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SOME RAMANUJAN INTEGRALS INVOLVING THE GENERALIZED ZETA FUNCTION AND THE $\tilde{H}$-FUNCTION (b)

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ABSTRACT

In this paper some Ramanujan integrals associated with generalized the Riemann Zeta function and $\tilde{H}$-function is evaluated.

Importance of the results established in this paper lies in the fact they involve $\tilde{H}$-function which is sufficiently general is nature and capable of yielding a large number of results merely by specializing the parameters their in.

Keywords: *The $\tilde{H}$-function, the Zeta-function, Ramanujan Integral,*

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1. INTRODUCTION

The $\tilde{H}$-function, a generalization of Fox's (2) H-function introduced by Inayat Hussain [5], and studied by Bachman and Shrivastava [1] and others, is defined and represented in the following manner:

$$\tilde{H}^{M,N}_{P,Q}[z] = \tilde{H}^{M,N}_{P,Q}\left[z\left|\begin{matrix}(e_j,E_j;\in_j)_{1,N},(e_j;E_j)_{N+1,P}\\(f_j,F_j)_{1,M},(F_j,f_j;\tau_j)_{M+1,Q}\end{matrix}\right.\right]$$

$$=\frac{1}{2\pi\omega}\int_L \tilde{\phi}(\xi)z^{\xi}d\xi, \tag{1.1}$$

Where,

$$\tilde{\phi}(\xi)=\frac{\prod_{J=1}^{M}\Gamma(f_j-F_j\xi_j)\prod_{J=1}^{N}\Gamma(1-e_j+E_j\xi)^{\varepsilon}}{\prod_{i=M+1}^{Q}\Gamma(1-F_j+F_j\xi)^{\tau}\prod_{J=N+1}^{P}\Gamma(e_j-E_j\xi)} \tag{1.2}$$

And the contour L is the line from. c-$\omega\infty$ to c+$\omega\infty$ suitability indented to keep the poles $\left|\overline{\left(f_j-F_j\xi\right)}.\ j=(1,2,.......M)\right.$ bof to the right of the path and the singularities $\left|\overline{\left(1-e_j+E_j.\xi\right)}\right., j=(1,2..........,N)$ of to the left path.

The followings sufficient condition for absolute convergence of the integral defined in equation (1.2) have been recently given by Sharma Gupta, and Jain (6, P 169-172)

(i) $\quad |arg(z)|<\frac{1}{2}\Omega\pi$, and $\Omega>0$,

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(ii) $\left|arg(z)\right| \le \frac{1}{2}\,\Omega\pi$ and $\Omega \ge 0$,

where,

$$\Omega = \sum_{J=1}^{M} F_j + \sum_{J=1}^{N} \varepsilon E_j - \sum_{J=M+1}^{P} F_j \tau_j - \sum_{J=N+1}^{P} E_j, \qquad (1.3)$$

2. FORMULA REQUIRED

In the sequal of the work we will use (6,P402) following formula–

$$\int_0^\infty x^{\rho-1}[2/(1+\sqrt{1+4x})]^n dx = \frac{\lceil n+1 \, \Gamma(\rho)\,\Gamma(n-2\rho)}{\lceil n \; \Gamma(n-\rho+1}$$

where n > o , o < p < (n / 2) or (n > 2p >o) (2.1)

The Reminn. zeta function, mention by Goyal and Laddha [3 p.99] are used for drive the integrals

$$\phi_\mu(z,a,s,g) = \sum_{g=0}^{\infty} (\mu)_g.(a+g)^{-s} z^{g}/g!, \qquad \mu \ge 1, |z|<1, Re(a)>0, \qquad (2.2)$$

When μ =1 in (2.2) then we get generalized Riemann zeta function

$$\phi_1(z,s,a,g) = \sum_{g=0}^{\infty} (a+g)^{-s} z^g, |z|<1, Re(a)>0 \qquad (2.3)$$

When μ =1 and s=1 then (2.1) Reduce to Hyper geometric function

$$\phi(z.1,a,g) = \frac{1}{\alpha}\sum_{r=0}^{\infty} \frac{(1)_r\,(x)_r}{(1+a)_r}\frac{z^r}{\lfloor r}, \qquad (2.4)$$

3. INTEGRALS

In this section we will establish few integrals involving product of $\widetilde{H}$ -function and Reminn zeta function.

