Research Journal of Pure Algebra -3(3), 2013, Page: 126-133
Available online through www.rjpa.info ISSN 2248–9037

GROUP SYMMETRY ANALYSIS FOR MHD BOUNDARY LAYER FLOW WITH DIFFUSION AND CHEMICAL REACTION PAST A STRETCHING

R. M. Darji*
**Dept of Mathematics, Sarvajanik College of Engg. and Tech, Surat-395001, Gujarat, INDIA*

M. G. Timol**
***Dept. of Mathematics, Veer Narmad South Gujarat University, Surat-395007, Gujarat, INDIA*

(Received on: 07-03-13; Revised & Accepted on: 19-03-13)

ABSTRACT

The electrically conducting magnetohydrodynamics (MHD) laminar incompressible boundary layer for the Newtonian fluid past a stretching sheet with diffusion and chemical reaction using group symmetry method is investigated. The symmetry groups admitted by the governing boundary value problem are obtained. Particular attention is paid on the deductive group technique which provides the general group transformation and hence similarity solution of the problem. Finally, with the use of the entailed similarity variables the problem is transformed into a boundary value problem of ODEs and is solved numerically using collocation method.

Keywords: *Group symmetry, boundary layer flow, similarity solution.*

AMS Subject classification: *76A05, 76M55, 54H15.*

1. INTRODUCTION

The group symmetry method so-called deductive group method is applied for an analysis of a particular problem of fluid mechanics. The main advantage of such method is that they can successfully be applied to non-linear partial differential equations (See Darji and Timol [1, 2, 3]). The symmetries of a differential equation are those continuous groups of transformations under which the differential equation remains invariant, that is, a symmetry group maps any solution to another solution. The interesting point is that, having obtained the symmetries of a specific problem, one can proceed further to find out the group-invariant solutions, which, in the case of the group of transformations, are nothing but the well-known similarity solutions. The similarity methods are quite popular because they result in the reduction of the independent variables of the problem. In our case, the problem under investigation is two-dimensional. Hence, any similarity transformations will transform the system of PDEs into a system of ODEs. In this context, we have systematically developed the group transformation and hence similarity solution of the problem under the similarity requirement by employing the general group symmetry method so-called deductive method [4]. This technique probably provides the group transformation without any adhoc assumptions and that will transform the governing equations in to the ordinary differential equations.

The flow past a linearly stretching sheet is a standard problem in fluid mechanics. The problem has huge practical applications in polymer processing industries, paper production, several biological process and many others. The diffusion of chemically reactive species in boundary layer flow around the stretching sheet is very important in chemical industries, metal and polymer processing industries during extrusion process.

Crane [5] was the first person to study the laminar boundary layer flow caused by a stretching sheet with stretching velocity varying linearly with distance from a fixed point. The heat and mass transfer in Newtonian boundary layer flow past a stretching sheet with suction or blowing was studied by Gupta and Gupta [6].Grubka and Bobba [7] investigated the heat transfer characteristics of the stretching sheet problem with variable temperature. Tapanidis et al. [8] discussed the application of scaling group of transformations to viscoelastic second-grade fluid flow. Recently, Mukhopadhyay et al. [9] studied the MHD boundary layer flow and heat transfer over a stretching sheet with variable viscosity using the scaling group of transformations.

However, there are a several investigations on chemical reaction effects on flowing fluid done by many researchers. Das et al. [10] investigated the effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. Anderson et al. [11] studied the diffusion of a chemically reactive species

***Corresponding author:** *R. M. Darji**
**Dept of Mathematics, Sarvajanik College of Engg. and Tech, Surat-395001, Gujarat, INDIA*

from a stretching sheet. The similarity solutions of mixed convection with diffusion and chemical reaction over a horizontal moving plate were obtained by Fan et al. [12]. The study of constant MHD hall effect for free convective flow and mass transfer over a stretching sheet with chemical reaction was done by Afify [13]. Recently, Cortell [14, 15] discussed the effects of magnetic field on the flow and mass transfer of second grade fluid in a porous medium over a stretching sheet with chemically reactive species and also analyzed the motion and mass transfer for two classes of viscoelastic fluid past a porous stretching sheet with suction or blowing.

