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RESULTS OF RD-ALGORITHM IN DIVISIBILITY OF NUMBERS

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ABSTRACT

Let $z=a_n a_{n-1} \dots a_1$ *and* $w=b_m b_{m-1} \dots b_1$ *are dividend and odd divisor respectively. In this paper, we introduce a new algorithm entitled Reduce Digits Algorithm (RD-Algorithm) for divisibility of numbers and we show that which there is a direct relationship between X and the speed of algorithm and calculations (X is n right digits of odd divisor).*

AMS subject classification: *13AXX, 13F15.*

Keywords: *Divisibility, Partition of numbers, RD- Algorithm.*

1. INTRODUCTION

In various section of number theory, different and difficult methods are considered for dividing special numbers. Also, some algorithms are introduced because of importance of this matter. We introduced a new algorithm entitled "RD-Algorithm" in the paper "A New Method for Divisibility of Numbers" for the first time in the world which we can represent the dividing of numbers to each other in a simple and rapid way. In the other words, this algorithm reduced the number of digits of dividend. In this paper, in order to fasten dividing numbers, we fixed 5 right digits of odd divisor, and use the extracted methods of the algorithm for the numbers which its 5 right digits of them are 00001,57143,66667,88889. The more the X is higher, the more the speed of algorithm is high(X is n right digits of odd divisor). A lot of study on divisibility methods has been conducted for many years. In number theory, divisibility methods of whole numbers are very useful because they help us to quickly determine if a number can be divided by n. There are several different methods for divisibility of numbers with many variants and some of them can be found in [5, 6, 7, 8, 9, 11, 12]. For example in [10] presented that numbers which are dividable to 11 should have the (sum of the odd numbered digits) - (sum of the even numbered digits) is divisible by 11. Similarly some studies are presented for special numbers such as 15, 17, 19, etc. In this paper, we suppose that $z=a_n a_{n-1} \dots a_1$ and $w=b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively. Also, we show that in [1, 2], if $z=a_n a_{n-1} \dots a_1$ and $w=b_m b_{m-1} \dots b_1$ are dividend and prime divisor respectively, then:

1. If $w|(b_m b_{m-1} \dots b_2)a_1 - (a_n a_{n-1} \dots a_2)$ and $b_1 = 1$, then $w|z$.

2. If $w|(w - (7w-1)/10)a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 3$, then $w|z$.

3. If $w|\ w - (3w-1)/10)\ a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 7$, then $w|z$.

4. If $w|w\ - ((9w\ -1)/10)\ a_1 + (a_n a_{n-1} \dots a_2)$ and $b_1 = 9$, then $w|z$.

5. If $w\ = 5$ is prime divisor then the proof of $w|z$ is clear.

6. If $w=b_m b_{m-1} \dots b_1$ is composite divisor, then with using of fundamental theorem of arithmetic, the proof of $w|z$ is obvious.

Also, we show that in [3], if $z=a_n a_{n-1} \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively, then:

a. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=1, b_2=0, then $w|z$ if $w||(b_m b_{m-1} \dots b_3)a_2 a_1 - a_n a_{n-1} \dots a_3|$.

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b. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=3, b_2=4, then w|z if $w||((7w-1)/100)a_2a_1 - (a_n a_{n-1} \dots a_3)|$.

c. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, b_2=6, then w|z if $w||((3w-1)/100)\ a_2a_1\text{-}(a_n a_{n-1} \dots a_3)|$.

d. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9, b_2=8, then w|z if $w||((9w\text{-}1)/100)a_2a_1\text{-}(a_n a_{n-1} \dots a_3)|$.

Also, we show that in [4], if $z = a_n \dots a_1$ and $w = b_m b_{m-1} \dots b_1$ are dividend and odd divisor respectively, then:

a. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=1, b_2=0, b_3=0, then w|z if $w||(b_m b_{m-1} \dots b_4)a_3a_2a_1 - a_n a_{n-1} \dots a_4$.

b. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=3, b_2=4, b_3=1, then w|z if $w||((7w-1)/1000)a_3a_2a_1 - (a_n a_{n-1} \dots a_4)|$.

c. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=7, b_2=6, b_3=6, then w|z if $w||((3w-1)/1000)\ a_3a_2a_1 - (a_n a_{n-1} \dots a_4)|$.

d. If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that b_1=9 , b_2=8, b_3=8, then w|z if $w||((9w\text{-}1)/1000)\ a_3a_2a_1 - (a_n a_{n-1} \dots a_4)|$.

Now, we present in this paper that with increase of X the speed of RD-Algorithm increase, and there is a direct relationship between X, the speed of algorithm and calculations. (X is n right digits of odd divisor).

