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A COMMON FIXED POINT THEOREM FOR FOUR SELF MAPS ON A FUZZY METRIC SPACE WITH HADZIC TYPE t − NORM UNDER S-B PROPERTY

K. P. R. Sastry[1], G. A. Naidu[2], N. Umadevi[3*] and R. V. Bhaskar[4]

[1]8-28-8/1, Tamil Street, Chinna Waltair, Visakhapatnam-530017, India
[2]Department of Mathematics, Andhra university, Visakhapatnam-530003, India
[3]Department of Mathematics, Raghu Engineering College, Visakhapatnam-531 162, India
[4]Department of Mathematics, Raghu Engineering College, Visakhapatnam-531 162, India

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ABSTRACT

In this paper the concept of weak compatibility in a fuzzy metric space with Hadzic type t −norm and S-B property has been applied to obtain a common fixed point theorem for four self maps on a fuzzy metric space.

Keywords: *Fuzzy metric space, weak compatible maps, S-B property and Hadzic type t −norm.*

Mathematical Subject Classification (2010): *47H10, 54H25.*

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [13] in 1965. Since then, to use this concept in topology and analysis, many authors have extensively developed the theory of fuzzy sets and its applications. Kramosil and Michalek [6] have introduced the concept of fuzzy metric spaces in different ways. In 1988, Grabiec [4] extended the fixed point theorem of Banach [1] to fuzzy metric spaces. George and Veeramani [3] have modified the concept of fuzzy metric space introduced by Kramosil and Michalek [6].They have also shown that every metric induces a fuzzy metric. Singh *et. al.* [12] proved various fixed point theorems using the concepts of semi-compatibility, compatibility and implicit relations in fuzzy metric spaces. Rajinder Sharma [8] obtained a common fixed point theorem for four self maps on a fuzzy metric space with $min\ t$-norm under S-B property. Sastry *et.al.* [9] proved a fixed point theorem in fuzzy metric spaces with $min\ t$-norm and obtained the result in [8] as a corollary. Further, in[9] an open problem is raised regarding the validity of their result in fuzzy metric spaces with general continuous t-norm (not necessary $min\ t$-norm).In this paper, we partially answer the open problem in the affirmative by proving their common fixed point theorem in fuzzy metric spaces with Hadzic type t −norm under S-B property. However, the general solution to the open problem is still open.

Definition 1.1: (Zadeh.L.A. [13]) A fuzzy set A in a nonempty set X is a function with domain X and values in $[0,1]$.

Definition 1.2: (Schweizer.B. and Sklar. A. [10]) A function $*: [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if $*$ satisfies the following conditions:
For $a, b, c, d \in [0,1]$,
(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a * 1 = a \ for\ all\ a \in [0,1]$
(iv) $a * b \le c * d \ whenever\ a \le c\ and\ b \le d$
We observe that $a * b = \min\{a, b\}$ is a t-norm.

Note: $a * b = \min\{a, b\}$ is a t − norm $\Leftrightarrow t * t \ge t. \forall t \in [0,1]$.

Different types of t-norms: (Schweizer.B. and Sklar. A. [10])
(1) $a * b = a.b$ (product t-norm T_p)

(2) $a * b = \max\{a + b - 1, 0\}$ Lukasieswict t-norm

(3) $$T(x,y) = \begin{cases} \min\{x,y\}, \text{if } \max\{x,y\} = 1 \\ 0, \quad \text{otherwise} \end{cases}$$

***Corresponding author:** N. Umadevi[3*]*
[3]Department of Mathematics, Raghu Engineering College, Visakhapatnam-531 162, India

(4) $T(x,y)=\begin{cases}\min\{x,y\}, \text{if } x+y>1\\ 0, \quad \text{otherwise}\end{cases}$

(5) $T^{\Delta}(x,y)=\begin{cases}\dfrac{xy}{2}, \text{if } \max\{x,y\}<1\\ xy, \quad \text{otherwise}\end{cases}$

Definition 1.3: (Kramosil. I. and Michalek. J. [6]) A triple $(X,M,*)$ is said to be a fuzzy metric space (FM space, briefly) if X is a nonempty set, $*$ is a continuous $t-$norm and M is a fuzzy set on $X^2\times[0,\infty)$ satisfying the following conditions:
For $x,y,z\in X$ and $s,t>0$.
(i) $M(x,y,0)=0$
(ii) $M(x,y,t)=1$ *if and only if* $x=y$.
(iii) $M(x,y,t)=M(y,x,t)$
(iv) $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$
(v) $M(x,y,\cdot):[0,\infty)\longrightarrow[0,1]$ is left continuous.