As in all above work the similarity transformations are to be assumed by adhoc manner. Motivated by this work, we propose to investigate the application of deductive group of transformations to analyze the steady boundary layer flow past a stretching sheet with diffusion and chemically reactive species in presence of transverse magnetic field, undergoing a first order reaction. By systematically searching group of transformations, the set of governing partial differential equations for the flow and concentration distribution are transformed into a set of self-similar ordinary differential equations. The appropriate boundary conditions for the velocity and mass concentration fields are also transformed. The solution of the momentum equation is obtained analytically and the concentration equation is solved numerically by collocation algorithm in Matlab.

2. PROBLEM FORMULATION:

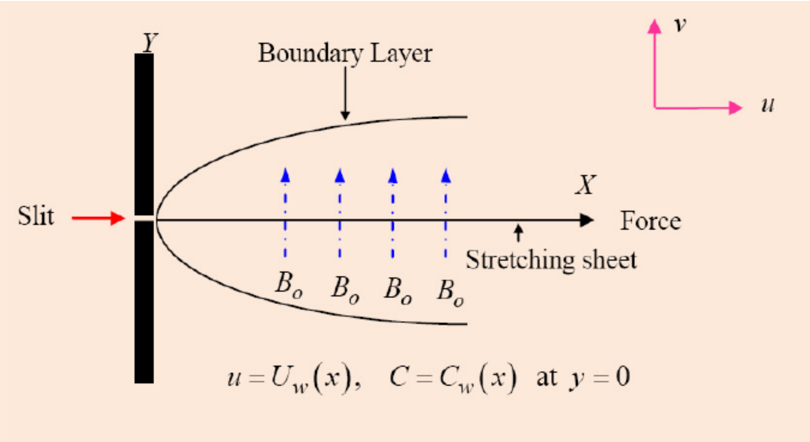

Figure 1: Schematic of two-dimensional stretching sheet

The physical situation considered for the investigation is that of a steady state, laminar boundary layer flow of an electrically conducting incompressible viscous fluid in the presence of a transverse magnetic field B_0 due to a stretching horizontal sheet as shown in Fig. 1. The flow is generated by the action of two equal and opposite forces along the *X*-axis and the sheet is stretched with a velocity that is proportional to the distance from the slit. Let the concentration at the stretching sheet is $C_w(x) = cx$, *c* is constant. The stretching sheet assumed velocity of the form $U_w(x) = bx$ where *b* is the stretching constant and x is the distance from the slit. It is also assumed that the magnetic Reynolds number Re_m is very small; i. e. $\mathrm{Re}_m = \mu_0 \sigma b L \ll 1$ where μ_0 is the magnetic permeability, *L* is the reference length and σ is the electric conductivity. We neglect the induced magnetic field, which is small in comparison with the applied magnetic field. Applying boundary layer approximation the governing continuity, momentum and concentration equations may be written in usual notation as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B_0^2}{\rho}u \tag{2}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D\frac{\partial^2 C}{\partial y^2} - kC \tag{3}$$

In the above equations, u and v are the velocity components along the X and Y axes, respectively, μ is the kinematic velocity, ρ is the fluid density, *C* is the concentration, *D* is the diffusion coefficient, *k* denotes the reaction rate constant.

The relevant boundary conditions applicable to the flow are:

$$\left.\begin{aligned} &u(x,0)=U_w(x)\quad [=bx],\quad v(x,0)=0,\\ &C(x,0)=C_w(x)\quad [=cx],\\ &u(x,y)=C(x,y)=0\quad \text{as}\quad y\to\infty \end{aligned}\right\} \tag{4}$$

We introduce following dimensionless quantity:

$$x^*=\frac{x}{L},\quad y^*=\sqrt{\mathrm{Re}}\,\frac{y}{L},\quad u^*=\frac{u}{U_0},\quad v^*=\sqrt{\mathrm{Re}}\,\frac{v}{U_0},\quad C^*=\frac{C}{C_0}$$

Where $U_0=bL$ is the reference moving speed, $\mathrm{Re}=\dfrac{U_0L}{\nu}$ is the Reynolds number, $C_0=cL$ is the reference concentration. Dropping the asterisks (for simplicity) the boundary layer equations become

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{5}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}-Mu \tag{6}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-\delta C \tag{7}$$

In which $Sc=\dfrac{\nu}{D}$ is the Schmidt number, $\delta=\dfrac{kL}{U_0}$ is the reaction rate parameter, $M=\dfrac{\sigma B_0^2}{\rho L}$ is refereed as magnetic field strength parameter.