(I^ST) Integral:

$$\int_0^\infty x^{\rho-1}[2/(1+\sqrt{1+4x})]^n \;(\phi_\mu(cx^\nu)[2/(1+\sqrt{1+4x})]^\xi \;\widetilde{H}^{M,N}_{P,Q}[z\; x^{\sigma_1}[2/<1+\sqrt{1+4x}>]^{\rho_1}]dx$$

$$= \sum_{g=0}^{\varepsilon} (\mu)_g (a+g)^{-s}\frac{c^g}{\lfloor g}\;\widetilde{H}^{M,\;N+3}_{P+3,\;Q+2}\left[z\,\middle|\,\begin{matrix}R1\\R2\end{matrix}\right]$$

where,

R_1 = {(-n- ξ g: p_1, 1) , (1-n - ξ g ; 2 ρ + 2 vg , ρ -2 σ_1, 1)(1- ρ - ν g, σ ,1), $\left(\varepsilon_j\,,\,E_j\,;\,\varepsilon_j\right)_{1,N}$, $\left(\varepsilon_j\,.E_j\right)_{N+1,P}$

R_2 = $\left(f_j\,,\,F_j\right)_{1,M}$, $\left(F_j\,,\,F_j\,;\,\tau_i\right)_{M+1,Q}$ (1-n- ξ ρ g: , 1) , (ξ -n-q+p+ ν g:- ρ $_1\sigma$,2}

Provided n+ ξ g + ρ_1 ξ Co, Re (ρ_1 (f$_j$ / F$_j$) > 0,
ρ + $^\nu$ g + σ_1 (f$_j$ / F$_j$) > 0 , and

$$\left|\arg(z)\right| \le \frac{1}{2}\Omega\pi \qquad \left|\arg(z)\right| \le \frac{1}{2}\Omega\pi \qquad (3.1)$$

IInd Integral:

$$\int_0^\infty x^{\rho-1}[2/(1+\sqrt{1+4x})]^n\ (\phi_\mu(cx^\nu)[2/(1+\sqrt{1+4x})]^\xi\ \widetilde{H}_{P,\ Q}^{M,N}[z\ x^{-\sigma_1}\ [2/<1+\sqrt{1+4x}>]^{-\rho_1}]dx$$

$$=\sum_{g=0}^{\varepsilon}(\mu)_g(a+g)^{-s}\frac{c^g}{\lfloor g}\ \widetilde{H}_{P+2,Q+3}^{M+3N}\left[z\middle|_{R_4}^{R_3}\right]$$

where,

R_3= {1+n+ξg:ρ_1) (n + ξg - 2 ρ- 2vg ,ρ_1+2σ1) (ρ+νg,σ1), $\left(\varepsilon_j,\ E_j;\varepsilon_j\right)_{1,N},\left(\varepsilon_j.E_j\right)_{N+1,P}$

$R_4\equiv\left(f_j,\ F_j\right)_{1,M},\ \left(F_j\ ,F_j\ ;\tau_i\right)_{M+1,Q}$ (n+ξg; ρ1) , (n+ξg-p-νg+ρ_1-σ_1)

Provided n+ξg +ρ_1 ξ >0, p + vg +σ_1 > o,

$$|\arg(z)|>\frac{1}{2}\Omega\pi\qquad |\arg(z)|\geq\frac{1}{2}\Omega\pi \tag{3.2}$$

IIIrd Integral:

$$\int_0^\infty x^{\rho-1}[2/(1+\sqrt{1+4x})]^n\ (\phi_\mu(x^\nu)[2/(1+\sqrt{1+4x})]^\xi\ \widetilde{H}_{P,\ Q}^{M,N}[z\ x^{\sigma_1}[2/<1+\sqrt{1+4x}>]^{-\rho_1}]dx$$

$$=\sum_{g=0}^{\varepsilon}(\mu)_g(a+g)^{-s}\frac{c^g}{\lfloor g}\ \widetilde{H}_{P+3,Q+2}^{M\ ,\ N+3}\left[z\middle|_{R_6}^{R_5}\right]$$

where,

R_5 = {(1+n+ξg: ρ_1) , (n+ ξg -ρ_1) , (n+ξg -2ρ-2vρ_1+2σ_1) $\left(\varepsilon_j\ ,E_j\ ;\varepsilon_j\right)_{1,N},\left(\varepsilon_j.E_j\right)_{N+1,P}$