2. RESULTS OF RD-ALGORITHM IN DIVISIBILITY OF NUMBERS

Theorem 2.1: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5b_4b_3b_2b_1 = 00001$, then w|z if $w||(b_m b_{m-1} \dots b_6)a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$.

Proof: If $w||(b_m b_{m-1} \dots b_6)a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$, then there exists an integer k such that kw $= (b_m b_{m-1} \dots b_6)a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6$. Therefore,100000kw= $(b_m b_{m-1} \dots b_6) * 100000a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6$, hence we have (100000kw-$a_5a_4a_3a_2a_1$)w = -z , so w|z.

Remark 2.2: In this paper, with using theorems for dividend and odd divisor, we can introduce the new numbers (as same as 0 in follow Example). If we don't know whether a new number is divisible by odd divisor, we should apply the theorems again. In this case, the new numbers are dividend.

Example 2.3: Is 17904689523 divisible by 200001?

Solution: with using above theorem | (2*89523)-179046 | = 0. Therefore, 17904689523 is divisible by 200001.

Theorem 2.4: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5b_4b_3b_2b_1 = 57143$, then w|z if $w||((7w-1)/100000)a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$.

Proof: If $w||((7w-1)/100000)a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$, then there exists an integer k such that kw=$((7w-1)/100000)a_5a_4a_3a_2a_1 - (a_n a_{n-1} \dots a_6)$. Therefore, (100000k-7$a_5a_4a_3a_2a_1$)$w = -z$, so w|z.

Example 2.5: Is 143115479503 divisible by 257143?

Solution: with using above theorem | (18*79503)-1431154 | = 100<w. Therefore, 143115479503 is not divisible by 257143.

Theorem 2.6: If $z = a_n a_{n-1} \dots a_1$ is dividend and $w = b_m b_{m-1} \dots b_1$ is odd divisor such that $b_5b_4b_3b_2b_1 = 66667$, then w|z if $w||((3w-1)/100000)\ a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$.

Proof: If $w||((3w-1)/100000)\ a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6|$, then there exists an integer k such that kw= $((3w-1)/100000)\ a_5a_4a_3a_2a_1 - a_n a_{n-1} \dots a_6$. Therefore, (100000k-3$a_5a_4a_3a_2a_1$)$w = -z$, so w|z.

Example 2.7: Is 3170383011738 divisible by 2566667?

Solution: with using above theorem | (77*11738)-31703830 | = 30800004.
But the divisibility 53173334 by 26586667 is not clear. Therefore, with using above theorem for 30800004 to 2566667, we have | (77*4)-308 |=0. Therefore, 3170383011738 is divisible by 2566667.

Remark 2.8: $3*\underbrace{66\ldots6}_{n-th}7-1 \overset{10^{n+1}}{\equiv} 0$. [6]

Corollary 2.9: If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=7, $b_2 = b_3 = \ldots = b_j$=6, then w|z if w|| $((3w-1)/10^j)\, a_j \ldots a_2 a_1 - (a_n a_{n-1} \ldots a_{j+1})$ |.

Theorem 2.10: If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that $b_5 b_4 b_3 b_2 b_1 = 88889$, then w|z if w|| ((9w-1)/ 100000) $a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_6$|.

Proof: If w w|| ((9w-1)/ 100000) $a_5 a_4 a_3 a_2 a_1 - a_n a_{n-1} \ldots a_6$|, then there exists an integer k such that

kw= ((9w-1)/100000) $a_5 a_4 a_3 a_2 a_1 - (a_n a_{n-1} \ldots a_6)$. Therefore, (100000k-9$a_5 a_4 a_3 a_2 a_1$)$w = -z$, so w|z.

Example 2.11: Is 1822249046 divisible by 1288889?

Solution: with using above theorem | (116*49046)-18222 | = 5671114. But the divisibility 5671114 by 1288889 is not clear. Therefore, with using above theorem for 1822249046 to 1288889, we have | (116*71114)-56 |=8249168. But the divisibility 8249168 by 1288889 is not clear. Therefore, with using above theorem for 8249168 to 1288889, we have | (116*49168)-82 |=5703406. But the divisibility 5703406 by 1288889 is not clear. Therefore, with using above theorem for 5703406 to 91288889, we have | (116*3406)-57 |=395039<1288889. Therefore, 1822249046 is not divisible by 1288889.

Remark 2.12: $9*\underbrace{88\ldots8}_{n-th}9-1 \overset{10^{n+1}}{\equiv} 0$.[6]

Corollary 2.13: If z = $a_n a_{n-1} \ldots a_1$ is dividend and w = $b_m b_{m-1} \ldots b_1$ is odd divisor such that b_1=9,$b_2 = b_3 = \ldots = b_j$=8, then w|z if w|| $((9w-1)/10^j)\, a_j \ldots a_2 a_1 - (a_n a_{n-1} \ldots a_{j+1})$ |.

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