A COMMON FIXED POINT THEOREM FOR FOUR SELF MAPS ON A FUZZY METRIC SPACE WITH HADZIC TYPE t − NORM UNDER S-B PROPERTY

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ABSTRACT

In this paper the concept of weak compatibility in a fuzzy metric space with Hadzic type t −norm and S-B property has been applied to obtain a common fixed point theorem for four self maps on a fuzzy metric space.

Keywords: Fuzzy metric space, weak compatible maps, S-B property and Hadzic type t −norm.

1. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [13] in 1965. Since then, to use this concept in topology and analysis, many authors have extensively developed the theory of fuzzy sets and its applications. Kramosil and Michalek [6] have introduced the concept of fuzzy metric spaces in different ways. In 1988, Grabiec [4] extended the fixed point theorem of Banach [1] to fuzzy metric spaces. George and Veeramani [3] have modified the concept of fuzzy metric space introduced by Kramosil and Michalek [6].They have also shown that every metric induces a fuzzy metric. Singh *et. al.* [12] proved various fixed point theorems using the concepts of semi-compatibility, compatibility and implicit relations in fuzzy metric spaces. Rajinder Sharma [8] obtained a common fixed point theorem for four self maps on a fuzzy metric space with *min t*-norm under S-B property. Sastry *et.al.* [9] proved a fixed point theorem in fuzzy metric spaces with *min t*-norm and obtained the result in [8] as a corollary. Further, in[9] an open problem is raised regarding the validity of their result in fuzzy metric spaces with general continuous t-norm (not necessary *min t*-norm).In this paper, we partially answer the open problem in the affirmative by proving their common fixed point theorem in fuzzy metric spaces with Hadzic type t −norm under S-B property. However, the general solution to the open problem is still open.

Definition 1.1: (Zadeh.L.A. [13]) A fuzzy set A in a nonempty set X is a function with domain X and values in $[0,1]$.

Definition 1.2: (Schweizer.B. and Sklar. A. [10]) A function $*: [0,1] \times [0,1] \to [0,1]$ is said to be a continuous t-norm if $*$ satisfies the following conditions:
For $a, b, c, d \in [0,1]$,
(i) $*$ is commutative and associative
(ii) $*$ is continuous
(iii) $a * 1 = a \ for \ all \ a \ \in [0,1]$
(iv) $a * b \leq c * d \ whenever \ a \ \leq c \ and \ b \leq d$
We observe that $a * b = \min\{a, b\}$ is a t-norm.

Note: $a * b = \min\{a, b\}$ is a $t − \text{norm} \Leftrightarrow t * t \geq t. \forall t \in [0,1]$.

Different types of *t*-norms: (Schweizer.B. and Sklar. A. [10])
(1) $a * b = a.b$ (product t-norm T_p)

(2) $a * b = \max\{a + b − 1, 0\}$ Lukasieswict t-norm

(3) $T(x, y) = \begin{cases} \min\{x, y\}, \text{if } \max\{x, y\} = 1 \\ 0, \qquad \text{otherwise} \end{cases}$

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(4) $T(x, y) = \begin{cases} \min\{x, y\}, & \text{if } x + y > 1 \\ 0, & \text{otherwise} \end{cases}$

(5) $T^{\Delta}(x, y) = \begin{cases} \dfrac{xy}{2}, & \text{if } \max\{x, y\} < 1 \\ xy, & \text{otherwise} \end{cases}$

Definition 1.3: (Kramosil. I. and Michalek. J. [6]) A triple $(X, M, *)$ is said to be a fuzzy metric space (FM space, briefly) if X is a nonempty set, $*$ is a continuous t −norm and M is a fuzzy set on $X^2 \times [0, \infty)$ satisfying the following conditions:

For $x, y, z \in X$ and $s, t > 0$.

(i) $M(x, y, 0) = 0$

(ii) $M(x, y, t) = 1$ *if and only if* $x = y$.

(iii) $M(x, y, t) = M(y, x, t)$

(iv) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$

(v) $M(x, y, \cdot) : [0, \infty) \to [0,1]$ is left continuous.