The boundary conditions become

$$\left.\begin{aligned} &u(x,0)=x,\\ &v(x,0)=0,\\ &C(x,0)=x,\\ &u(x,y)=C(x,y)=0\quad \text{as}\quad y\to\infty \end{aligned}\right\} \tag{8}$$

Introducing dimensionless stream function $\psi(x,y)$ such that $u=\dfrac{\partial\psi}{\partial y}$ and $v=-\dfrac{\partial\psi}{\partial x}$ which satisfies equation (5) identically. Then equations (6) and (7) become

$$\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y^2}=\frac{\partial^3\psi}{\partial y^3}-M\frac{\partial\psi}{\partial y} \tag{9}$$

$$\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x}-\frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial y^2}-\delta C \tag{10}$$

The boundary conditions become

$$\left.\begin{aligned} &\frac{\partial\psi}{\partial y}(x,0)=x,\quad \sqrt{2}\,\frac{\partial\psi}{\partial x}(x,0)=0,\\ &C(x,0)=x,\\ &\frac{\partial\psi}{\partial y}(x,y)=C(x,y)=0\quad \text{as}\quad y\to\infty \end{aligned}\right\} \tag{11}$$

3. DEDUCTIVE GROUP SYMMETRY ANALYSIS

The method of solution depends on the application of the class of a one-parameter continuous deductive group of transformations to the system of partial differential equations (9) and (10). Under this class, first, we search the subgroup of transformations, through which one will reduce the two independent variables by one and the system of non-linear partial differential equations (9) to (11) will transform to the system of ordinary differential equations.

$$C_G : \quad T_a(Q) = \aleph^Q(a)s + \Re^Q(a) = \bar{Q} \tag{12}$$

Where Q stands for x, y, ψ, C whereas $\aleph's$ and $\Re's$ are real-valued and are at least differentiable in the real argument a.

To transform the differential equation, transformations of the derivatives of ψ are obtained from C_G via chain-rule operations:

$$\left.\begin{aligned} \bar{s}_{\bar{i}} &= \left(\frac{\aleph^Q}{\aleph^i}\right) Q_i \\ \bar{Q}_{\bar{i}\,\bar{j}} &= \left(\frac{\aleph^Q}{\aleph^i \aleph^j}\right) Q_{ij} \end{aligned}\right\} \quad Q = \psi, C\ ; \quad i, j = x, y \tag{13}$$

Equation (9) and (10) are said to be invariantly transformed, for some function $\xi_1(a)$ and $\xi_2(a)$ whenever,

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial^2\bar{\psi}}{\partial\bar{y}^2} - \frac{\partial^3\bar{\psi}}{\partial\bar{y}^3} + M\frac{\partial\bar{\psi}}{\partial\bar{y}} = \xi_1(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial^3\psi}{\partial y^3} + M\frac{\partial\psi}{\partial y}\right] \tag{14}$$

$$\frac{\partial\bar{\psi}}{\partial\bar{y}}\frac{\partial\bar{C}}{\partial\bar{x}} - \frac{\partial\bar{\psi}}{\partial\bar{x}}\frac{\partial\bar{C}}{\partial\bar{y}} - \frac{1}{Sc}\frac{\partial^2\bar{C}}{\partial\bar{y}^2} + \delta\bar{C} = \xi_2(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y} - \frac{1}{Sc}\frac{\partial C}{\partial y} + \delta C\right] \tag{15}$$

