Research Journal of Pure Algebra -3(3), 2013, Page: 119-125
Available online through www.rjpa.info ISSN 2248–9037

NEIGHBORHOOD CONNECTED EQUITABLE EDGE DOMINATION IN GRAPHS

***Sivakumar. S., N. D. Soner and Anwar Alwardi**
Department of Studies in Mathematics, University of Mysore, Mysore 570 006, India

(Received on: 07-03-13; Revised & Accepted on: 20-03-13)

ABSTRACT

Let G = (V, E) be a graph, for any edge f∈E(G) the degree of f=uv in G is defined by deg(f)=deg(u)+deg(v) –2. A set F⊆E for edges is an equitable edge dominating set of G if every edge f not in F is adjacent to at least one edge $f' \in F$ *such that* $|\deg(f) - \deg(f')| \le 1$. *The minimum cardinality of such equitable edge dominating set is denoted by* $\gamma_e^{'}(G)$ *and is called equitable edge domination number of G. In this paper we introduced The connected equitable edge domination and neighbourhood connected equitable edge domination in a graphs exact value for the some standard graphs bounds and some interesting results are obtained.*

Mathematics subject classification: *05C69.*

Key words and phrases: *Equitable edge dominating set, connected equitable edge dominating set, Neighborhood connected equitable edge dominating set.*

1. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite, undirected with neither loops nor multiple edges the order and size of G are denoted by p and q respectively for graph theoretic terminology we refer to Chartrand and Lesnaik [2] A subset S of V is called a dominating set if $N[S] = V$ the minimum (maximum) cardinality of a minimal dominating set of G is called the domination number (upper domination number) of G and is denoted by $\gamma(G)$, $(\Gamma(G))$. An excellent treatment of the fundamentals of domination is given in the book by Haynes etal [6] A survey of several advanced topics in domination is given in the book edited by Haynes et al. [7]. Various types of domination have been defined and studied by several authors and more than 75 models of domination are listed in the appendix of Haynes et al. [6]. Sampathkumar and Walikar [9] introduced the concept of connected domination in graphs. Let $G = (V, E)$ be a graph and let $v \in V$ the open neighborhood and the closed neighborhood of v are denoted by $N(v)$ and $N[v] = N(v) \cup v$ respectively. If $S \subseteq V$ then $N(S) = \cup_{v \in S} N(v)$ and $N[S] = N(S) \cup S$. If $S \subseteq V$ and $u \in S$ then the private neighbor set of u with respect to S is defined by $Pn[u, S] = \{v : N[v] \cap S = \{u\}\}$.

A dominating set S of G is called a connected dominating set if the induced subgraph $\langle S \rangle$ is connected the minimum cardinality of a connected dominating set of G is called the connected domination number of G and is denoted by $\gamma_c(G)$. Arumugam.S and Sivagnanam.C.[1] introduced the concept of neighborhood connected domination in graphs, A dominating set S of a connected graph G is called a neighborhood connected dominating set (ncd-set) if the induced subgraph $\langle N(S) \rangle$ is connected. The minimum cardinality of a ncd-set of G is called the neighborhood connected domination number of G and is denoted by $\gamma_{nc}(G)$. A ncd-set S is said to be minimal if no proper subset of S is a ncd-set. A coloring of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color. The minimum integer K for which a graph G is k – colorable is called the chromatic number of G and is denoted by $\chi(G)$.

A matching in G=(V, E) is a set M⊆E of pairwise non-adjacent edges. Let Y bea subset of the reals, a function f: V→Y is a Y-dominating function if for every vertex v∈V, f(N(v))≥1.

***Corresponding author:** *Sivakumar. S.*
Department of Studies in Mathematics, University of Mysore, Mysore 570 006, India

a function f: E→Y is a Y-edge dominating function if for every vertex e∈E, f(N(e))≥1.