Then M is called a fuzzy metric on X.

The function $M(x, y, t)$ denotes the degree of nearness between x and y with respect to t.

Definition 1.4: (George.A. and Veeramani.P. [3]) Let $(X, M, *)$ be a fuzzy metric space. Then,

(i) A sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if $\lim_{n \to \infty} M(x_n, x, t) = 1 \ \forall \ t > 0$.

(ii) A sequence $\{x_n\}$ in X is called a Cauchy sequence if

$$\lim_{n \to \infty} M(x_{n+p}, x_n, t) = 1 \ \forall \ t > 0 \text{ and } p = 1, 2, \dots$$

A FM –space in which every Cauchy sequence is convergent is said to be complete.

Definition 1.5: (Singh,B. and Jain,S. [12]) Two self maps S and T of a fuzzy metric space $(X, M, *)$ are said to be weakly compatible if they commute at their coincidence points, that is, $Sx = Tx$ implies $STx = TSx$.

Definition 1.6: ([11]) Let S and T be two self mappings of a fuzzy metric space $(X, M, *)$.

We say that S and T satisfy the property S-B if there exists a sequence $\{x_n\}$ in X such that $\lim_{n \to \infty} Sx_n = \lim_{n \to \infty} Tx_n = z$ for some $z \in X$.

In the rest of the paper, we assume that a fuzzy metric space $(X, M, *)$ satisfies the following condition:

$\lim_{t \to \infty} M(x, y, t) = 1$ for all $x, y \in X$. (I)

Definition 1.7: (Hadzic[5]) Let $*$ be a t −norm. For any $a \in [0,1]$, write $*^0(a) = 1$ and

$*^1(a) = *(*^0(a), a) = *(1, a) = a$. In general, define $*^{n+1}(a) = *(*^n(a), a)$ for $n = 0, 1, 2 \dots \dots \dots$

If $\{*^n\}$ is equicontinuous at 1, that is, given $\varepsilon > 0 \ \exists \ \delta > 0$ such that $x > 1 - \delta$ implies $*^n(x) > 1 - \varepsilon \ \forall n \in N$, then we say that $*$ is a Hadzic type t −norm.

We observe that min t −norm is of Hadzic type.

Rajinder Sharma [8] proved the following:

Theorem 1.8: Let $(X, M, *)$ be a fuzzy metric space with $t * t \geq t$ for all $t \in [0,1]$ and condition (I). Let A, B, S and T be mappings of X into itself such that

(1.8.1) $A(X) \subset T(X)$ *and* $B(X) \subset S(X)$,

(1.8.2) (A, S) *or* (B, T) satisfies the property $(S - B)$,

(1.8.3) there exists a constant $k \in (0,1)$ such that

$$M^{2p}(Ax, By, kt) \geq \min\{M^{2p}(Sx, Ty, t), M^q(Sx, Ax, t) \cdot M^{q'}(Ty, By, t), M^r(Sx, By, t) \cdot M^{r'}(Ty, Ax, (2 - \alpha)t),$$
$$M^s(Sx, Ax, t) \cdot M^{s'}(Ty, Ax, (2 - \alpha)t), \ M^l(Sx, By, t) \cdot M^l(Ty, By, t)\}$$

for all $x, y \in X, \alpha \geq 0, \alpha \in (0,2), t > 0$ and $0 < p, q, q', r, r', s, s', l, l' \leq 1$ such that

$$2p = q + q' = r + r' = s + s' = l + l'.$$

(1.8.4) the pairs (A, S) and (B, T) are weakly compatible

(1.8.5) one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X.

Then A, B, S and T have a unique common fixed point in X.

Sastry *et.al.* [9] proved the following theorem 1.9 and corollary 1.10 and obtained theorem 1.8 as a corollary to corollary 1.10.

Theorem 1.9: Let $(X, M, *)$ be a fuzzy metric space and $*$ be $\min t - norm$ with condition (I). Let A, B, S and T be mappings of X into itself such that

(1.9.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$, and one of $A(X), B(X), S(X)$ $or T(X)$ is a closed subset of X.

