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A NEW GENERALISATION OF SAM-SOLAI'S MULTIVARIATE WIGNER DISTRIBUTION OF KIND-1 OF TYPE-B*

Dr. G. S. David Sam Jayakumar*
Assistant Professor, Jamal Institute of Management, Jamal Mohamed College, Tiruchirappalli – 620 020, South India, India

Dr. A. Solairaju
Associate Professor, Dept. of Mathematics, Jamal Mohamed College, Tiruchirappalli – 620 020, South India, India

A. Sulthan
Research scholar, Jamal Institute of Management, Jamal Mohamed College, Tiruchirappalli – 620 020, South India, India

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ABSTRACT

This paper proposed a new generalization of Sam-Solai's Multivariate Wigner distribution of kind-1 of Type-B from the univariate case. Further, we find its Cumulation, Marginal, Conditional distributions, Generating functions and also discussed its special case. The special cases include the transformation of Sam-solai's Multivariate Wigner distribution of Kind-1 of Type-B into Multivariate one parameter Wigner distribution of Kind-1 of Type-B, Multivariate Wigner distribution of Kind-1 of Type-A, Multivariate log-Wigner distribution of Kind-1 of Type-B and Multivariate Inverse -Wigner distribution of Kind-1 of Type-B. It is found that the conditional variance of Sam-Solai's Multivariate conditional Wigner distribution is heteroscedastic and the correlation was found to be +0.16. Area values of the bi-variate Wigner surface also extracted and bi-variate Wigner surfaces, contours are visualized.

Keywords: *Sam-Solai's Multivariate Wigner distribution of Kind-1 of Type-B, Multivariate one parameter Wigner distribution of Kind-1 of Type-B, Multivariate Wigner distribution of Kind-1 of Type-A,Multivariate log-Wigner distribution of Kind-1 of Type-B and Multivariate Inverse -Wigner distribution of Kind-1 of Type-B.*

Mathematics Subject Classification. *Primary 62H10; Secondary 62E15.*

INTRODUCTION

The origin of the conical distributions was first studied by the famous Noble laureate Wigner by introducing the semi-circle law and its distribution in the statistical literature. In this modern scenario, the Wigner's semi-circle distribution was used in diverse fields. Many authors studied this distribution in Trigonometric perspective and some authors tried to extend the semi-circle law to the power law as well as to the higher dimensional circles such as spheres and hyper-spheres. Wigner [1957] proposed the distribution of the roots of certain symmetric matrices and Berezin [1973] highlighted some remarks about the Wigner distribution with special reference to the semi-circular law. Moreover, Mardia [1975] studied the Von mises distribution function and Watson [1982, 1983] proposed some distributions on the circles and the spheres. Some authors studied the application of Wigner's semi-circle distribution such as Accardi et al [1996] and Hiafi [2000] with reference to the quantum electro dynamics and entropy functions respectively. On the other hand, Bai at al [1988], Fang at al [1990], Boutet de Monvel *et al.* [1999] and Evans [2000] explored and studied the convergence law of Wigner's semicircle law, symmetric multivariate distributions which include circular distributions, distribution of large random matrices and von mises distribution for higher dimensions respectively. Similarly, shimizu et al [2002], Jones et al [2005] and Arthur pewsey [2007]studied and proposed the Pearson Type VII distribution with reference to spheres, explored a family of distributions on circle and highlighted the wrapped t-family of circular distributions respectively.Moreover,Soltani[2009] introduced the two-sided power distributions which includes wigner's sem-circle distribution is a special case and Toshihiro et al[2009] proposed the Sine skewed circular distributions. Finally, Arizmendi et al [2010] and Shogo Kato et al [2010] investigated the classical G-type distribution and origin of the family of distributions on the circle arising from the Mobius transformations respectively. Based on the past reviews, the authors noted only little work is done in the multivariate generalization of the Wigner's semi-circle distribution. By utilizing this research gap, the authors proposed a new generalization of Multivariate Wigner distribution and its properties, form are discussed in the next section.

