Math. Analysis, Vol. 3, 2009, no. 8, 393 – 398.

10. S.S.Chauhan and Nidhi Joshi, Fixed Point Theorems in M-Fuzzy metric space, International Journal of Theoretical & Applied Sciences, 1(1): 82-86(2009).

11. S.S.Chauhan and Kiran Utreja, Some Results on M-Fuzzy Metric Space, International Journal OF Theoretical & Applied Sciences, 2 (2):19-21 (2010).

12. Subrahmanyam, P.V., 1995. Common fixed point theorem in fuzzy metric spaces. Inform. Sciences, 83: 105-112.

13. Vasuki, R., 1999. Common Fixed points for R-weakly computing maps in fuzzy metric spaces. Indian J. Pure Appl. Math., 30(4): 419-423.

14. T. Veerapandi G. UthayaSankar, A. Subramania, Some Fixed Point Theorems in M- Fuzzy Metric Space, International Journal of Mathematical Archive-2(11), 2011, Page: 2205-2214.

15. L. A. Zadeh, Fuzzy sets, Inform and Control, 8 (1965), 338-353.

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