Then M is called a fuzzy metric on X.

The function $M(x,y,t)$ denotes the degree of nearness between x and y with respect to t.

Definition 1.4: (**George.A. and Veeramani.P. [3])** Let $(X,M,*)$ be a fuzzy metric space. Then,
(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x\in X$ if $\lim_{n\to\infty}M(x_n,x,t)=1\ \forall\, t>0$.
(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if

$$\lim_{n\to\infty}M\big(x_{n+p},x_n,t\big)=1\ \forall\, t>0 \text{ and } p=1,2,\ldots$$

A FM –space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.5: (Singh,B. and Jain,S. [12]) Two self maps S and T of a fuzzy metric space $(X,M,*)$ are said to be weakly compatible if they commute at their coincidence points, that is, $Sx=Tx$ implies $STx=TSx$.

Definition 1.6: ([11]) Let S and T be two self mappings of a fuzzy metric space$(X,M,*)$.

We say that S and T satisfy the property S-B if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Sx_n=\lim_{n\to\infty}Tx_n=z$ for some $z\in X$.

In the rest of the paper, we assume that a fuzzy metric space $(X,M,*)$ satisfies the following condition:

$\lim_{t\to\infty}M(x,y,t)=1$ for all $x,y\in X$. (I)

Definition 1.7: (Hadzic[5]) Let $*$ be a $t-$norm. For any $a\in[0,1]$, write $*^0(a)=1$ and

$*^1(a)=*(*^0(a),a)=*(1,a)=a$. In general, define $*^{n+1}(a)=*(*^n(a),a)$ for $n=0,1,2\ldots\ldots\ldots$

If $\{*^n\}$ is equicontinuous at 1, that is, given $\varepsilon>0\ \exists\,\delta>0$ such that $x>1-\delta$ implies $*^n(x)>1-\varepsilon\ \forall n\in N$, then we say that $*$ is a Hadzic type $t-$norm.

We observe that min $t-$norm is of Hadzic type.

Rajinder Sharma [8] proved the following:

Theorem 1.8: Let $(X,M,*)$ be a fuzzy metric space with $t*t\ \ge t$ for all $t\in[0,1]$ and condition (I). Let A,B,S *and* T be mappings of X into itself such that

(1.8.1) $A(X)\subset T(X)$ *and* $B(X)\subset S(X)$,
(1.8.2) (A,S) *or* (B,T) satisfies the property $(S-B)$,
(1.8.3) there exists a constant $k\in(0,1)$ such that

$$M^{2p}(Ax,By,kt)\ \ge\ \min\{M^{2p}(Sx,Ty,t),M^{q}(Sx,Ax,t).M^{q'}(Ty,By,t),M^{r}(Sx,By,t).M^{r'}(Ty,Ax,(2-\alpha)t),$$
$$M^{s}(Sx,Ax,t).M^{s'}(Ty,Ax,,(2-\alpha)t),\ M^{l}(Sx,By,t).M^{l'}(Ty,By,t)\}$$

for all $x, y \in X$, $\alpha \geq 0, \alpha \in (0,2), t > 0$ and $0 < p, q, q', r, r', s, s', l, l' \leq 1$ such that

$$2p = q + q' = r + r' = s + s' = l + l'.$$

(1.8.4) the pairs (A, S) and (B, T) are weakly compatible
(1.8.5) one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X.

Then A, B, S and T have a unique common fixed point in X.

Sastry *et.al.* [9] proved the following theorem 1.9 and corollary 1.10 and obtained theorem 1.8 as a corollary to corollary 1.10.

Theorem 1.9: Let $(X, M, *)$ be a fuzzy metric space and $*$ be min $t-norm$ with condition (I). Let A, B, S and T be mappings of X into itself such that

(1.9.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$, and one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X.

(1.9.2) (B, T) satisfies the property $(S - B)$,

(1.9.3) there exists a constant $k \in (0,1)$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$ and satisfies

$M(Ax, By, kt) \geq min\{M(Sx, Ty, t), M(Sx, Ax, t). M(Ty, By, t), M(Sx, By, \alpha t), M(Ty, Ax, (2-\alpha)t)\}$ $\forall t > 0$

(1.9.4) (A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Corollary1.10: Let $(X, M, *)$ be a fuzzy metric space and $*$ be min $t-$norm with condition (I). Let A, B, S and T be mappings of X into itself such that

(1.10.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$,and one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X.