Corresponding author: *Dr. G. S. David Sam Jayakumar*, Assistant Professor, Jamal Institute of Management, Jamal Mohamed College, Tiruchirappalli – 620 020, South India, India*

SECTION-1: SAM-SOLAI'S MULTIVARIATE WIGNER DISTRIBUTION

Definition 1.1: Let $X_1, X_2, X_3, \ldots X_q$ are the random variables followed Continuous univariate Wigner semi-circle distribution with mean 0 and variance ($R_i^2/4$) for all i (i=1 to p). Then the density of Multivariate Sam-Solai's Wigner distribution of Kind-1 of Type-B is defined as

$$f(x_1, x_2, x_3 \ldots, x_p) = \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})} \quad (1)$$

where $i \neq j \quad -R_i \leq x_i \leq +R_i \quad R_i > 0$

Definition 1.2: -From (1) and if $Z_i = x_i / R_i$, then using multi-dimensional Jacobian of transformation, the Sam-Solai's multivariate standard Wigner distribution and its density is defined as

$$f(Z_1, Z_2, Z_3 \ldots, Z_p) = \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1 - Z_i}{2}}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\sqrt{\frac{1 + Z_i}{2}} \quad (2)$$

where $i \neq j \quad -1 \leq Z_i \leq +1$

Theorem1.3: -The cumulative distribution function of the Sam-Solai's Multivariate Wigner distribution is defined by

$$F(Z_1, Z_2, Z_3 \ldots, Z_p) = \int_{-R_1}^{z_1}\int_{-R_2}^{z_2}\int_{-R_3}^{z_3}\cdots\int_{-R_p}^{z_p}\{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - u_i}{R_i})}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + u_i}{R_i})} du_i$$

where $i \neq j \quad -R_i \leq u_i \leq Z_i$

$$F(Z_1, Z_2, Z_3 \ldots, Z_p) = (\frac{1}{8})^{p/2}\prod_{i=1}^{p}((R_i + Z_i)/R_i)^{3/2})\{(\sum_{i=1}^{p}\frac{\frac{1}{2} + \frac{Z_i\sqrt{(R_i^2 - Z_i^2)}}{\pi R_i^2} + \frac{\arcsin(Z_i/R_i)}{\pi}}{\frac{1}{\sqrt{8}}((R_i + Z_i)/R_i)^{3/2})}) - (p-1)\} \quad (3)$$

Theorem1.4: -The cumulative distribution function of the Sam-Solai's Multivariate standard Wigner distribution is defined by

$$F(Z_1, Z_2, Z_3 \ldots, Z_p) = \int_{-1}^{z_1}\int_{-1}^{z_2}\int_{-1}^{z_3}\cdots\int_{-1}^{z_p}\{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1 - u_i}{2}}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\sqrt{\frac{1 + u_i}{2}} du_i$$

where $i \neq j \quad -1 \leq u_i \leq Z_i$

$$F(Z_1, Z_2, Z_3 \ldots, Z_p) = (\frac{1}{8})^{p/2}\prod_{i=1}^{p}(1 + Z_i)^{3/2})\{(\sum_{i=1}^{p}\frac{\frac{1}{2} + \frac{Z_i\sqrt{(1 - Z_i^2)} + \arcsin(Z_i)}{\pi}}{\frac{1}{\sqrt{8}}(1 + Z_i)^{3/2}}) - (p-1)\} \quad (4)$$

Theorem 1.5: The Probability density function of Sam-Solai's Multivariate wigner distribution of X_1 on $X_2, X_3, \ldots X_p$ is

$$f(x_1 / x_2, x_3 \ldots, x_p) = \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (p-1)\}\frac{3}{4R_1}\sqrt{\frac{1}{2}(\frac{R_1 + x_1}{R_1})}}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (p-2)\}} \quad (5)$$

where $i \neq j \quad -R_1 \leq x_1 \leq +R_1$

Proof: It is obtained from

$$f(x_1 / x_2, x_3 \ldots, x_p) = \frac{f(x_1, x_2, x_3 \ldots, x_p)}{f(x_2, x_3 \ldots, x_p)}$$

Theorem 1.6- Mean and Variance of Sam-Solai's Multivariate Conditional Wigner distribution are

$$E(x_1 / x_2, x_3 \ldots, x_p) = \frac{\frac{R_1}{5}\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\}}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}}$$

$$V(x_1 / x_2, x_3 \ldots, x_p) = E(x_1^2 / x_2, x_3 \ldots, x_p) - (E(x_1 / x_2, x_3 \ldots, x_p))^2 \tag{6}$$

where $E(x_1^2 / x_2, x_3 \ldots, x_p) = \dfrac{R_1^2(\frac{1}{4}+\frac{11}{35}\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\})}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}}$