(1.9.2) (B, T) satisfies the property $(S - B)$,

(1.9.3) there exists a constant $k \in (0,1)$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$ and satisfies

$$M(Ax, By, kt) \geq min \{M(Sx, Ty, t), M(Sx, Ax, t). M(Ty, By, t), M(Sx, By, \alpha t), M(Ty, Ax, (2 - \alpha)t)\} \quad \forall t > 0$$

(1.9.4) (A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Corollary1.10: Let $(X, M, *)$ be a fuzzy metric space and $*$ be $\min t - norm$ with condition (I). Let A, B, S and T be mappings of X into itself such that

(1.10.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$, and one of $A(X), B(X), S(X)$ $or T(X)$ is a closed subset of X.

(1.10.2) (B, T) satisfies the property $(S - B)$,

(1.10.3) there exists a constant $k \in (0,1)$, $\mu > 0$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$, satisfying

$$M^{\mu}(Ax, By, kt) \geq min \{M^{\mu}(Sx, Ty, t), M^{\mu}(Sx, Ax, t), M^{\mu}(Ty, By, t), M^{\mu}(Sx, By, \alpha t), M^{\mu}(Ty, Ax, (2 - \alpha)t)\}$$

(1.10.4) (A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Further, in [9], the following open problem is raised.

Open problem 1.11: Is theorem1.9 true if $\min t - norm$ is replaced by any continuous $t -$norm?

2. MAIN RESULT
In this section we present our main result and obtain theorem 1.9 (and consequently corollary1.10).This answers the open problem 1.11 partially, in the affirmative.

Theorem 2.1: Let $(X, M, *)$ be a fuzzy metric space and $*$ be Hadzic type $t - norm$ with condition (I). Let A, B, S and T be mappings of X into itself such that

(2.1.1) $A(X) \subset T(X)$ and $B(X) \subset S(X)$, and one of $A(X), B(X), S(X)$ or $T(X)$ is a closed subset of X.

(2.1.2) (A, S) or (B, T) satisfies the property $(S - B)$,

(2.1.3) there exists a constant $k \in (0,1)$ and $\alpha \in (0,2)$, such that $k < \alpha, k + \alpha < 2$ and satisfies

$$M(Ax, By, kt) \geq * \{M(Sx, Ty, t), M(Sx, Ax, t). M(Ty, By, t), M(Sx, By, \alpha t), M(Ty, Ax, (2 - \alpha)t)\}$$

(2.1.4)(A, S) and (B, T) are weakly compatible.

Then A, B, S and T have a unique common fixed point in X.

Proof: Without loss of generality we suppose that $T(X)$ is a closed and (B, T) satisfies the S-B property. Then there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \to \infty} Bx_n = \lim_{n \to \infty} Tx_n = z \text{ for some } z \in X. \tag{J}$$

Since $B(X) \subset S(X)$ there exists a sequence $\{y_n\}$ in X such that $Bx_n = Sy_n$.

Hence $\lim_{n \to \infty} Sy_n = z$. Now we prove that $\lim_{n \to \infty} Ay_n = z$. By (2.1.3),

$$M(Ay_n, Bx_n, kt) \geq * \{M (Sy_n, Tx_n, t), M(Sy_n, Ay_n, t), M(Tx_n, Bx_n, t), \ M(Sy_n, Bx_n, \alpha t), M(Tx_n, Ay_n, (2 - \alpha)t)\}$$

$$= * \{M(Bx_n, Tx_n, t), M(Bx_n, Ay_n, t), M(Tx_n, Bx_n, t), \ M(Bx_n, Bx_n, \alpha t), M(Tx_n, Ay_n, (2 - \alpha)t)\}$$

$$= * \{M (Tx_n, Bx_n, t), M(Bx_n, Ay_n, t), 1, M(Tx_n, Ay_n, (2 - \alpha)t)\}$$

$$\lim_{n \to \infty} \inf\left(M(Ay_n, Bx_n, kt) \right) \geq * \ \{\lim_{n \to \infty} \inf\left(M(Tx_n, Bx_n, t) \right), (Bx_n, Ay_n, t)) \lim_{n \to \infty} \inf (M(Tx_n, Ay_n, (2 - \alpha)t))\}$$