(1.10.2) (B, T) satisfies the property $(S - B)$,

(1.10.3) there exists a constant $k \in (0,1)$, $\mu > 0$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$, satisfying

$M^{\mu}(Ax, By, kt) \geq min\{M^{\mu}(Sx, Ty, t), M^{\mu}(Sx, Ax, t), M^{\mu}(Ty, By, t), M^{\mu}(Sx, By, \alpha t), M^{\mu}(Ty, Ax, (2-\alpha)t)\}$

(1.10.4) (A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Further, in [9], the following open problem is raised.

Open problem 1.11: Is theorem1.9 true if min $t-$ norm is replaced by any continuous $t-$norm?

2. MAIN RESULT

In this section we present our main result and obtain theorem 1.9 (and consequently corollary1.10).This answers the open problem 1.11 partially, in the affirmative.

Theorem 2.1: Let $(X, M, *)$ be a fuzzy metric space and $*$ be Hadzic type $t-$ norm with condition (I). Let A, B, S and T be mappings of X into itself such that

(2.1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$, and one of $A(X), B(X), S(X)$or $T(X)$ is a closed subset of X.

(2.1.2) (A, S)or (B, T) satisfies the property $(S - B)$,

(2.1.3) there exists a constant $k \in (0,1)$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$ and satisfies

$M(Ax, By, kt) \geq *\{M(Sx, Ty, t), M(Sx, Ax, t). M(Ty, By, t), M(Sx, By, \alpha t), M(Ty, Ax, (2-\alpha)t)\}$

(2.1.4)(A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Proof: Without loss of generality we suppose that $T(X)$ is a closed and (B,T)satisfies the S-B property. Then there exists a sequence $\{x_n\}$ in X such that

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = z$ for some $z \in X$. (J)

Since $B(X) \subset S(X)$ there exists a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$.

Hence $\lim_{n\to\infty} Sy_n = z$. Now we prove that $\lim_{n\to\infty} Ay_n = z$. By (2.1.3),

$$M(Ay_n, Bx_n, kt) \geq *\{M(Sy_n, Tx_n, t), M(Sy_n, Ay_n, t), M(Tx_n, Bx_n, t), M(Sy_n, Bx_n, \alpha t), M(Tx_n, Ay_n, (2-\alpha)t)\}$$

$$= *\{M(Bx_n, Tx_n, t), M(Bx_n, Ay_n, t), M(Tx_n, Bx_n, t), M(Bx_n, Bx_n, \alpha t), M(Tx_n, Ay_n, (2-\alpha)t)\}$$

$$= *\{M(Tx_n, Bx_n, t), M(Bx_n, Ay_n, t), 1, M(Tx_n, Ay_n, (2-\alpha)t)\}$$

$$\lim_{n\to\infty}\inf\big(M(Ay_n, Bx_n, kt)\big) \geq * \{\lim_{n\to\infty}\inf\big(M(Tx_n, Bx_n, t)\big), (Bx_n, Ay_n, t))\lim_{n\to\infty}\inf(M(Tx_n, Ay_n, (2-\alpha)t))\}$$

$$= *\{1, \lim_{n\to\infty}\inf(M(z, Ay_n, t)), \lim_{n\to\infty}\inf(M(z, Ay_n, (2-\alpha)t))\} \text{ (by (J))}$$

$$\geq *\{\lim_{n\to\infty}\inf\big(M(z, Ay_n, \lambda t)\big), \lim_{n\to\infty}\inf\big(M(z, Ay_n, \lambda t)\big)\}$$

where $\lambda = \min\{1, (2-\alpha)\}$

$$\lim_{n\to\infty}\inf\big(M(Ay_n, z, t)\big) \geq * \{\lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)t\right)\right), \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)t\right)\right)\}$$

$$\geq *\{\lim_{n\to\infty}\inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)), \lim_{n\to\infty}\inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)),$$
$$\lim_{n\to\infty}\inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)), \lim_{n\to\infty}\inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right))\}$$

$$= *^4 \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)\right)$$

… … … … … … … … … … … … … … … … … …

$$\geq *^{2^m} \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right)\right)$$

Since $*$ is of Hadzic type $t-$norm to $\varepsilon > 0 \ni \delta > 0 \ni x > 1-\delta \Rightarrow *^p(x) > 1-\varepsilon$ for $p \in N$

$$*^{2^m} \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\frac{\lambda}{k}\right)^m t\right)\right) > 1-\varepsilon$$

