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FIXED POINT THEOREM IN MENGER PROBABILISTIC METRIC SPACE

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ABSTRACT

In this paper Fixed point results and menger probabilistic metric space for multivalued maps.

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INTRODUCTION

The probabilistic metric spaces is an important part of Stochastic Analysis, to develop the fixed point theory in such spaces. There are many results in fixed point theory in probablisitc metric space. Metric spaces were introduced by Gahler in 1964, and since then there have been many fixed point theorems proved in metric spaces and as a generalization of metric spaces, there have been only a few results in fixed point theory.

A coincidence point theorem for multivalued mappings satisfying generalized Hicks' contraction principle in Menger spaces. A probabilistic metric space is introduced by Menger. Many fixed point results have been obtained for single valued in probabilistic metric spaces. Fixed point theorem is proved for multi-valued version of the strict probabilistic (b_n)-contractions by Mihet. Hadzic introduced the notion of a multi-valued probabilistic ψ-contraction and by using the notion of the function of non compactness, a fixed point theorem was proved. Radu in generalized C-contraction which was presented by Hicks. A multi-valued generalization of the notion of a C-contraction and fixed point theorem are introduced in. Hadzic generalized fixed point theorem for multi-valued in. Zikic proved a coincidence point theorem for three mappings, which is a generalization of Hicks theorem.

2. PRELIMINARIES

2.1 Definition: [18] A t-norm is a function $\Delta: [0, 1] \times [0, 1] \to [0, 1]$ which satisfies the following conditions.

i) $\Delta(1, a) = a$
ii) $\Delta(a, b) = \Delta(b, a)$
iii) $\Delta(c, d) \geq \Delta(a, b)$ whenever $c \geq a$ and $d \geq b$,
iv) $\Delta(\Delta(a, b), c) = \Delta(a, \Delta(b, c))$

2.2 Definition: [18] A mapping $F: R \to R^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf_{t\in R} F(t) = 0$ $\sup_{t\in R} F(t) = 1$, where R is the set of real numbers and R^+ denotes the set of non-negative real numbers.

2.3 Definition: Menger Space [18] A Menger space is a triplet (M. F, Δ) where M is a non empty set, F is a function defined on $M \times M$ to the set of distribution functions and Δ is a t-norm, such that the following are satisfied.

i) $F_{xy}(0) = 0$ for all $x, y \in M$,
ii) $F_{xy}(s) = 1$ for all $s > 0$ and $x, y \in M$ if and only if $x = y$
iii) $F_{xy}(s) = F_{yx}(s)$ for all $x, y \in M$, $s > 0$ and
iv) $F_{xy}(u+v) \geq \Delta(F_{xz}(u), F_{xy}(v))$ for all $u, v \geq 0$ and $x, y, z \in M$.

A sequence $\{x_n\} \subset M$ converges to some point $x \in M$ if for given $\varepsilon \in M$ if for given $\in > 0, > 0$ we can find a positive integer $N_{\varepsilon,\lambda}$ such that for all $n > N_{\varepsilon,\lambda}$.

$$F_{x_n x}(\varepsilon) > 1 - \lambda$$

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Fixed point theory in Menger spaces is a developed branch of mathematics. Sehgal and Bharucha-Reid first introduced the contraction mapping principle in probabilistic metric spaces. [Hadzic and Pap].

2.4 Definition: (Cauchy sequence) A sequence $\{x_n\}$ in a Menger space (M, F,Δ) is called a Cauchy sequence if for each E ∈ (0.1) and t > 0. There exists $\eta_0 \in N$ such that $F_{xn}, x_m (t) > 1 - \in$ for all m, $n \geq \eta_0$. The Menger space (M, F, Δ) is said to be complete if every cauchy sequence in M convergent.