$$= * \{1, \lim_{n \to \infty} \inf(M(z, Ay_n, t)), \lim_{n \to \infty} \inf(M(z, Ay_n, (2 - \alpha)t))\} \ (by \ (J))$$

$$\geq * \{ \lim_{n \to \infty} \inf \left(M(z, Ay_n, \lambda t) \right), \lim_{n \to \infty} \inf \left(M(z, Ay_n, \lambda t) \right)\}$$

where $\lambda = \min\{1, (2 - \alpha)\}$

$$\lim_{n \to \infty} \inf\left(M(Ay_n, z, t) \right) \geq * \{\lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)t\right) \right), \lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)t\right) \right)\}$$

$$\geq * \{\lim_{n \to \infty} \inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)), \lim_{n \to \infty} \inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)),$$
$$\lim_{n \to \infty} \inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)), \lim_{n \to \infty} \inf(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)) \}$$

$$= *^4 \lim_{n \to \infty} \inf (M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^2 t\right)$$

$$\ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$\geq *^{2^m} \lim_{n \to \infty} \inf (M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right)$$

Since $*$ is of Hadzic type $t-norm$ to $\varepsilon > 0 \ \exists \ \delta > 0 \ \exists \ x > 1 - \delta \Rightarrow *^p (x) > 1 - \varepsilon$ for $p \in N$

$$*^{2^m} \lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right) \right) > 1 - \varepsilon$$

Whenever $\lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right) \right) > 1 - \delta$

To δ corresponds a $q \in z^+ \ni \lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right) \right) > 1 - \delta$ if $m \geq q$ by (I)

$$\therefore *^{2^m} \lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right) \right) > 1 - \varepsilon \text{ if } \ m \geq q$$

$$\therefore \lim_{n \to \infty} \inf\left(M(Ay_n, z, t) \right) \geq *^{2^m} \lim_{n \to \infty} \inf \left(M\left(z, Ay_n, \left(\tfrac{\lambda}{k}\right)^m t\right) \right) > 1 - \varepsilon \quad \text{whenever } m \geq q \ \forall \ t > 0$$

$$\therefore \lim_{n \to \infty} \inf\left(M(Ay_n, z, t) \right) \geq 1$$

$$\therefore \lim_{n \to \infty} \inf\left(M(Ay_n, z, t) \right) = 1$$

$$\therefore Ay_n \to z$$

Since $T(X)$ is a closed subset of $X, \exists v \in X \ni Tv = z \in X$.

We have $\lim_{n \to \infty} Ay_n = \lim_{n \to \infty} Tx_n = \lim_{n \to \infty} Sy_n = \lim_{n \to \infty} Bx_n = Tv$.

By (2.1.3), $M(Ay_n, Bv, kt) \geq * \{M(Sy_n, Tv, t), M(Sy_n, Ay_n, t).M(Tv, Bv, t), M(Sy_n, Bv, \alpha t), M(Tv, Ay_n, (2 - \alpha)t)\}$

$\therefore$ On letting $n \to \infty$, we get

$M(z, Bv, kt) \geq * \{M(z, Tv, t), M(z, z, t) M(z, Bv, t)M(z, Bv, \alpha t), M(z, z,, (2 - \alpha)t)\}$

$\qquad \geq * \{1, 1, M(z, Bv, t)M(z, Bv, \alpha t), 1\}$

$\qquad = * \{M(z, Bv, t), M(z, Bv, \alpha t)\}$

$\qquad \geq * \{M(z, Bv, \lambda t), M(z, Bv, \lambda t)\}$ where $\lambda = min\{1, \alpha\}$

$M(z, Bv, t) \geq *^2 M\left(z, Bv, \left(\frac{\lambda}{k}\right)t\right) \geq *^{2^2} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^2 t\right) \geq \ldots \ldots \geq *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0 \exists \delta > 0 \ni *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon$ if $M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$

To δ corresponds a $q \in z^+ \ni M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{2^m} M\left(z, Bv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon \; \forall \; t > 0$

$\therefore M(z, Bv, t) \geq 1 - \varepsilon$. This is true for every $t > 0$.

