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APPLICATION OF RD-ALGORITHM IN DIVISIBILITY OF SPECIAL NUMBERS

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ABSTRACT

Let $z = a_n a_{n-1} \dots a_1$ be a dividend and $w = b_m b_{m-1} \dots b_1$ be a odd divisor. In this paper, we give a new algorithm entitled Reduce Digits Algorithm (RD-Algorithm) for divisibility of numbers and we introduce some new methods for high speed of divisibility in RDA with using form digits of w. This algorithm reduced the number of digits for z.

AMS subject classification: *13AXX, 13F15.*

Keywords: *Divisibility, Partition of numbers, RD- Algorithm.*

1. INTRODUCTION

A lot of study on divisible of method have been conducted for many years. In number theory, divisibility methods of whole numbers are very useful because they help us to quickly determine if a number can be divided by n. There are several different methods for divisibility of numbers with many variants and some of them can be found in [6, 8, 9, 10, 11, 12].For example in [10] presented that numbers which are dividable to 11 should have the (sum of the odd numbered digits) - (sum of the even numbered digits) is divisible by 11. Similarly some studies are presented for special numbers such as 15, 17, 19, etc ([2, 5, 7]). In this paper, we suppose that $z = a_n a_{n-1} \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively. Also, we show that in [1], if $z = a_n a_{n-1} \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and prime divisor respectively, then:

1. If $w|(b_m b_{m-1} \dots b_2)a_1 - (a_n a_{n-1} \dots a_2)$ and $b_1 = 1$ then $|z$.
2. If $w|(w - (7w-1)/10)a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 3$ then $w|z$.
3. If $w|(w - (3w-1)/10)a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 7$ then $w|z$.
4. If $w|(w - ((9w-1)/10)\, a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 9$ then $w|z$.
5. If $w = 5$ is prime divisor then the proof of $w|z$ is clear.
6. If $w = b_m b_{m-1} \dots b_1$ is composite divisor, then with using of fundamental theorem of arithmetic, the proof of $w|z$ is obvious.

Also, we show that in [3], if $z = a_n a_{n-1} \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively, then:

1. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=1$, $b_2=0$, then $w|z$ if $w||(b_m b_{m-1} \dots b_3)a_2 a_1 - a_n a_{n-1} \dots a_3|$.
2. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=3$, $b_2=4$, then $w|z$ if $w||((7w-1)/100)a_2 a_1 - (a_n a_{n-1} \dots a_3)|$.
3. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=7$, $b_2=6$, then $w|z$ if $w||\ ((3w-1)/100)a_2 a_1 - (a_n a_{n-1} \dots a_3)|$.
4. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=9$, $b_2=8$, then $w|z$ if $w||\ ((9w-1)/100)a_2 a_1 - (a_n a_{n-1} \dots a_3)|$.

Also, we show that in [4], if $z = a_n a_{n-1} \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively, then:

1. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=1$, $b_2=0$, $b_3=0$, then $w|z$ if $w||(b_m b_{m-1} \dots b_4)a_3 a_2 a_1 - a_n a_{n-1} \dots a_4|$.
2. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_1=3$, $b_2=4$, $b_3=1$, then $w|z$ if $w||((7w-1)/1000)a_3 a_2 a_1 - (a_n a_{n-1} \dots a_4)|$.

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3 If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, b_2=6, b_3=6, then w|z if $w||((3w-1)/1000)a_3a_2a_1 - (a_n a_{n-1} \dots a_4)|$.

4 If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9, b_2=8, b_3=8 , then w|z if $w||((9w\text{-}1)/1000)a_3a_2a_1 - (a_n a_{n-1} \dots a_4)|$.

In this paper, in order to fasten dividing numbers, we fixed 4 right digits of odd divisor.

2. APPLICATION OF REDUCE DIGITS ALGORITHM IN DIVISIBILITY OF NUMBERS AND RESULTS

Theorem 2.1: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4b_3b_2b_1 = 0001$, then w|z if $w||(b_m b_{m-1} \dots b_5)a_4a_3a_2a_1 - a_n a_{n-1} \dots a_5|$.