Proof: The Mean and variance of Multivariate Conditional Wigner distribution are given as

$$E(x_1 / x_2, x_3 \ldots, x_p) = \int_{-R_1}^{+R_1} x_1 f(x_1 / x_2, x_3 \ldots, x_p) dx_1$$

$$E(x_1 / x_2, x_3 \ldots, x_p) = \int_{-R_1}^{+R_1} x_1 \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\}\frac{3}{4R_1}\sqrt{\frac{1}{2}(\frac{R_1 + x_1}{R_1})}}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}} dx_1$$

$$E(x_1 / x_2, x_3 \ldots, x_p) = \frac{\frac{R_1}{5}\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\}}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}} \tag{7}$$

Similarly

$$E(x_1^2 / x_2, x_3 \ldots, x_p) = \int_{-R_1}^{+R_1} x_1^2 \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\}\frac{3}{4R_1}\sqrt{\frac{1}{2}(\frac{R_1 + x_1}{R_1})}}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}} dx_1$$

$$E(x_1^2 / x_2, x_3 \ldots, x_p) = \frac{R_1^2(\frac{1}{4}+\frac{11}{35}\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-1)\})}{\{\frac{16}{3\pi}(\sum_{i=2}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i}}))-(p-2)\}} \tag{8}$$

Substitute (7) and (8) in (6) we obtained the Conditional variance of the Multivariate conditional Wigner distribution.

Theorem1.7-If there are ***p=(q+k)*** random variables, such that ***q*** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depends on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$,then the density function of Sam-Solai's multivariate conditional Wigner distribution is

$$f(x_1, x_2, x_3 \ldots, x_q / x_{q+1}, x_{q+2}, x_{q+3} \ldots, x_{q+k}) = \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (q+k-1)\}(\frac{3}{4})^q \prod_{i=1}^{q} \frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})}}{\{\frac{16}{3\pi}(\sum_{i=q+1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (k-1)\}} \tag{9}$$

where $i \neq j \quad -R_i \leq x_i \leq +R_i \quad p = q+k$

Proof: Let the multivariate conditional law for ***q*** random variables $X_1, X_2, X_3, \ldots X_q$ conditionally depends on the ***k*** variables $X_{q+1}, X_{q+2}, X_{q+3}, \ldots X_{q+k}$ is given as

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{f(x_1, x_2, x_3, \ldots x_q, x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}{f(x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k})}$$

$$f(x_1, x_2, x_3, \ldots x_q / x_{q+1}, x_{q+2}, x_{q+3}, \ldots x_{q+k}) = \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (q+k-1)\}(\frac{3}{4})^{q+k} \prod_{i=1}^{q+k} \frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})}}{\int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\cdots\int_{-R_p}^{+R_p}\{\frac{16}{3\pi}(\sum_{i=1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (q+k-1)\}(\frac{3}{4})^{q+k} \prod_{i=1}^{q+k} \frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})} \prod_{i=1}^{q} dx_i}$$

$$f(x_1, x_2, x_3 \ldots, x_q / x_{q+1}, x_{q+2}, x_{q+3} \ldots, x_{q+k}) = \frac{\{\frac{16}{3\pi}(\sum_{i=1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (q+k-1)\}(\frac{3}{4})^q \prod_{i=1}^{q} \frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})}}{\{\frac{16}{3\pi}(\sum_{i=q+1}^{q+k}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (k-1)\}}$$

where $i \neq j \quad -R_i \leq x_i \leq +R_i \quad p = q+k$

SECTION: 2-CONSTANTS OF SAM-SOLAI'S MULTIVARIATE WIGNER DISTRIBUTION

Theorem 2.1 The Marginal Co-variance and Population Correlation Co-efficient between the random variables X_1 and X_2 is given as

$$COV(x_1, x_2) = \frac{R_1 R_2}{25} \tag{10}$$

Proof: Let the product moment of the Sam-Solai's multivariate Wigner distribution in terms of Co-variance from the origin is given as

$$COV(x_1, x_2) = \int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\cdots\int_{-R_p}^{+R_p} x_1 x_2 f(x_1, x_2, x_3, \ldots x_p)\prod_{i=1}^{p} d_{i}$$

$$COV(x_1, x_2) = \int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\cdots\int_{-R_p}^{+R_p} x_1 x_2 \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p} \frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})} \prod_{i=1}^{p} dx_i$$

$$COV(x_1, x_2) = R_1 R_2 / 25$$

Correlation Co-efficient of a distribution is $\rho(x_1, x_2) = COV(x_1, x_2) / \sigma_1 \sigma_2$ (11)