Lemma 2.1: Let X be a metric space and (x, F, Δ) be a Menger probabilistic metric space with metric d and let w be w-distance, t-norm in Δ, on x. Let $\{x_n\}$ and $\{y_n\}$ be a sequence in X. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[0, \infty)$ converging to 0, and let x, y, z ∈ X Then, the following hold:

a) If $w(x_n, y) \leq \alpha_n$ and $w(x_n, z) \leq \beta_n$ for any n ∈ N, then y = z; in particular, if w(x, y) = 0 and w(x, z) = 0, then y = z;

b) If $w(x_n, y_n) \leq \alpha_n$ and $w(x_n, z) \leq \beta_n$ for any n ∈ N, then $\{y_n\}$ converges to z;

c) If $w(x_n, x_m) \leq \alpha n$ for any n, m ∈N with m > n, then $\{x_n\}$ is a cauchy sequence;

d) If w $(y, x_n) \leq \alpha_n$ for any n ∈ N, then $\{x_n\}$ is a cauchy sequence.

Theorem: Let (X, F, Δ) be a Menger probabilistic metric space with the t-norm Δ satisfying the condition:

$$\sup_{t<1} \Delta(t, t) = 1 \tag{2.1}$$

For any α ∈ (0, 1], we define $d_\alpha : X \times X \to R^+$ as follows:

$$d_\alpha (x, y) = \inf \{t > 0: F_{x,y} (t) > 1 - \alpha\}. \tag{2.2}$$

Then $\{x, d_\alpha : \alpha \in (0, 1))$ is a generating space of quasi metric family; (2.3)

The topology $\tau_{\{d\alpha\}}$ on $(x, d_\alpha : \alpha \in (0, 1])$ coincides with the (ε,λ) – topology τ on (X, F,Δ)

3. MAIN RESULTS

In this section, we consider X complete and M is a non empty closed subset of X.

Theorem: 3.1 Let (×, M, Δ) be complete menger probabilistic metric space with the t-norm let T: M → cl(M) be a multivalued k_w – map such that

$$\text{Inf } \{ t > 0 : M_w(x, u, \alpha) + M_w(x, t, \alpha)(x) > 1 - \alpha : x \in X, t \in T \}$$

For every u ∈ X with u ∉ T(u) x ∈ X and α ∈ (0, 1] where 0 ≤ h < ½. Then "T" has a fixed point.

Proof: Let u_o be an arbitrary element of M and $u_1 \in T(u_0)$ since T is k_w-map. There exists $u_2 \in T(u_1)$ such that

$$M_w (u_1, u_2\ \alpha) \leq r\, M_w (u_0, u_1, \alpha) + r\, M_w (u_1, u_2, \alpha)$$

Where $r \in [0, 1/2)$ and consequently

$$M_w (u_1, u_2\ \alpha) \leq \frac{r}{1-r} M_w (u_9, u_1\ \alpha)$$

Thus we get a sequence $\{u_n\}$ in M such that for every n ∈ N, $u_{n+1} \in T(u_n)$

$$M_w (u_n\ .\ u_{n+1,} \alpha) \leq \left[\frac{1}{1-r}\right] M_w (un-1, u_n\ \alpha)$$

For some fixed point r, $0 < r < \frac{1}{2}$

Put $\lambda = \frac{r}{1-r}$ then $0 < \lambda < 1$

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For m and n positive Integers m > n, we have

$$M_w(u_n, u_{m,}\alpha) \le M_w(u_n, u_{n+1,}\alpha) + M_w(u_{n+1,} u_{n+2,}\alpha) + \ldots\ldots\ldots + M_w(u_{m-1}, u_{n,}\alpha)$$
$$\le \lambda^n M_w(u_0, u_{1,}\alpha)$$
$$\le \frac{\lambda^n}{1-\lambda} M_w\left(u_0, u_1\ \alpha\right)$$

Which implies that $M_w(u_n, u_m\ \alpha) \to 0$ as $n \to \infty$ and by lemma 2.1, $\{ u_n \}$ is a Cauchy sequence.

From the completeness of $\times$, we get that $\{u_n\}$ converges to some $V_0 \in x$. M being closed we have $v_o \in M$.