$\therefore M(z, Bv, t) \geq 1$.

$\therefore M(z, Bv, t) = 1$.

$\therefore Bv = z$.

$\therefore Tv = Bv = z$.

Since (B, T) is weakly compatible, $BTv = TBv \Rightarrow Bz = Tz$.

Since $B(X) \subset S(X), \exists u \in X \ni Su = Bv$. By (2.1.3)

$M(Au, Bv, kt) \geq * \{M(Su, Tv, t), M(Su, Au, t).M(Tv, Bv, t), M(Su, Bv, \alpha t), M(Tv, Au, (2 - \alpha)t)\}$

$M(Au, Tv, kt) \geq * \{M(Bv, Tv, t), M(Tv, Au, t).M(Tv, Bv, t), M(Tv, Bv, \alpha t), M(Tv, Au, (2 - \alpha)t)\}$

$\qquad = * \{1, M(Tv, Au, t), 1, 1, M(Tv, Au, (2 - \alpha)t)\}$

$\qquad \geq * \{M(Au, Tv, \lambda t), M(Au, Tv, \lambda t)\}$ where $\lambda = min\{1, 2 - \alpha\}$

$M(Au, Tv, t) \geq *^2 M\left(Au, Tv, \left(\frac{\lambda}{k}\right)t\right) \geq *^{2^2} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^2 t\right) \geq \ldots \geq *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0 \exists \delta > 0 \ni *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon$ if $M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$

To δ corresponds a $q \in z^+ \ni M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{2^m} M\left(Au, Tv, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon \; \forall \; t > 0$

$\therefore M(Au, Tv, kt) \geq 1 - \varepsilon$. This is true for every $t > 0$.

$\therefore M(Au, Tv, t) \geq 1$.

$\therefore M(Au, Tv, t) = 1$.

$\therefore Au = Tv.$

Since (A, S) is weakly compatible, we have $ASu = SAu \Rightarrow Az = Sz.$

By (2.1.3), $M(Au, Bz, kt) \geq * \{M(Su, Tz, t), M(Su, Au, t). M(Tz, Bz, t), M(Su, Bz, \alpha t), M(Tz, Au, (2-\alpha)t)\}$

$\therefore M(z, Tz, kt) \geq * \{M(z, Tz, t), M(Au, Au, t). M(Tz, Tz, t), M(z, Tz, \alpha t), M(Tv, Au, (2-\alpha)t)\}$

$= * M(z, Tz, t), 1, 1, M(z, Tz, \alpha t), M(Tz, z, (2-\alpha)t)\}$

$= * \{M(z, Tz, t), M(z, Tz, \alpha t), M(Tz, z, (2-\alpha)t)\}$

$\geq * \{M(z, Tz, \lambda t), M(z, Tz, \lambda t), M(z, Tz, \lambda t)\}$ where $\lambda = min \{1, \alpha, (2-\alpha)\}$

$= *^3 M(z, Tz, \lambda t)$

$\therefore M(z, Tz, t) \geq *^3 M\left(z, Tz, \left(\frac{\lambda}{k}\right)t\right) \geq *^{3^2} M\left(z, Tz, \left(\frac{\lambda}{k}\right)^2 t\right) \geq ... \geq *^{3^m} M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0 \ \exists \delta > 0 \ni *^{3^m} M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon$ if $M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$

To δ corresponds a $q \in z^+ \ni M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{3^m} M\left(z, Tz, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon \ \forall \ t > 0$

$\therefore M(z, Tz, t) \geq 1 - \varepsilon$

$\therefore M(z, Tz, t) \geq 1$

$\therefore M(z, Tz, t) = 1$

$\therefore z = Tz.$

Now, in a similar way we can show that $Az = z.$

$\therefore z = Tz = Bz = Sz = Az.$

$\Rightarrow z$ is a common fixed point of A, B, S and T.

Uniqueness: Let p, q be two common fixed points of A, B, S and T.