It observes that $\sigma_1 = R_1 / 2$ and $\sigma_2 = R_2 / 2$ (12)

From (10), (11) and (12), it follows that $\rho(x_1, x_2) = +4/25$ (13)

Remark 2.1: The result can be generalized to the Co-variance and Correlation between the i^{th} and j^{th} random variable are given as

$$COV(x_i, x_j) = R_i R_j / 25 \qquad (14)$$

$$\rho(x_i, x_j) = +4/25 \quad \text{where } i \neq j$$

Theorem 2.2: The Moment generating function of Sam-Solai's Multivariate Wigner distribution is

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p) = \{(\frac{3}{16})^p \prod_{i=1}^{p} \frac{e^{-R_i t_i}}{(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})(\sum_{i=1}^{p} \frac{2I_1(R_i t_i)/R_i t_i}{\frac{3e^{-R_i t_i}}{16(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})}) - (p-1)\} \qquad (15)$$

where $I_1(R_i t_i)$ is the modified Bessel function and $erf(i\sqrt{2R_i t_i})$ is the complex error function.

Proof: Let the moment generating function of the Multivariate distribution is given as

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p) = \int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\dots\int_{-R_p}^{+R_p} e^{\sum_{i=1}^{p} t_i x_i} f(x_1,x_2,x_3,\dots x_p)\prod_{i=1}^{p} dx_i$$

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p) = \int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\dots\int_{-R_p}^{+R_p} e^{\sum_{i=1}^{p} t_i x_i} \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i - x_i}{R_i})}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})}dx_i \qquad (16)$$

From (16), it observes that

$$\int_{-R_i}^{+R_i} e^{t_i x_i}\frac{2}{\pi R_i^2}\sqrt{R_i^2 - x_i^2}\,dx_i = \frac{2I_1(R_i t_i)}{R_i t_i} \qquad (17)$$

$$\int_{-R_i}^{+R_i} e^{t_i x_i}\frac{3}{4R_i}\sqrt{\frac{1}{2}(\frac{R_i + x_i}{R_i})}dx_i = \frac{3e^{-R_i t_i}}{16(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i}) \qquad (18)$$

$$erf(i\sqrt{2R_i t_i}) = \frac{2}{\sqrt{\pi}}\int_{0}^{i\sqrt{2R_i t_i}} e^{-x_i^2}dx_i \qquad (19)$$

From (16), (17), (18) and (19), it follows that

$$M_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p) = \{(\frac{3}{16})^p \prod_{i=1}^{p} \frac{e^{-R_i t_i}}{(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})(\sum_{i=1}^{p} \frac{2I_1(R_i t_i)/R_i t_i}{\frac{3e^{-R_i t_i}}{16(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})}) - (p-1)\}$$

By integration.

Theorem 2.3: The Cumulant of the Moment generating function of the Sam-Solai's Multivariate Wigner distribution is

$$C_{x_1,x_2,x_3,\dots x_p}(t_1,t_2,t_3,\dots t_p) = p\log(\frac{3}{16}) + \sum_{i=1}^{p}\log(\frac{e^{-R_i t_i}}{(-R_i t_i)^{3/2}}) + \sum_{i=1}^{p}\log((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})$$
$$+\log(\sum_{i=1}^{p}\frac{2I_1(R_i t_i)/R_i t_i}{\frac{3e^{-R_i t_i}}{16(-R_i t_i)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2R_i t_i}) - 4ie^{2R_i t_i}\sqrt{R_i t_i})}) - (p-1)) \qquad (20)$$

where $I_1(R_i t_i)$ is the modified Bessel function and $erf(i\sqrt{2R_i t_i})$ is the complex error function

Proof: It is found from

$$C_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)=\log(M_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p))$$

Theorem 2.4: The Characteristic function of the Sam-Solai's Multivariate Wigner distribution is

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)=(\frac{3}{16})^p\prod_{j=1}^{p}\frac{e^{-iR_jt_j}}{(-iR_jt_j)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2iR_jt_j})-4ie^{2iR_jt_j}\sqrt{iR_jt_j})(\sum_{i=1}^{p}\frac{2J_1(R_jt_j)/R_jt_j}{\frac{3e^{-iR_jt_j}}{16(-iR_jt_j)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2iR_jt_j})-4ie^{2iR_jt_j}\sqrt{iR_jt_j})})-(p-1)) \tag{21}$$

where $J_1(R_jt_j)$ is the Bessel function and $erf(i\sqrt{2iR_jt_j})$ is the complex error function.