Let $n \in N$ be fixed since $\{u_m\}$ converges to some v_0 and $w^M(u_n, .)$ is lower semi continuous, we have

$$M_w(u_n, v_0\ \alpha) \le \lim_{n\to\infty} \inf\ M_w\left(u_n . u_m \alpha\right) \le \frac{\lambda^n}{1-\lambda} M_w\left(u_0, u_1 \alpha\right)$$

So, as $n \to \infty$, we have $M_w(u_n, v_0\ \alpha) \to 0$

Assume $v_0 \notin T(v_0)$. Then by hypothesis, we have.

$$0 < \text{Inf}\ \{M_w(u, v_0\ \alpha) + M_w(u, t(u)\ \alpha) > 1 - \alpha, u \in x\}$$
$$\le \text{Inf}\ \{M_w(u_n, v_0\ \alpha) + M_w(u_n, T(u_n))\ ; > 1 - \alpha, n \in N\}$$
$$\le \text{Inf}\ \{ M_w(u_n, v_{0,\alpha}) + M_w(u_n . T(u_n), \alpha) > 1 - \alpha, : n \in N\}$$
$$\le \text{Inf} \left\{ \frac{\lambda^n}{1-\lambda} M_w\left(u_0, u_1, \alpha\right) + \lambda^n\ M_w\left(u_0, u_1, \alpha\right) > 1-\alpha; n \in N \right\}$$
$$= 0$$

Which is impossible and hence $v_0 \in T(v_0)$

Theorem: 3.2 Let $(\times . M. \Delta)$ be complete Menger probabilistic Metric space, Each k_w – map $T: M \to cl(M)$ has a fixed point, provided that for any iterative sequence $\{u_n\}$ in M with $u_n \to v_0 \in M$. The sequence of real number $\{M_w(v_0, u_n, \alpha\}$ converges to zero.

Proof: as in theorem 3.1. There exists a convergent iterative sequence $\{u_n\}$ such that $u_n \to v_0 \in M$. with.

$$M_w(u_n\ v_0 .\ \alpha) \le \lim_{m\to\infty} \inf M_w(u_n, u_m, \alpha) \le \frac{\lambda^n}{1-\lambda} M_w(u_0, u_1, \alpha)$$

and

$$M_w(u_n . u_{n+1}, \alpha) \le \lambda^n M_w(u_0 . u_1\ \alpha) : > 1 - \alpha$$

where $\lambda = \dfrac{r}{1-r} < 1$

Note that $M_w(u_n . v_0 . \alpha) \to 0$ as $n \to \infty$ further. Since $u_n \in T(u_{n-1})$ and T is a k_w – map. There is $v_n \in T(V_0)$ such that

$$M_w(u_n, v_n, \alpha) \le r\ [M_w(u_{n-1}, u_n, \alpha) + M_w(v_0, v_n, \alpha)] > 1 - \alpha$$
$$\le \{r \int M_w(u_{n-1}, u_n) + r\ M_w(v_0, v_n) + r\ M_w(u_n, v_n) : > 1 - \alpha\}$$
$$\le \frac{r}{1-r}\ \{M_w(u_{n-1}, u_n, \alpha) > 1 - \alpha\} + \frac{r}{1-r}\ \{M_w(v_0, u_n, \alpha) > 1 - \alpha\}$$

And Thus $M_w(u_n, v_n\ \alpha) \to 0$ as $n \to \infty$

Thus by lemma 2.1 we get that $V_0 \to V_0$ and since $V_n \in T(V_0)$ which is closed, $S_0\ V_0 \in T(V_0)$

Now we prove some results on existence of common fixed points.

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Theorem 3.3: Let (x . M. Δ) be complete Menger probabilistic metric space with t-norm and Δ satisfying condition Let $\{T_n\}$ be a sequence of multivalued maps of M in to cl (M) suppose that. There exists a constant $0 \le r < \frac{1}{2}$ such that for any two maps $T_i, T_j \in \{T_n\}$ and for any $x \in M$. $u \in T_i(x)$. There exists $V \in T_j(u)$ for all $y \in M$ with

$$W_M(u, v, \alpha) \le r\,\{M_w(x, u, \alpha) + M_w(x . T_n(x). \alpha) > 1 - \alpha : x \in x\} > 0$$

For any $u \notin T_n(u)$. Then $\{T_n\}$ has a common fixed point.