Then by (2.1.3), $M(Ap, Bq, kt) \geq * \{M(Sp, Tq, t), M(Sp, Ap, t), M(Tq, Bq, t), M(Sp, Bq, \alpha t), M(Tq, Ap, (2-\alpha)t)\}$

$\geq * \{M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$

$\geq * \{M(p, q, t), 1, 1, M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$

$\geq * \{M(p, q, t), M(p, q, \alpha t), M(q, p, (2-\alpha)t)\}$

$\geq * \{M(p, q, \lambda t), M(p, q, \lambda t), M(p, q, \lambda t)\}$ where $\lambda = min\{1, \alpha, (2-\alpha)\}$

$M(p, q, t) \geq *^3 M\left(p, q, \left(\frac{\lambda}{k}\right)t\right) \geq *^{3^2} M\left(p, q, \left(\frac{\lambda}{k}\right)^2 t\right) \geq ... \geq *^{3^m} M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right)$

To $\varepsilon > 0 \ \exists \delta > 0 \ni *^{3^m} M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon$ if $M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$

To δ corresponds a $q \in z^+ \ni M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \delta$ if $m \geq q$

$\therefore m \geq q \Rightarrow *^{3^m} M\left(p, q, \left(\frac{\lambda}{k}\right)^m t\right) > 1 - \varepsilon \ \forall \ t > 0$

$\therefore M(p, q, t) \geq 1 - \varepsilon$

$\therefore M(p, q, t) \geq 1$

$\therefore M(p, q, t) = 1$

$\therefore p = q$. This completes the proof of the theorem.

Note: Theorem 1.9 follows as a corollary to theorem (2.1), if $k < \alpha < 1 \ and \ k + \alpha < 2$, since $t * t \ \geq t$ for all $t \ > 0$ $\Rightarrow *$ is the $mint - norm$.

We conclude the paper with two open problems.

Open problem1: Is theorem 2.1 true if either $k < \alpha \ or \ k + \alpha < 2$ is violated.

Open problem2: Is theorem 2.1 true if $*$ is continuous $t-$norm (not necessarily Hadzic type $t -$ norm)?

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REFERENCES

[1] Banach.S. TheoriedesoperationsLineaires, MonografieMathematyczne, Warsawa, Poland, 1932.

[2] Cho,Y.J., Fixed points in fuzzy metric spaces, J. Fuzzy Math., 5(4) (1997), 949-962.

[3] George.A and Veeramani.P, On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64(1994), 395-399.

[4] Grabiec.M, Fixed points in fuzzy metric space, fuzzy sets and systems, 27(1988), 385-389.

[5] Hadzic.O. On the (ϵ, λ)topology in probabilistic locally convex spaces, Glasnik Matem, 1978, 13(33), 293-297.

[6] Kramosil.I and Michalek.J, Fuzzy metric and statistical metric spaces, Ky-bernetika, 11(1975), 336-344.

[7] Mishra, S. N., Sharma, N. and Singh, S. L. (1994). Common fixed points of maps in fuzzy metric spaces. *Internat. J. Math. Math. Sci.*, 17, 253-258.

[8] Rajinder Sharma, Common fixed point of weakly compatible mappings under a new property in fuzzy metric spaces, Vol.2, no4, (2012) 38-44, Net work and complex systems.

[9] Sastry.K.P.R, Naidu.G.A, Umadevi.N, Common fixed point theorem for four self maps on a fuzzy metric space under S-B property. (Communicated).

[10] Schweizer.B and Sklar.A, Probabilistic Metric Spaces, North Holland, Amsterdam, 1983.

[11] Sharma, Sushil and Bamboria, D. (2006). Some new common fixed point theorems in fuzzy metric space under strict contractive conditions, *J. Fuzzy Math.*, 14, No. 2, 1-11.

[12] Singh.B and Jain.S. Weak compatibility and fixed point theorems in fuzzy metric spaces, Ganita, 56(2) (2005), 167-176.

[13] Zadeh.L.A, Fuzzy sets, Infor. and Control, 8(1965), 338-353.

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