Proof: Let the characteristic function of a multivariate distribution is given as

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)=\int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\dots\int_{-R_p}^{+R_p}e^{i\sum_{j=1}^{p}t_jx_j}f(x_1,x_2,x_3,\dots x_p)\prod_{j=1}^{p}dx_j$$

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)=\int_{-R_1}^{+R_1}\int_{-R_2}^{+R_2}\dots\int_{-R_p}^{+R_p}e^{i\sum_{j=1}^{p}t_jx_j}\{\frac{16}{3\pi}(\sum_{j=1}^{p}\sqrt{\frac{1}{2}(\frac{R_j-x_j}{R_j}}))-(p-1)\}(\frac{3}{4})^p\prod_{j=1}^{p}\frac{1}{R_j}\sqrt{\frac{1}{2}(\frac{R_j+x_j}{R_j}})dx_j \tag{22}$$

From (22), it observes that

$$\int_{-R_j}^{+R_j}e^{it_jx_j}\frac{2}{\pi R_j^2}\sqrt{R_j^2-x_j^2}\,dx_j=\frac{2J_1(R_jt_j)}{R_jt_j} \tag{23}$$

$$\int_{-R_j}^{+R_j}e^{it_jx_j}\frac{3}{4R_j}\sqrt{\frac{1}{2}(\frac{R_j-x_j}{R_j}})dx_j=\frac{3e^{-iR_jt_j}}{16(-iR_jt_j)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2iR_jt_j})-4ie^{2iR_jt_j}\sqrt{iR_jt_j}) \tag{24}$$

$$erf(i\sqrt{2iR_jt_j})=\frac{2}{\sqrt{\pi}}\int_{0}^{i\sqrt{2iR_jt_j}}e^{-x_j^2}dx_j \tag{25}$$

From (22), (23), (24) and (25), it follows that

$$\phi_{x_1,x_2,x_{3,\dots}x_p}(t_1,t_2,t_{3,\dots}t_p)=(\frac{3}{16})^p\prod_{j=1}^{p}\frac{e^{-iR_jt_j}}{(-iR_jt_j)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2iR_jt_j})-4ie^{2iR_jt_j}\sqrt{iR_jt_j})(\sum_{i=1}^{p}\frac{2J_1(R_jt_j)/R_jt_j}{\frac{3e^{-iR_jt_j}}{16(-iR_jt_j)^{3/2}}((\sqrt{2\pi})erf(i\sqrt{2iR_jt_j})-4ie^{2iR_jt_j}\sqrt{iR_jt_j})})-(p-1))$$

(by integration).

SECTION-3: SOME SPECIAL CASES

Result 3.1: From (1) and if ***P= 1***, then the Sam-Solai's multivariate Wigner density of Kind-1 of Type-B is reduced into product of the density function of uni-variate wigner semi-circle distributions.

Result 3.2: From (1) and if ***P*=2**, then the density of Sam-Solai's Multivariate Wigner distribution was reduced into

$$f(x_1,x_2)=\{\frac{16}{3\pi}\sqrt{\frac{1}{2}(\frac{R_1-x_1}{R_1})}+\frac{16}{3\pi}\sqrt{\frac{1}{2}(\frac{R_2-x_2}{R_2})}-1\}\frac{3}{4R_1}\sqrt{\frac{1}{2}(\frac{R_1+x_1}{R_1})}\frac{3}{4R_2}\sqrt{\frac{1}{2}(\frac{R_2+x_2}{R_2})} \tag{26}$$

where $-R_1\le x_1\le +R_1\quad -R_2\le x_2\le +R_2\qquad R_1,R_2>0$

This is called Sam-Solai's Bi-variate Wigner distribution of Kind-1 of Type-B

Result 3.3: From (2) and if ***P* =*2***, then the Sam-Solai's Bi-variate standard Wigner distribution and its density is given as

$$f(Z_1, Z_2) = \{\frac{16}{3\pi}\sqrt{\frac{1-Z_1}{2}} + \frac{16}{3\pi}\sqrt{\frac{1-Z_2}{2}} - 1\}(\frac{3}{4}\sqrt{\frac{1+Z_1}{2}})(\frac{3}{4}\sqrt{\frac{1+Z_2}{2}}) \tag{27}$$

where $-1 \le Z_1 \le +1$ ' $-1 \le Z_2 \le +1$

This is called Sam-Solai's Bi-variate standard Wigner distribution of Kind-1 of Type-B

Result 3.4: Below the diagram shows the Bi-variate probability surface and contour plot of the Sam-Solai's Bi-variate standard Wigner distribution are given.