Whenever $\lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\frac{\lambda}{k}\right)^m t\right)\right) > 1-\delta$

To δ corresponds a $q \in z^+ \ni \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\frac{\lambda}{k}\right)^m t\right)\right) > 1-\delta$ if $m \geq q$ by (I)

$\therefore$ $*^{2^m} \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\frac{\lambda}{k}\right)^m t\right)\right) > 1-\varepsilon$ if $m \geq q$

$\therefore$ $\lim_{n\to\infty}\inf\big(M(Ay_n, z, t)\big) \geq *^{2^m} \lim_{n\to\infty}\inf\left(M\left(z, Ay_n, \left(\frac{\lambda}{k}\right)^m t\right)\right) > 1-\varepsilon$ whenever $m \geq q \;\forall\, t > 0$

$\therefore$ $\lim_{n\to\infty}\inf\big(M(Ay_n, z, t)\big) \geq 1$

$\therefore$ $\lim_{n\to\infty}\inf\big(M(Ay_n, z, t)\big) = 1$

$\therefore$ $Ay_n \to z$

Since $T(X)$ is a closed subset of X, $\exists v \in X \ni Tv = z \in X$.

We have $\lim_{n\to\infty} Ay_n = \lim_{n\to\infty} Tx_n = \lim_{n\to\infty} Sy_n = \lim_{n\to\infty} Bx_n = Tv$.

By (2.1.3), $M(Ay_n, Bv, kt) \geq *\{M(Sy_n, Tv, t), M(Sy_n, Ay_n, t).M(Tv, Bv, t),\ M(Sy_n, Bv, \alpha t), M(Tv, Ay_n, (2-\alpha)t)\}$

$\therefore$ On letting $n \to \infty$, we get

$M(z, Bv, kt) \geq *\{M(z, Tv, t), M(z, z, t)\ M(z, Bv, t)M(z, Bv, \alpha t), M(z, z,, (2-\alpha)t)\}$

$\geq *\{1,1, M(z, Bv, t)M(z, Bv, \alpha t), 1\}$

$= *\{M(z, Bv, t), M(z, Bv, \alpha t)\}$

$\geq *\{M(z, Bv, \lambda t), M(z, Bv, \lambda t)\}$ where $\lambda = min\{1, \alpha\}$

$M(z, Bv, t) \geq *^2 M\left(z, Bv, \left(\frac{\lambda}{k}\right)t\right) \geq *^{2^2} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^2 t\right) \geq \ldots\ldots\ldots \geq *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0\ \exists \delta > 0 \ni *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon$ if $M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$

To δ corresponds a $q \in z^+ \ni M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon\ \forall\ t > 0$

$\therefore M(z, Bv, t) \geq 1-\varepsilon$. This is true for every $t > 0$.

$\therefore M(z, Bv, t) \geq 1$.

$\therefore M(z, Bv, t) = 1$.

$\therefore Bv = z$.

$\therefore Tv = Bv = z$.

Since (B, T) is weakly compatible, $BTv = TBv \Rightarrow Bz = Tz$.

Since$B(X) \subset S(X), \exists u \in X \ni Su = Bv$. By (2.1.3)

$M(Au, Bv, kt) \geq *\{M(Su, Tv, t), M(Su, Au, t).M(Tv, Bv, t),\ M(Su, Bv, \alpha t), M(Tv, Au, (2-\alpha)t)\}$

$M(Au, Tv, kt) \geq *\{M(Bv, Tv, t), M(Tv, Au, t).M(Tv, Bv, t), M(Tv, Bv, \alpha t), M(Tv, Au, (2-\alpha)t)\}$

$= *\{1, M(Tv, Au, t), 1,1, M(Tv, Au, (2-\alpha)t)\}$

$\geq *\{M(Au, Tv, \lambda t), M(Au, Tv, \lambda t)\}$ where $\lambda = min\{1,2-\alpha\}$

$M(Au, Tv, t) \geq *^2 M\left(Au, Tv, \left(\frac{\lambda}{k}\right)t\right) \geq *^{2^2} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^2 t\right) \geq \ldots \geq *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0\ \exists \delta > 0 \ni *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon$ if $M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$

To δ corresponds a $q \in z^+ \ni M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon\ \forall\ t > 0$

$\therefore M(Au, Tv, kt) \geq 1-\varepsilon$. This is true for every $t > 0$.

$\therefore M(Au, Tv, t) \geq 1$.

$\therefore M(Au, Tv, t) = 1$.