A subset S of V is called an equitable dominating set if for every $v \in V - S$ there exist a vertex $u \in S$ such that $uv \in E(G)$ and $|d(u) - d(v)| \leq 1$. The minimum cardinality of such an equitable dominating set is denoted by γ_e and is called the equitable domination number of G. A vertex $u \in V$ is said to be degree equitable with a vertex $v \in V$ if $|d(u) - d(v)| \leq 1$. If S is an equitable dominating set then any super set of S is an equitable dominating set. An equitable set S is said to be a minimal equitable dominating set if no proper subset of S is an equitable dominating set. The minimal upper equitable dominating number is Γ_e the upper equitable dominating set of G. If $u \in V$ such that $|d(u) - d(v)| \geq 2$ for every $v \in N(u)$ then u is in every equitable dominating set such points are called an equitable isolated. I_e denotes the set of all equitable isolates. An equitable dominating S of connected graph G is called a connected equitable dominating set (ced-set) if the induced subgraph $\langle S \rangle$ is connected. The minimum cardinality of a ced-set of G is called the connected equitable domination number of G and is denoted by $\gamma_{ce}(G)$. Let $G = (V, E)$ be a graph and let $u \in V$ the equitable neighborhood of u denoted by $N_e(u)$ is defined as $N_e(u) = \{v \in V : | v \in N(u), | d(u) - d(v)| \leq 1$.}The maximum and minimum equitable degree of a point in G are denoted by $\Delta_e(G)$ and $\delta_e(G)$ that is $\Delta_e(G) = max_{u \in V(G)} |N_e(u)|$ and $\delta_e(G) = min_{u \in V(G)} |N_e(u)|$. The open equitable neighbourhood and closed equitable neighbourhood of v are denoted by $N_e(v)$ and $N_e[v] = N_e(v) \cup \{v\}$ respectively. If $S \subseteq V$ then $N_e(S) = \cup_{v \in S} N_e(v)$ and $N[S] = N_e(S) \cup S$. If $S \subseteq V$ and $u \in S$ then the private equitable neighbor set of u with respect to S is defined by $pne[u, S] = N_e[u] - N_e[S - \{u\}]$.

If G is connected graph, then a vertex cut of G is a subset R of $V(G)$ with the property that the subgraph of G induced by $V(G) - R$ is disconnected.

EQUITABLE EDGE DOMINATION NUMBER

Anwar Alwardi and N. D. Soner introduce the Edge Equitable Domination in graphs [3]. Let $G = (V, E)$ be a graph. for any edge $f \in E$ The degree of $f = uv$ in G is defined by $deg(f) = deg(u) + deg(v) - 2$. A set $S \subseteq E$ of edges is equitable edge dominating set of G if every edge f not in S is adjacent to at least one edge $f' \in S$ such that $|deg(f) - deg(f')| \leq 1$.

The minimum cardinality of such equitable edge dominating set is denoted by $\gamma_e'(G)$ and is called equitable edge domination number of G. S is minimal if for any edge $f \in S$, $S - \{f\}$ is not an equitable edge dominating set of G. A subset S of E is called an equitable edge independent set, if for any $f \in S$, $f \notin N_e(g)$, for all $g \in S - \{f\}$. If an edge $f \in E$ be such that $|deg(f) - deg(g)| \geq 2$ for all $g \in N(f)$ then f is in any equitable dominating set. Such edges are called equitable isolates. The equitable neighbourhood of f denoted by $N_e(f)$ is defined as $N_e(f) = \{g \in N(f), |deg(f) - deg(g)| \leq 1\}$. The cardinality of $N_e(f)$ is called the equitable degree of f and denoted by $deg_e(f)$. The maximum and minimum equitable degree of edge in G are denoted respectively by $\Delta_e'(G)$ and $\delta_e'(G)$. That is $\Delta_e'(G) = max_{f \in E(G)} |N_e(f)|$, $\delta_e'(G) = min_{f \in E(G)} |N_e(f)|$. The equitable degree of an edge f in a graph G denoted by $deg_e(f)$ is equal to the number of edges which is equitable adjacent with f. the minimum equitable edge dominating set is denoted by γ_e'-set. In this paper if f and g any two edges in $E(G)$ we say that f and g are equitable adjacent if f and g are adjacent and $|deg(f) - deg(g)| \leq 1$ where $deg(f), deg(g)$ is the degree of the edges f and g respectively. The degree of the edge $f = uv$, $deg(f) = deg(v) + deg(u) - 2$.

2. MAIN RESULT

Definition2.1: An equitable edge dominating set F of a connected graph G is called the connected equitable edge dominating set (ceed-set) if the induced subgraph $\langle F \rangle$ of G is connected. The minimum cardinality of a Ceed-set is called the connected equitable edge domination number and is denoted by $\gamma'_{ce}(G)$.

Observation2.2: A connected equitable edge dominating set of G exits if and only if G is a connected graph G.