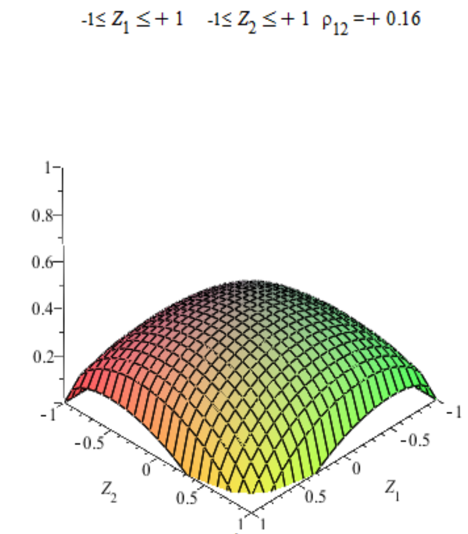

Fig.1

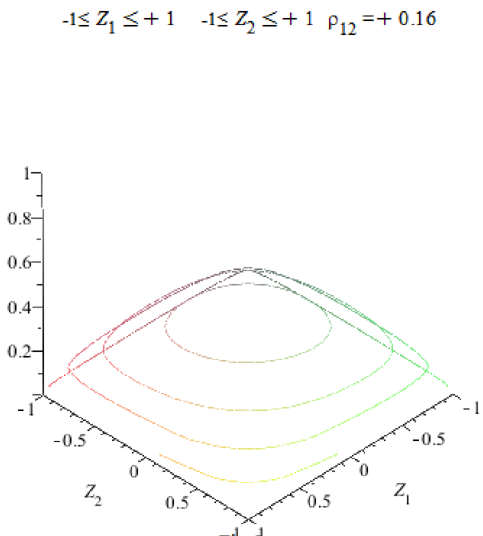

Fig.2

Result 3.5: From (4) and if ***P=2***, then the Sam-Solai's Bi-variate Cumulative standard wigner distribution is given as

$$F(Z_1, Z_2) = (\frac{1}{\sqrt{8}}(1+Z_1)^{3/2})(\frac{1}{\sqrt{8}}(1+Z_2)^{3/2})\{\frac{\frac{1}{2} + \frac{Z_1\sqrt{(1-Z_1^2)} + \arcsin(Z_1)}{\pi}}{(\frac{1}{\sqrt{8}}(1+Z_1)^{3/2})} + \frac{\frac{1}{2} + \frac{Z_2\sqrt{(1-Z_2^2)} + \arcsin(Z_2)}{\pi}}{(\frac{1}{\sqrt{8}}(1+Z_2)^{3/2})} - 1\} \tag{28}$$

Result 3.6: Below the diagram shows the Bi-variate cumulative probability surface and Cumulative Contour plot of the Sam-Solai's Bi-variate cumulative standard Wigner distribution are given.

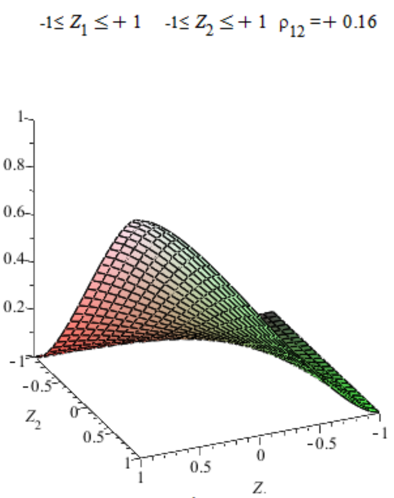

Fig.3

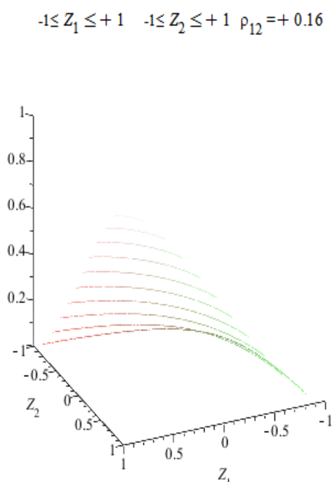

Fig.4

Result 3.7-Using the software Maple version 14, the table values from -0.9 to +0.9 with interval value 0.1 for Sam-Solai's Bi-variate standard Wigner distribution are obtained. Area under the Sam-Solai's Bi-variate Wigner surfaces based on cumulative distribution function is given.