Proof: If $w||(b_m b_{m-1} \dots b_5)a_4a_3a_2a_1 - a_n a_{n-1} \dots a_5|$, then there exists an integer k such that kw $= (b_m b_{m-1} \dots b_5)a_4a_3a_2a_1 - a_n a_{n-1} \dots a_5$. Therefore 10000kw= $(b_m b_{m-1} \dots b_5) * 10000a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5) * 10000$, hence we have (10000kw-$a_4a_3a_2a_1$)w = -z, so w|z.

Remark 2.2: In this paper, with using theorems for dividend and odd divisor, we can introduce the new numbers (as same as 0 in follow Example). If we don't know whether a new number is divisible by odd divisor, we should apply the theorems again. In this case, the new numbers are dividend.

Example 2.3: Is 691601235 divisible by 560001?

Solution: with using above theorem | (56*1235)-69160 | = 0. Therefore, 691601235 is divisible by 560001.

Theorem 2.4: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4b_3b_2b_1 = 7143$, then w|z if $w||((7w-1)/10000)a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$.

Proof: If $w||((7w-1)/10000)a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$, then there exists an integer k such that kw=$((7w-1)/10000)a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)$. Therefore, (10000k-7$a_4a_3a_2a_1$)w $= -z$, so w|z.

Example 2.5: Is 4073760369 divisible by 1577143?

Solution: with using above theorem | (1104*376)-407376 | = 0.

Therefore, 4073760369 is divisible by 1577143.

Theorem 2.6: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4b_3b_2b_1 = 6667$, then w|z if $w|| ((3w-1)/10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$.

Proof: If w|z if $w|| ((3w-1)/10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$, then there exists an integer k such that kw= $((3w-1)/10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)$. Therefore, (10000k-3$a_4a_3a_2a_1$)w $= -z$, so w|z.

Example 2.7: Is 6247866745 divisible by 26586667?

Solution: with using above theorem | (7976*6745)-624786 | = 53173334.

But the divisibility 53173334 by 26586667 is not clear. Therefore, with using above theorem for 53173334 to 26586667, we have | (7976*3334)-5317 |=26586667. Therefore, 6247866745 is divisible by 26586667.

Remark 2.8: $3*\underbrace{66\dots6}_{n-th}7-1 \overset{10^{n+1}}{\equiv} 0$. [6]

Corollary 2.9: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, $b_2 = b_3 = \dots = b_m$=6, then w|z if $w|| ((3w-1)/10^m)\, a_m \dots a_2a_1 - (a_n a_{n-1} \dots a_{m+1})$ |.

Theorem 2.10: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_4b_3b_2b_1 = 8889$, then w|z if $w|| ((9w\text{-}1)/\, 10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$.

Proof: If $w|| ((9w\text{-}1)/\, 10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)|$, then there exists an integer k such that

kw= $((9w\text{-}1)/10000)\, a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_5)$. Therefore, (10000k-9$a_4a_3a_2a_1$)w $= -z$, so w|z.

Example 2.11: Is 618523619128 divisible by 918889?

Solution: with using above theorem | (827*9128)-61852361 | = 54303505. But the divisibility 54303505 by 918889 is not clear. Therefore, with using above theorem for 54303505 to 918889, we have | (827*3505)-5430 |=2893205. But the divisibility 2893205 by 918889 is not clear. Therefore, with using above theorem for 2893205 to 918889, we have |(827*3205)-289 |=2650246. But the divisibility 2650246 by 918889 is not clear. Therefore, with using above theorem for 2650246 to 918889, we have | (827*246)-265 |=203177<918889. Therefore, 90292806 is not divisible by 918889.

Remark 2.12: $9*\underbrace{88\ldots8}_{n-th}9-1 \overset{10^{n+1}}{\equiv} 0$.[6]

Corollary 2.13: If $z = a_n a_{n-1} \ldots a_1$ is dividend and $w = b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=9,$b_2 = b_3$=…=b_m=8 , then w|z if w|| $((9w-1)/10^m)\, a_m \ldots a_2 a_1 - (a_n a_{n-1} \ldots a_{m+1})$ |.

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