Substituting the values from the equation (12) and (13) in above system of equations, yields

$$\frac{(\aleph^\psi)^2}{\aleph^x(\aleph^y)^2}\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2}\right] - \frac{\aleph^\psi}{(\aleph^y)^3}\frac{\partial^3\psi}{\partial y^3} + M\frac{\aleph^\psi}{\aleph^y}\frac{\partial\psi}{\partial y} = \xi_1(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial^2\psi}{\partial y\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial^2\psi}{\partial y^2} - \frac{\partial^3\psi}{\partial y^3} + M\frac{\partial\psi}{\partial y}\right] \tag{16}$$

$$\frac{\aleph^\psi\aleph^C}{\aleph^x\aleph^y}\left[\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y}\right] - \frac{1}{Sc}\frac{\aleph^C}{(\aleph^y)^2}\frac{\partial^2 C}{\partial y^2} + \delta\left(\aleph^C C + \Re^C\right) = \xi_2(a)\left[\frac{\partial\psi}{\partial y}\frac{\partial C}{\partial x} - \frac{\partial\psi}{\partial x}\frac{\partial C}{\partial y} - \frac{1}{Sc}\frac{\partial^2 C}{\partial y^2} + \delta C\right] \tag{17}$$

The invariance of equations (16) and (17) together with boundary conditions, implies that

$$\frac{(\aleph^\psi)^2}{\aleph^x(\aleph^y)^2} = \frac{\aleph^\psi}{(\aleph^y)^3} = \frac{\aleph^\psi}{\aleph^y} = \xi_1(a)$$

$$\frac{\aleph^\psi\aleph^C}{\aleph^x\aleph^y} = \frac{\aleph^C}{(\aleph^y)^2} = \aleph^C = \xi_2(a)$$

These yields,

$$\aleph^\psi = \aleph^x = \aleph^C, \quad \aleph^y = 1, \quad \Re^x = \Re^y = \Re^\psi = \Re^C = 0 \tag{18}$$

The one-parameter group *G*, that transforms invariantly the differential equations (9) and (10) with the auxiliary conditions (7) is,

$$G : \left\{\begin{array}{l} G_H : \left\{\begin{array}{l} \bar{x} = \aleph^x x \\ \bar{y} = y \end{array}\right. \\ \bar{\psi} = \aleph^x \psi \\ \bar{C} = \aleph^x C \end{array}\right. \tag{19}$$

3.1 Complete set of absolute invariants:

If $\eta=\eta(x,y)$ is the absolute invariant of the independent variables then the absolute invariants for the dependent variables ψ and C are:

$$\Gamma_j(x,y,\psi,C)=f_j(\eta),\quad j=1,2 \tag{20}$$

And are given by the first-order linear partial differential equation, [See (4)]

$$\sum_{i=1}^{4}(\alpha_i Q_i+\beta_i)\frac{\partial\Gamma}{\partial Q_i}=0, Q_i=x,y,\psi,C \tag{21}$$

Where

$$\alpha_i=\left.\frac{\partial\aleph^i}{\partial a}\right|_{a=a^0}\quad \text{and}\quad \beta_i=\left.\frac{\partial\Re^i}{\partial a}\right|_{a=a^0}\qquad i=1,2,3,4 \tag{22}$$

(15) and 'a^0' denotes the value of 'a' which yields the identity element of the group *G*.

Using the definition of $\alpha's$ and $\beta's$, the characteristic equation of (21) is given by

$$\frac{dx}{x}=\frac{dy}{0}=\frac{d\eta}{0}=\frac{d\psi}{\psi}=\frac{dC}{C}=\frac{d\Gamma}{0} \tag{23}$$

The absolute invariants of the group (19) owing the equation (23) are given by,

$$\eta=y,\quad \psi=xf(\eta),\quad C=xg(\eta) \tag{24}$$

3.2 Reduction to ordinary differential equation:

Substituting the values of partial derivatives from (24) in equation (9) and (10), we get

$$f'''-(f')^2+ff'-Mf'=0 \tag{25}$$

$$\frac{1}{Sc}g''+fg'-\delta g=0 \tag{26}$$

Together with boundary conditions,

$$f(0)=0,\quad f'(0)=1,\quad f'(\infty)=0 \tag{27}$$

$$g(0)=1,\quad g(\infty)=0 \tag{28}$$

3.3 Physical quantities of interest:

The momentum boundary layer equation (25) subjected to the boundary conditions (27) has an exact solution (see Pavlov [16]) of the form

$$f(\eta)=\frac{1-e^{-m\eta}}{m},\ \text{where } m=\sqrt{1+M}$$

(29)