Z_1	Z_2																		
	-0.9	-0.8	-0.7	-0.6	-0.5	-0.4	-0.3	-0.2	-0.1	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
-0.9	-0.003	-0.006	-0.008	-0.011	-0.013	-0.015	-0.017	-0.018	-0.019	-0.02	-0.02	-0.02	-0.02	-0.019	-0.017	-0.015	-0.012	-0.007	0.000
-0.8	-0.002	-0.006	-0.01	-0.013	-0.016	-0.019	-0.021	-0.023	-0.024	-0.025	-0.025	-0.024	-0.023	-0.02	-0.017	-0.012	-0.006	0.003	0.016
-0.7	0.001	-0.002	-0.006	-0.009	-0.012	-0.015	-0.017	-0.018	-0.019	-0.019	-0.018	-0.016	-0.014	-0.01	-0.004	0.003	0.012	0.025	0.044
-0.6	0.007	0.005	0.003	0.000	-0.002	-0.004	-0.005	-0.005	-0.005	-0.004	-0.002	0.001	0.006	0.011	0.018	0.028	0.04	0.056	0.08
-0.5	0.0146	0.0158	0.0155	0.0149	0.0143	0.0141	0.0144	0.0153	0.017	0.0194	0.0229	0.0274	0.0333	0.0407	0.0499	0.0615	0.0761	0.0953	0.1227
-0.4	0.0246	0.0297	0.0323	0.0341	0.0358	0.0375	0.0397	0.0423	0.0457	0.0498	0.0549	0.0611	0.0687	0.0778	0.0889	0.1024	0.1193	0.141	0.1717
-0.3	0.0365	0.0464	0.0527	0.0576	0.0619	0.0661	0.0705	0.0753	0.0806	0.0866	0.0935	0.1015	0.1108	0.1217	0.1345	0.1498	0.1685	0.1924	0.2256
-0.2	0.0502	0.0657	0.0763	0.0848	0.0923	0.0994	0.1063	0.1135	0.121	0.1291	0.138	0.1478	0.1589	0.1714	0.1859	0.2028	0.2231	0.2486	0.2836
-0.1	0.0654	0.0873	0.1029	0.1155	0.1266	0.1368	0.1466	0.1564	0.1663	0.1766	0.1875	0.1993	0.2121	0.2262	0.2422	0.2605	0.2821	0.3087	0.3449
0	0.0821	0.1111	0.1321	0.1492	0.1642	0.1779	0.1909	0.2034	0.2159	0.2286	0.2416	0.2552	0.2698	0.2855	0.3028	0.3222	0.3447	0.3721	0.4087
0.1	0.1001	0.1368	0.1636	0.1856	0.2048	0.2223	0.2385	0.2541	0.2692	0.2843	0.2994	0.315	0.3312	0.3483	0.3668	0.3872	0.4103	0.438	0.4743
0.2	0.1192	0.164	0.1971	0.2243	0.248	0.2693	0.289	0.3077	0.3256	0.3431	0.3604	0.3778	0.3956	0.414	0.4335	0.4546	0.4781	0.5056	0.541
0.3	0.1392	0.1926	0.2322	0.2648	0.2931	0.3185	0.3418	0.3636	0.3843	0.4043	0.4237	0.443	0.4623	0.4819	0.5022	0.5237	0.5472	0.5741	0.6081
0.4	0.16	0.2222	0.2686	0.3068	0.3398	0.3694	0.3963	0.4213	0.4448	0.4672	0.4887	0.5097	0.5304	0.551	0.572	0.5937	0.6169	0.6428	0.6747
0.5	0.1813	0.2526	0.3058	0.3496	0.3875	0.4212	0.4518	0.48	0.5062	0.531	0.5545	0.5771	0.599	0.6205	0.6419	0.6636	0.6861	0.7106	0.7399
0.6	0.2028	0.2832	0.3433	0.3928	0.4355	0.4733	0.5075	0.5388	0.5677	0.5947	0.6201	0.6442	0.6672	0.6893	0.7109	0.7323	0.7539	0.7767	0.8028
0.7	0.2242	0.3137	0.3806	0.4356	0.483	0.5249	0.5625	0.5967	0.6282	0.6573	0.6844	0.7097	0.7336	0.7562	0.7778	0.7986	0.8189	0.8395	0.8621
0.8	0.2451	0.3434	0.4168	0.4772	0.5291	0.5747	0.6156	0.6525	0.6863	0.7172	0.7458	0.7722	0.7967	0.8195	0.8408	0.8607	0.8795	0.8976	0.9161
0.9	0.265	0.3714	0.451	0.5162	0.5721	0.6212	0.6649	0.7043	0.7399	0.7724	0.8021	0.8293	0.8541	0.8768	0.8974	0.9162	0.9331	0.9484	0.9623