Proposition2.3: For any graph G. $\gamma'(G) \le \gamma'_e(G) \le \gamma'_{ce}(G)$

Proof: From the definition of the connected equitable edge dominating set of a graph G, it is clearly that for any graph G any connected equitable edge dominating set F is also an equitable edge dominating set and every equitable edge dominating set is also edge dominating set.

Hence $\gamma'(G) \le \gamma'_e(G) \le \gamma'_{ce}(G)$.

Theorem2.4: For any connected graph G of order p ≥ 3, $\gamma'_{ce}(G) \le p-2$.

Proof: Suppose T be a spanning tree of G. If u is an end vertex of T then p-2 edges of T other than that incident with u from a connected equitable edge dominating set of G, hence the result

The following propositions are straight forward from the definition of Ceed-set.

1) $\gamma'_{ce}(K_p) = p-2, \quad \text{if } p \ge 3$
2) $\gamma'_{ce}(C_p) = p-2, \quad \text{if } p \ge 3$
3) $\gamma'_{ce}(P_p) = p-2 \quad \text{if } p \ge 3$
4) $\gamma'_{ce}(K_{r,\ s}) = \min\{r,\ s\}$

For any tree T of order p at least two cut vertices

$$\gamma'_{ce}(T) = p-1-n$$

Where n is the number of end vertices of T.

Theorem 2.5: For any graph G, $\gamma'_{ce}(G) \le q - \Delta'_e(G)$.

Proof: Let f be an edge in G of an equitable degree $\Delta'_e(G)$ then clearly E(G)–N_e(f) is an connected equitable edge set hence $\gamma'_{ce}(G) \le q - \Delta'_e(G)$.

Propostion2.6: For any graph G without any equitable isolated edges, if F is minimal connected equitable edge dominating set then E–F is equitable edge dominating set.

Proof: Let F be minimal connected equitable edge dominating set of G. Suppose E–F is not an equitable edge dominating set. Then there exist an edge f such that f∈F is not an equitable adjacent to any edge in E–F. Since G has no equitable isolated edges then f is equitable dominated by at least one edge in F–{f}. Thus F–{f} is an equitable edge dominating set a contradiction to the minimality of F, Therefore E–F is an equitable edge dominating set.

Theorem 2.7: For any γ'_{ce}-set F of a graph G = (V, E)

$$|E - F| \le \sum_{f \in F} \deg_e(f),$$

The equality holds if and only if. For every edge f∈E–F, there exists only one edge g∈F such that $N_e(f) \cap F = \{g\}$.

Proof: Since each edge in E–F is equitable adjacent to at least one edge of F. Therefore each edge in E–F contributes at least one to the sum of the equitable degrees of the edges of F. $\left|E-F\right| \le \sum_{f \in F} \deg_e\left(f\right)$,

Suppose the condition is not true, Then $N_e(f) \cap F \ge 2$, For some edge $f \in E-F$. Let f_1 and f_2 belong to $N_e(f) \cap F$. Hence $\sum_{f \in F} \deg_e\left(f\right)$ exceeds E–F by at least one since f_1 counted twice once in $\deg_e(f_1)$ and the once in $\deg_e(f_2)$. Hence if the equality holds then the condition must be true. The converse is obvious.

Theorem 2.8: For any (p, q) graph G, $\left\lceil \frac{q}{\Delta'_e\left(G\right)+1} \right\rceil \le \gamma'_{ce}\left(G\right)$ without equitable isolated edges.

Proof: From the above theorem

$$\left|E-F\right| \le \gamma'_{ce}\left(G\right)\Delta'_e\left(G\right)$$
$$q-\gamma'_{ce}\left(G\right) \le \gamma'_{ce}\left(G\right)\Delta'_e\left(G\right)$$
$$q \le \gamma'_{ce}\left(G\right)\left(\Delta'_e\left(G\right)+1\right)$$

There fore $$\left\lceil \frac{q}{\Delta'_e\left(G\right)+1} \right\rceil \le \gamma'_{ce}\left(G\right)$$

Theorem 2.9: A connected equitable edge dominating set F is minimal if and only if for each edge $f \in F$ one of the following conditions holds.

1) $N_e(f) \cap F \ne \phi$.
2) There exist an edge $g \in E-F$ such that $N_e(g) \cap F = \{f\}$.