The similarity function related to horizontal velocity is given by

$$f'(\eta)=e^{-m\eta} \tag{30}$$

The expression for local skin-friction coefficient C_{fx} is given by

$$\frac{1}{2}\sqrt{\mathrm{Re}_x}C_{fx}=-f''(0)=\sqrt{1+M} \tag{31}$$

where $\sqrt{\mathrm{Re}_x}$ is the local Reynolds number. Equation (26) subject to the boundary conditions (28) becomes

$$\frac{1}{Sc}g'' + \left(\frac{1-e^{-m\eta}}{m}\right)g' - \delta g = 0 \tag{32}$$

4. NUMERICAL SOLUTION OF THE PROBLEM

We use the collocation method (see [17]) in finding the numerical solutions of the resulting boundary value problems. The numerical procedure is treated using computational appropriate algorithm in Matlab. To solve the boundary value problems by the collocation method Eq. (32) is transformed into a system of two first order differential equations as follows:

$$\begin{aligned} \frac{df_0}{d\eta} &= f_1 \\ \frac{df_1}{d\eta} &= f_2 = Sc\left\{\delta f_0 - \left(\frac{1-e^{-m\eta}}{m}\right)f_1\right\} \end{aligned} \tag{33}$$

Subsequently the boundary conditions (28) take the form

$$f_0(0) = 0, \quad f_0(\infty) = 1, \quad \text{where} \quad f_0 = g(\eta) \tag{34}$$

We cut the infinite interval at a finite, large enough point and insert additional, so-called asymptotic boundary conditions at the far (right) end and then solve the resulting two-point boundary value problem by an A-stable symmetric collocation method. The resulting system of equations (32) is transformed into a system of non-linear algebraic equations (finite difference equations) using a central difference scheme with uniform mesh points. The boundary conditions in (32) form a part of the system of finite difference equations. The transformed system of non-linear algebraic equations is then linearized by Newtons method. This system of linear algebraic equations is then solved by the Gauss elimination method. Shooting method (see [18]) is used to obtain the initial guess solution. The results are presented in several graphs to steady an effect of Schmidt number *Sc*, reaction parameter δ and magnetic field strength *M* on the concentration.

5. RESULTS AND DISCUSSIONS:

- MHD boundary layer flow with diffusion and chemical reaction in an electrically conducting Newtonian fluid over a stretching sheet is investigated by systematically searching the group of transformations.
- Analytical solution for the momentum boundary layer equation shows that the horizontal velocity component is exponentially decreased where as the local skin friction coefficient is increased as the magnetic field strength increased.
- Numerical solution for chemical concentration obtained under the effect of several flow parameters *M*, *Sc* and δ by using the collocation method.
- Fig. 2 depicts the effect of Schmidt number by controlling the magnetic field strength and concentration parameter. It shows that the concentration decreases as the Schmidt number increase that warrants the fact of chemical reaction phenomenon.
- Fig. 3 projects the intensity of magnetic field on the concentration. From this profile it is evident that as the intensity of the magnetic field parameter *M* increases the chemical concentration of fluid is sharply increase. It shows the importance of magnetic field during chemical reaction.
- Fig. 4 depicts the influence of reaction parameter. This graph suggest that in presence of magnetic field and controlling the Schmidt number, an increase in reaction parameter will decrease the chemical concentration. Fig. 2 and Fig. 3 display the co-related effects of Schmidt number and reaction parameter in presence of magnetic field.