Proof: Let u_0 be an asbitrary element of M and let $u_1 \in T_1(u_0)$. Then these is an $u_2 \in T_2(u_1)$ such that

$$M_w(u_1, u_2, \alpha) \le \frac{r}{1 = r} M_w(u_0, u_1, \alpha)$$

So there exist a sequence $\{u_n\}$ such that $u_{n+1} \in T_{n+1}(u)$ and for all $n \ge 1$.

$$M_w(u_n, u_{n+1}, \alpha) \le \left[\frac{r}{1-r}\right]^n M_w(u_0, u_1, \alpha)$$

Put $\lambda = \frac{r}{1-r}$ Note that $0 < \lambda < 1$ and

$M_w(u_n. u_{n+1}, \alpha) \le \lambda^n M_w(u_0, u_1, \alpha)$ for all $n \ge 1$. Then, as $n \to \infty$, we get that $\{u_n\}$ is a cauehy sequence in X

Let $P = \lim_{n\to\infty} u_n$ in M.

Now we show that $P \in n_{n\ge1}\, T_n(P)$.

Let T_m be an arbitrary member of $\{T_n\}$. Since $u_n \in T_n(u_{n-1})$. By hypothesis there is $S_n \in T_m(P)$ such that.

$$M_w(u_n, s_n, \alpha) \le r\{M_w(u_{n-1}, u_n, \alpha) + M_w(P, s_n . \alpha) : > 1 - \alpha\}$$

We proceed as in the proof of theorem 3.1 and get

$$\begin{aligned} M_w(u_n.p.\alpha) &\le \lim_{m\to\infty} \inf M_w(u_n, u_m, \alpha) \\ &\le \frac{\lambda^n}{1-\lambda} M_w(u_0, u_1, \alpha) \end{aligned}$$

Which is converges to O as $n \to \infty$. Now assume that $P \notin T_m(P)$. Then, by hypothesis and for $n > m$ and $M \ge 1$.

We have

$$\begin{aligned} 0 &\le \text{Inf}\,\{M_w(u . p . \alpha) + M_w(u, T_m(u), \alpha) > 1 - \alpha : u \in x\} \\ &\le \text{Inf}\,\{M_w(u_{m-1}, p, \alpha) + M_w(u_{m-1}, T_m(u_{m-1})\, \alpha) > 1 - \alpha : m \in N\} \\ &\le \text{Inf}\,\{M_w(u_{m-1}, p, \alpha) + M_w(u_{m-1}, T_m(u_{m-1}), \alpha) > 1 - \alpha : m \in N\} \\ &\le \inf \left\{\frac{\lambda^{m-1}}{1-\lambda} M_w(u_0, u_1, \alpha) + \lambda^{m-1} M_w(u_0, u, \alpha) > 1 - \alpha : m \in N\right\} \end{aligned}$$

Which is impossible and hence $p \in T_m(P)$. But T_m is an arbitrary hence P is a common fixed point.

Theorem: 3.4: Let (X, F, Δ) be complete Menger probabilistic Metric space with the t-norm and Δ satisfying the condition let $\{T_n\}$ be a sequence of multivalued maps of M into cl(M). Suppose that there exists a constant r with $0 \le r$ $\frac{1}{2}$ and such that for any two maps T_i, T_j and for any $x \in M$, $u \in T_i(x)$. There exists $V \in T_j(u)$ for all $y \in M$ $w^m(u, v, \alpha) \le r\,\{M_w(x, u, \alpha) + M_w(u, v, \alpha) > 1 - \alpha : x \in X\}$.

Then $\{T_n\}$ has a common fixed point provided that any iterative sequence $\{u_n\}$ in M with $u_n \to v_0 \in M$ the sequence of real number $\{M_w (v_0, u_n, \alpha)\}$ converges to zero.

Proof: can be proved early in above

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