Proof: Suppose F is minimal connected equitable edge dominating set. Assume that (1) and (2) do not hold. Then for some $f \in F$ there exist an edge $g \in N_e(f) \cap F$ and for every edge $h \in E-F$. $N_e(h) \cap F = \{f\}$. Therefore $F-\{f\}$ is not an equitable edge dominating set contradiction to minimality of F. Therefore (1) or (2) holds.

Conversely, Suppose for every $f \in F$. One of the conditions holds. Suppose F is not minimal. Then there exist $f \in F$ such that $F-\{f\}$ is not an equitable edge dominating set. Therefore there exist an edge $g \in F-\{f\}$ such that $g \in N_e(f)$. Hence f does not satisfy (1). Then f must satisfy (2). Then there exist an edge $g \in E-F$ such that $N_e(g) \cap F=\{f\}$ since $F-\{f\}$ is an equitable edge dominating set. There exist an edge $f' = F-\{f\}$ such that f' is equitable adjacent to g. Therefore $f' \in N_e(g) \cap F$ and $f' \ne f$, a contradiction to $N_e(g) \cap F=\{f\}$. Hence F is minimal connected equitable edge dominating set.

3. MAIN RESULT

Definition 3.1: An equitable edge dominating set F of connected graph G is called the neighbourhood connected equitable edge dominating set (nceed-set) if the edge induced subgraph $\langle N(F)\rangle$ of G is connected. The minimum cardinality of a nceed-set is called the neighbourhood connected equitable edge domination number (nceed-number) and is denoted by $\gamma'_{nce}\left(G\right)$.

Proposition3.2: For any graph G, $\gamma'_e\left(G\right) \le \gamma'_{nce}\left(G\right) \le \gamma'_{ce}\left(G\right)$

Proposition 3.3: For any graph G, $\gamma'_e\left(G\right) \le \gamma'_{nce}\left(G\right) \le 2\gamma'_e\left(G\right)$

Proof: Let G be a connected graph and let F be an equitable edge dominating set of G. obviously pairing $e \in X$ with a private neighbour forms a nceed-set of cardinality $2\gamma'_e\left(G\right)$.

Theorem 3.4: For the path P_p, $p \ge 2$, $\gamma'_{nce}\left(P_p\right) = \left\lceil \frac{p-1}{2} \right\rceil$

Proof: Let $P_p = (v_1, v_2, \ldots, v_p)$ and let $e_i = v_i v_{i+1}$, if p is odd then F ={e_j: j=2k or 2k+1 and k is odd} is a nceed set of P_p and if p is even then $F_1 = F \cup \{e_{p-1}\}$ is a nceed-set of P_p. Hence $\gamma'_{nce}(P_p) \le \left\lceil \frac{p-1}{2} \right\rceil$. Further if F is any γ'_{nce}-set of P_p. Then $N_e(F)$ contains all the internal edges of P_p and hence $|F| \ge \left\lceil \frac{p-1}{2} \right\rceil$. Thus $\gamma'_{nce}(P_p) = \left\lceil \frac{p-1}{2} \right\rceil$.

Corollary 3.5: For any non-trivial path P_p,

a) $\gamma'_{nce}(P_p) = \gamma'_e(P_p)$ if and only if p=3 or 5.

b) $\gamma'_{nce}(P_p) = \gamma'_{ce}(P_p)$ if and only if p=2, 3, 5 or 6.

Proof: Since $\gamma'_e(P_p) = \left\lceil \frac{p-1}{3} \right\rceil$ and $\gamma'_e(P_p) = p-3$ the corollary follows.

Theorem 3.6: For the cycle C_p and p vertices.

$$\gamma'_{nce}(C_p) = \begin{cases} \left\lceil \frac{p}{2} \right\rceil & \text{if } p \not\equiv 3 (\text{mod } 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & \text{if } p \equiv 3 (\text{mod } 4) \end{cases}$$

Proof: Let $C_p = (v_1, v_2 \ldots v_p, v_1)$ and p=4k+r where $0 \le r \le 3$ and $e_i = v_i v_{i+1}$. Let F= {e_i, i=2j, 2j+1, j is odd} and
$1 \le j \le 2k-1$}

Let $$F_1 = \begin{cases} F & \text{if } p \equiv 0 (\text{mod } 4) \\ F \cup \{e_p\} & \text{if } p \equiv 1 \text{ or } 2 (\text{mod } 4) \\ F \cup \{e_{p-1}\} & \text{if } p \equiv 3 (\text{mod } 4) \end{cases}$$