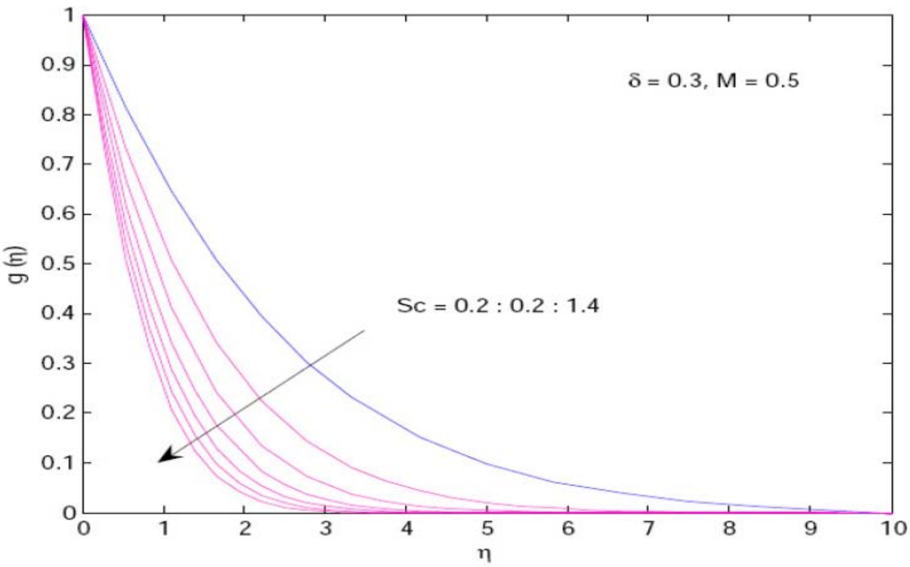

Figure 2: Effect of Schmidt number on dimensionless concentration

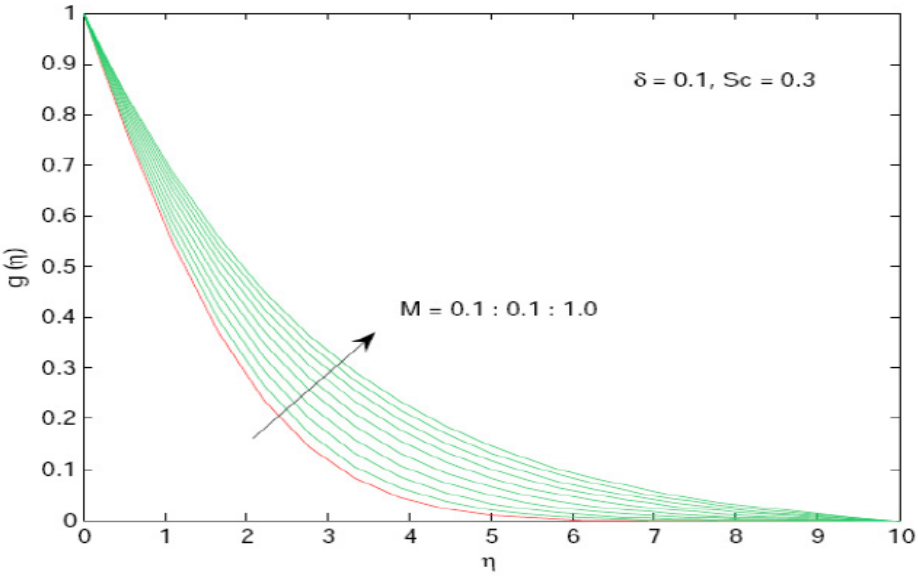

Figure 3: Intensity of magnetic strength on concentration

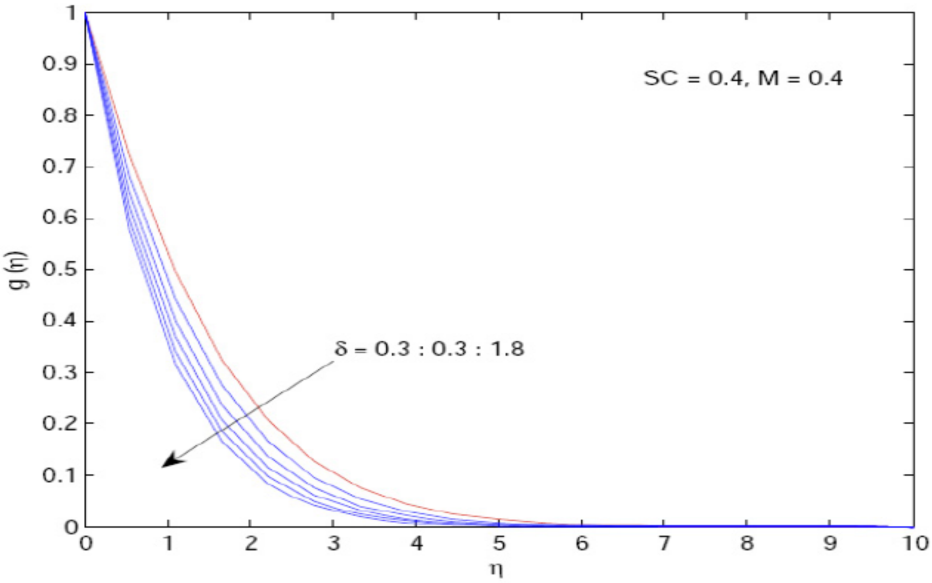

Figure 4: Influence of reaction parameter on concentration

6. CONCLUSION

The general group symmetry method based on general group transformations so-called deductive method is applied for the analysis of particular problem of boundary layer theory. The system of non-linear partial differential equations that governs the MHD boundary layer flow past a stretching sheet with diffusion and chemical reaction, is finally transformed into a single ordinary non-linear differential equation subject to the governing boundary conditions. The similarity solutions are produced numerically using A-stable collocation method. It is worth to notice that analysis has been made for non-dimensional quantities and hence it is valid for all types of under considered fluids

REFERENCES

[1] R. M. Darji and M. G. Timol, Deductive group theoretic analysis for MHD flow a sisko fluid in a porous medium, Int. J. Appl. Math. and Mech., 7(19) (2011), 49-58.

[2] R. M. Darji and M. G. Timol, Similarity solutions of laminar incompressible boundary layer equations of Non-Newtonian viscoinelastic fluids, Int. J. Mathematical Archive, 2(8) (2011), 1395-1404.

[3] R. M. Darji and M. G. Timol, Group-theoretic analysis for natural convection boundary layer flow of a class of Non-Newtonian fluids, Int. J. Adv. Sci. and Tech. Research, 2(1) (2011), 289-311.

[4] M. J. Moran and R. A. Gaggioli, Reduction of the number of variables in systems of partial differential equations with auxiliary conditions. SIAM J. Appl. Math., 16 (1968), 202-215.

[5] L. J. Crane, Flow past a stretching plate, ZAMP, 21 (1970), 645-647.

[6] P. S. Gupta and A. S. Gupta, Heat and mass transfer on a stretching sheet with suction or blowing, Can. J. Chem. Eng., 55 (1977), 744-746.