$\therefore Au = Tv$.

Since (A, S) is weakly compatible, we have $ASu = SAu \Rightarrow Az = Sz$.

By (2.1.3), $M(Au, Bz, kt) \geq *\{M(Su, Tz, t), M(Su, Au, t). M(Tz, Bz, t),\ M(Su, Bz, \alpha t), M(Tz, Au, (2-\alpha)t)\}$

$$\therefore M(z, Tz, kt) \geq *\{M(z, Tz, t), M(Au, Au, t). M(Tz, Tz, t), M(z, Tz, \alpha t), M(Tv, Au, (2-\alpha)t)\}$$

$$= * M(z, Tz, t), 1, 1, M(z, Tz, \alpha t), M(Tz, z, (2-\alpha)t)\}$$

$$= *\{M(z, Tz, t), M(z, Tz, \alpha t), M(Tz, z, (2-\alpha)t)\}$$

$$\geq *\{\ M(z, Tz, \lambda t), M(z, Tz, \lambda t), M(z, Tz, \lambda t)\} \text{ where } \lambda = min\,\{1, \alpha, (2-\alpha)\}$$

$$= *^3\ M(z, Tz, \lambda t)$$

$$\therefore M(z, Tz, t) \geq *^3\ M\left(z, Tz, \left(\tfrac{\lambda}{k}\right)t\right) \geq\ *^{3^2}\ M\left(z, Tz, \left(\tfrac{\lambda}{k}\right)^2 t\right) \geq \ldots \geq\ *^{3^m}\ M\left(z, Tz, \left(\tfrac{\lambda}{k}\right)^m t\right)$$

To $\varepsilon > 0\ \exists \delta > 0 \ni\ *^{3^m}\ M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon$ if $M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$

To δ corresponds a $q \in z^+ \ni M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow\ *^{3^m}\ M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon\ \forall\ t > 0$

$\therefore M(z, Tz, t) \geq 1-\varepsilon$

$\therefore M(z, Tz, t) \geq 1$

$\therefore M(z, Tz, t) = 1$

$\therefore z = Tz$.

Now, in a similar way we can show that $Az = z$.

$\therefore z = Tz = Bz = Sz = Az$.

$\Rightarrow z$ is a common fixed point of $A, B, S\ and\ T$.

Uniqueness: Let p, q be two common fixed points of $A, B, S\ and\ T$.

Then by (2.1.3), $M(Ap, Bq, kt) \geq *\{M\ (Sp, Tq, t), M(Sp, Ap, t), M(Tq, Bq, t),\ M(Sp, Bq, \alpha t), M(Tq, Ap, (2-\alpha)t)\}$

$$\geq *\{M\ (p, q, t), M(p, p, t), M(q, q, t),\ M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$$

$$\geq *\{M\ (p, q, t), 1, 1, M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$$

$$\geq *\{M\ (p, q, t), M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$$

$$\geq *\{\ M(p, q, \lambda t), M(p, q, \lambda t), M(p, q, \lambda t)\} \text{ where } \lambda = \min\{1, \alpha, (2-\alpha)\}$$

$$M(p, q, t) \geq *^3\ M\left(p, q, \left(\tfrac{\lambda}{k}\right)t\right) \geq\ *^{3^2}\ M\left(p, q, \left(\tfrac{\lambda}{k}\right)^2 t\right) \geq \ldots \geq\ *^{3^m}\ M\left(p, q, \left(\tfrac{\lambda}{k}\right)^m t\right)$$

To $\varepsilon > 0\ \exists \delta > 0 \ni\ *^{3^m}\ M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon$ if $M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$

To δ corresponds a $q \in z^+ \ni M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow\ *^{3^m}\ M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1-\varepsilon\ \forall\ t > 0$

$\therefore M(p, q, t) \geq 1-\varepsilon$

$\therefore M(p,q,t) \geq 1$

$\therefore M(p,q,t) = 1$

$\therefore p = q$. This completes the proof of the theorem.

Note: Theorem 1.9 follows as a corollary to theorem (2.1), if $k < \alpha < 1$ and $k + \alpha < 2$, since $t * t \geq t$ for all $t > 0$ $\Rightarrow *$ is the $mint - norm$.

We conclude the paper with two open problems.

Open problem1: Is theorem 2.1 true if either $k < \alpha$ or $k + \alpha < 2$ is violated.

Open problem2: Is theorem 2.1 true if $*$ is continuous $t-$norm (not necessarily Hadzic type $t-$ norm)?

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