Clearly F_1 is a nceed-set of C_p and hence

$$\gamma'_{nce}(C_p) = \begin{cases} \left\lceil \frac{p}{2} \right\rceil & \text{if } p \not\equiv 3 (\text{mod } 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & \text{if } p \equiv 3 (\text{mod } 4) \end{cases}$$

Now, Let F be and γ'_{nce}-set of C_p then $\langle F \rangle$ contains at most one isolated edge.

$$\langle N_e(F) \rangle = \begin{cases} C_p - \{e\} & \text{if } p \not\equiv 0 (\text{mod } 4) \\ C_p & \text{if } p \equiv 0 (\text{mod } 4) \end{cases}$$

Hence

$$|F| \ge \begin{cases} \left\lceil \frac{p}{2} \right\rceil & \text{if } p \not\equiv 3 (\text{mod } 4) \\ \left\lfloor \frac{p}{2} \right\rfloor & \text{if } p \equiv 3 (\text{mod } 4) \end{cases}$$

And the result follows

Corollary 3.7:

a) $\gamma'_{ne}(C_p) = \gamma'_e(C_p)$ if and only if p=3, 4, or 7.

b) $\gamma'_{nce}(C_p) = \gamma'_{ce}(C_p)$ if and only if p=3, 4, or 5.

Proof: since $\gamma'_{nce}(C_p) = \left\lceil \frac{p}{3} \right\rceil$ and $\gamma'_{ce}(C_p) = p-2$ the result follows.

Theorem 3.8: $\gamma'_{nce}(K_p) = \left\lfloor \frac{p}{2} \right\rfloor$, if p≥3.

Proof: Let F be an equitable edge dominating set of K_p also $\langle N_e(F)\rangle = K_p - F$ which is connected. Hence F is a nceed-set which implies $\gamma'_{nce}(K_p) \le |F| = \left\lfloor \frac{p}{2} \right\rfloor$. Since $\gamma'_e(K_p) = \left\lfloor \frac{p}{2} \right\rfloor$ the result follows.

Theorem 3.9: $\gamma'_{nce}(K_{r,s}) = \min\{r, s\}$, $|r-s| \le 1$

Proof: Let v be a vertex such that $deg_e(v)=\min\{r, s\}$. Let F be the set of all edges incident with v. It is clear that F is an equitable edge dominating set. Also $\langle N_e(F)\rangle = K_{r,s}$ if $K_{r,s}$ is a star and $\langle N_e(F)\rangle = K_{1,p-1}$ Thus F is a nceed-set. Hence $\gamma'_{nce}(K_{r,s}) \le |F| = deg_e(v) = \min\{r, s\}$ since $\gamma'_e(K_{r,s}) = \min\{r, s\}$ the result follows.

Theorem 3.10: For a tree T, $\gamma'_{nce}(T) = 1$ if and only if T is a star.

Proof: Let $\gamma'_{nce}(T) = 1$ and Let F={e} be the γ'_{nce}-set of G. Let e=uv and let $deg_e(u) \ge 2$.

If $deg_e(v)>1$. Then $\langle N_e(F)\rangle = T - e$ is disconnected. Hence $deg_e(v)=1$. Thus T is a star. The converse is obvious.

Lemma 3.11: A superset of a nceed-set is a nceed-set.

Proof: Let F be a nceed-set of graph G and let $F_1 = F \cup \{e\}$ where $e \in E-F$. Let e=uv clearly $e \in N_e(F)$ and F_1 is an equitable edge dominating set of G. Now Let f, $g \in V(\langle N_e(F_1)\rangle)$. If f, $g \in V(\langle N_e(F)\rangle)$ then any f-g path in $\langle N_e(F)\rangle$ is a f-g path in $\langle N_e(F_1)\rangle$. If $f \in V(\langle N_e(F)\rangle)$ and $g \notin V(\langle N_e(F)\rangle)$, Then without loss of generality we assume f-u path in $\langle N_e(F)\rangle$ and hence f-u path together with u-g path gives a f-g path in $\langle N_e(F_1)\rangle$. Also if f, $g \notin V(\langle N_e(F)\rangle)$ then (f, u, v, g) or (f, v, u, g) or (f, u, g) or (f, v, g) or (f, g) is a f-g path in $\langle N_e(F_1)\rangle$. Thus $\langle N_e(F_1)\rangle$ is connected so that F_1 is a nceed-set of G.