[7] L. G. Grubka and K. M. Bobba, Heat transfer characteristics of a continuous stretching surface with variable temperature, ASME J. Heat Transfer, 107 (1985), 248-250.

[8] T. Tapanidis, G. Tsagas and H. P. Mazumdar, Application of scaling group of transformations to viscoelastic second-grade fluid flow, Nonlinear Funct. Anal. Appl., 8 (2003), 345-350.

[9] S. Mukhopadhyay, G. C. Layek and Sk. A. Samad, Study of MHD boundary layer flow over a heated stretching sheet with variable viscosity, Int. J. Heat Mass Trans., 48 (2005), 4460-4466.

[10] U. N. Das, R. Deka and V. M. Soundalgekar, Effect of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, Forsch. Ingenieurwes., 60 (1994), 284-287.

[11] H. I. Anderson, O. R. Hansen and B. Holmedal. Diffusion of a chemically reactive species from a stretching sheet, Int. J. Heat Mass Trans., 37 (1994), 659-664.

[12] J. R. Fan, J. M. Shi and X. Z. Xu, Similarity solution of mixed convection with diffusion and chemical reaction over a horizontal moving plate, Acta Mech., 126 (1998), 59-69.

[13] A. Afify, MHD free convective flow and mass transfer over a stretching sheet with chemical reaction, Heat Mass Trans., 40 (2004), 495-500.

[14] R. Cortell, MHD flow and mass transfer of an electrically conducting fluid of second grade in a porous medium over a stretching sheet with chemically reactive species, Chem. Eng. Process., 46 (2007), 721-728.

[15] R. Cortell, Toward an understanding of the motion and mass transfer with chemically reactive species for two classes of viscoelastic fluid over a porous stretching sheet, Chem. Eng. Process., 46 (2007), 982-989.

[16] K.B. Pavlov, Magnetohydrodynamic flow of an impressible viscous fluid caused by deformation of a surface, Magnitnaya Gidrodinamika, 4 (1974), 146-147.

[17] P. A. Markowich and C. A. Ringhofer, Collocation Methods for Boundary Value Problems on 'Long' Intervals , Maths. of comp., 40(161) (1983), 123-150.

[18] S.D. Conte and C. de Boor, Elementary Numerical Analysis, McGraw-Hill, New York, (1972).

Source of support: Nil, Conflict of interest: None Declared