$R_6\equiv\left(f_j,\ F_j\right)_{1,M},\left(F_j\ ,F_j\ ;\tau_i\right)_{M+1,Q}$ (n+ξg: ρ_1) , (1+ρ+ξg-$\rho_1$rg ρ_1+σ_1}

Provided n+ξg +ρ_1 ξ >O , ρ + νg + σ_1 and

$$|\arg(z)|>\frac{1}{2}\Omega\pi,\qquad |\arg(z)|\geq\frac{1}{2}\Omega\pi \tag{3.3}$$

IVth Integral:

$$\int_0^\infty x^{\rho-1}[2/(1+\sqrt{1+4x})]^n\ (\phi_\mu(cx^\nu)[2/(1+\sqrt{1+4x})]^\xi\ \widetilde{H}_{P,\ Q}^{M,N}[z\ x^{-\sigma_1}[2/<1+\sqrt{1+4x}>]^{\rho_1}]dx$$

$$=\sum_{g=0}^{\varepsilon}(\mu)_g(a+g)^{-s}\frac{c^g}{\lfloor g}\widetilde{H}_{P+2,\ Q+3}^{M+1,N+2}\left[z\middle|_{R_8}^{R_7}\right]$$

where,

R_7 ={(n+vg ; σ_1), $\left(\varepsilon_j,E_j\ ;\varepsilon_j\right)_{1,N},\left(\varepsilon_j.E_j\right)_{N+1,P}$

(-n - ξ g ; ρ_1 ,1)(-1-n-ξg+2ρ-2rg ; ρ_1+2σ_1, 1)

$R_8\equiv(f_j\ ,\ F_j\ )_{1,M},\ (F_j\ ,\ F_j\ ;\ \tau_i\ )_{M+1,Q}$

(-1-n-ξg; ρ_1, 1) , (-ρ-ξg+ρ+rg ; ρ_1+ σ_1,1)

Provided n+ξg + ρ_1 ξ>o , P+vg+$\sigma_{1>o}$, and

$$|\arg(z)|>\frac{1}{2}\Omega\pi,\ |\arg(z)|\geq\frac{1}{2}\Omega\pi \tag{3.4}$$

Proof: To the establish (3.1) expressing the $\widetilde{H}$ function on left hand side using (1.1) as Mellin-Barnes. Contour integral and express the Remain zeta function in terms of the summation of series, then we have,

$$\sum_{g=o}^{\infty}(\mu)g\ (a+g)^{s}\,\frac{c^{g}}{\lfloor g}\,\frac{1}{2\pi i}\int_{L}\frac{\prod\limits_{J=1}^{M}\overline{\left(f_j-F_j\,\xi_j\right)}}{\prod\limits_{i=M+1}^{Q}\overline{\left(1-F_j+F_j\xi\right)^{r}}}\;\frac{\prod\limits_{J=1}^{N}\overline{\left(1-e_j+E_j\,\xi\right)^{\varepsilon}}}{\prod\limits_{J=N+1}^{P}\overline{\left(e_j-E_j\,\xi\right)}}$$

$$X\left\{\int_{o}^{\infty}x^{p+v\xi+\sigma_1 s-1}\,[2/(1+\sqrt{1+4x})]^{n+\xi g+\rho_1 s}dx\right\}.z^{\xi}d\xi.$$

evaluate the inner integrals with the help of (2.1) and expressing the Remain zeta function by (2.2) as a series and using (1.1) we get R.H.S. of (3.1)

Proceeding on similar lines the integrals (3.2) (3.3) and (3.4) have been obtained.

4. SPECIAL CASES

The importance of Ramanujan. Integrals involving $\widetilde{H}$ -function and Remain.zeta function lie in their manifold generality in view of the generality of the $\widetilde{H}$ function, on sepcializing the various parameters, we can obtain from our integrals, series and results involving a remarkable wide varity of useful function which are express able in terms of [2] Fox H-function G-function and Remain zeta. Function etc.

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