Theorem3.12: A nceed-set F of a graph G is a minimal nceed-set if and only if for every $e \in F$ one of the following holds.

1. $P_{ne}[e, F] \ne \phi$,
2. There exists two vertices f, $g \in \langle N_e(F)\rangle$ such that f-g path in $\langle N_e(F)\rangle$ contains at least one edge of $N_e(F) - N_e(F-\{e\})$.

Proof: Let F be a minimal nceed-set of G. Let $e \in F$ and Let $F_1 = F - e$. Then either F_1 is not an equitable edge dominating set of G or $\langle N_e(F_1)\rangle$ is disconnected. If F_1 is not an equitable edge dominating set of G, Then $P_{ne}[e, F] \ne \phi$. If $\langle N_e(F_1)\rangle$ is disconnected then there exists two vertices f, $g \in \langle N_e(F)\rangle$ such that there is no f-g path in $\langle N_e(F)\rangle$. Since $\langle N_e(F)\rangle$ is connected, it follows that every f-g path in $\langle N_e(F_1)\rangle$ contains at least one equitable edge of $N_e(F) - N_e(F-\{e\})$. Conversely F is a nceed-set of G satisfying the conditions of theorem, Then F is 1-minimal and hence the result follows from lemma.

Theorem 3.14: Let G be a graph with $\Delta'_e < q-1$ then $\gamma'_{nce}(G) \le q - \Delta'_e$.

Proof: Suppose $e \in E(G)$ and $deg_e(e) = \Delta_e$`, Since G is connected and $\Delta'_e < q-1$. There exists two equitable adjacent edges e_1 and e_2 such that $e_1 \in N_e(e)$ and $e_2 \in N_e[e]$. Now, Let $F = (N_e(e) - \{e_1\} \cup \{e_2\})$. Clearly E–F is a nceed-set of

G and hence $\gamma'_{nce}(G) \le q - \Delta'_e$. This bound is sharp for P_5, C_4.

Theorem 3.15: For any graph G $\gamma'_{nce}(G) \le \left\lfloor \frac{3p}{4} \right\rfloor$

Proof: Let F be a maximum matching of the graph G. Label the edges of F by $e_1, e_2 \ldots e_k, e_{k+1}, \ldots, e_r$ such that the edges e_i and e_{i+1}, i is odd $1 \le i \le k-1$ are equitable adjacent to common edge $f(e_i)$ with maximum value of k, Let $Y = \{f(ei) / i \text{ is odd}\}$. Then $F \cup Y$ is an equitable edge dominating set with $\langle N_e(F \cup Y) \rangle$ is connected and hence $\gamma'_{nce}(G) \le |F \cup Y| = \left\lfloor \frac{3p}{4} \right\rfloor$.

The above found is sharp the graph C_5

$$\gamma'_{nce}(C_5) = 3 = \left\lfloor \frac{3p}{4} \right\rfloor$$

ACKNOWLEDGMENTS
This research was supported by UGC-SAP, DRS-1, No F.510/2/DRS/201 (SAP-1). India

REFERENCES

[1] Arumugam.S and Sivagnanam.C., "Neighborhood connected domination in graphs", J. Combin. Math,Combin Compat.,Vol.73, PP.55-64, 2010.

[2] G. Chartrand and L. Lesnaik, Graphs and Digraphs, Chapman and Hall. CRC, 4th edition, 2005.

[3] Anwar Alwardi and N.D. Soner, Equitable edge domination in graphs (2012).

[4] K. D. Dharmalingam, Studies in Graph Theorey-Equitable domination and bottleneck domination, Ph.D Thesis (2006).

[5] Harary. F., and T.W., Haynes, Double domination in in Graphs, Ars Combin, 55,(2000), 201-213.

[6] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, (Fundamentals of Domination in Graphs) Marcel Dekker Inc., 1998.

[7] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs Advanced Topics, Marcel Dekker Inc., 1998.

[8] S.T. Hedetniemi and R. Laskar, Connected domination in graphs, Graph Theory and Combinatorics, B. Bollobas. Ed.. Academic Press, London. 1984, 209-217.

[9] E. Sampathkumar and H.B. Walikar, the connected domination number of a graph, J. Math. Phys. Sci., 13(1979), 607-613.

Source of support: UGC-SAP, DRS-1, No F.510/2/DRS/201 (SAP-1). India, Conflict of interest: None Declared