Result 3.8: From (1) and if $R_i = R$, then the Sam-Solai's Multivariate wigner distribution of Kind-1 of Type-B transformed into Multivariate one parameter Wigner distribution of Kind-1 of Type-B and its density is given as

$$f(x_1, x_2, x_3 \dots, x_p) = \{(\frac{16}{3\pi}\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R-x_i}{R}}))-(p-1)\}(\frac{3}{4R})^p \prod_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R+x_i}{R})} \tag{29}$$

where $i \neq j \quad -R \leq x_i \leq +R \qquad R > 0$

Result 3.9: From (1) and if $y_i = -x_i$, then the Sam-Solai's Multivariate wigner distribution of Kind-1 of Type-B transformed into Sam-solai's Multivariate Wigner distribution of Kind-1 of Type-A and its density is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i+y_i}{R_i}}))-(p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i}\sqrt{\frac{1}{2}(\frac{R_i-y_i}{R_i})} \tag{30}$$

where $i \neq j \quad -R_i \leq y_i \leq +R_i \qquad R_i > 0$

Result 4.0: From (1) and if $y_i = e^{x_i}$, then the Sam-Solai's Multivariate wigner distribution of Kind-1 of Type-B transformed into Sam-solai's Multivariate log- Wigner distribution of Kind-1 of Type-B and its density is given as

$$f(y_1, y_2, y_3 \dots, y_p) = \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{R_i-\log y_i}{R_i}}))-(p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i y_i}\sqrt{\frac{1}{2}(\frac{R_i+\log y_i}{R_i})} \tag{31}$$

where $i \neq j \quad e^{-R_i} \leq y_i \leq e^{R_i} \qquad R_i > 0$

Result 4.1: From (2) and if $y_i = 1/x_i$, then the Sam-Solai's Multivariate Wigner distribution of Kind-1of Type-B transformed into Sam-solai's Multivariate Inverse Wigner distribution of Kind-1 of Type-B and its density is given as

$$f(y_1, y_2, y_3 \ldots, y_p) = \{\frac{16}{3\pi}(\sum_{i=1}^{p}\sqrt{\frac{1}{2}(\frac{1-R_i y_i}{R_i y_i})}) - (p-1)\}(\frac{3}{4})^p \prod_{i=1}^{p}\frac{1}{R_i y_i^{\,2}}\sqrt{\frac{1}{2}(\frac{1+R_i y_i}{R_i y_i})} \quad (32)$$

where $i \neq j \quad -1/R_i \leq y_i \leq +1/R_i$, $R_i > 0$

CONCLUSION

The Sam-Solai's generalization of multivariate Wigner distribution of Kind-1 of Type-B is having some interesting features. At first, the marginal uni-variate distributions of the Sam-Solai's Multivariate Wigner distribution are uni-variate and enjoyed the symmetric property. Secondly, the standard correlations co-efficient between any two circular variables found to be +0.16. Finally, the multivariate generalization of Wigner's semi-circle distribution of Kind-1 of Type-B open the way for the transformation of the distribution into Multivariate one parameter Wigner distribution of Kind-1 of Type-B, Multivariate Wigner distribution of Kind-1 of Type-A, Multivariate log-Wigner distribution of Kind-1 of Type-B and Multivariate Inverse -Wigner distribution of Kind-1 of Type-B.

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