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SIMPLE RIGHT ALTERNATIVE RINGS WITH (x y) z = (x z) y

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ABSTRACT

In this paper, first we prove that in a simple right alternative ring R with (x y)z = (x z)y, the square of every element of R is in the nucleus. Using this we prove that R is alternative.

***Key words:** Simple, 2-divisible ring, Nucleus, Right Alternative Ring, Alternative Ring.*

Simple Ring: A ring R is said to be simple if whenever A is an ideal of R, then either A = 0 or A =R

2-divisible Ring: We define a ring R to be 2-divisible if 2 x = 0 implies x = 0, for all x in R.

Nucleus:
The nucleus N of a ring R, we mean the set of all elements n in R such that (n, R, R) =(R, n, R) = (R, R, n) = 0.

Right Alternative Ring: A right alternative ring R is a ring in which y (x x) = (y x) x, for all x, y, in R.

Alternative Ring: A right alternative ring R is a ring in which (x x) y = x (x y), y (x x) = (y x) x, for all x, y in R.

INTRODUCTION

Kleinfeld and Smith[1,2] studied simple alternative rings with the assumption that either commutators are in the nucleus or all squares x^2 are in the nucleus in order to see that whether these rings are alternative or associative. In this paper, first we prove that in a simple right alternative ring R with (x y) z = (x z) y, the square of every element of R is in the nucleus. Using this we prove that R is alternative.

Let R be a 2-divisible non associative right alternative ring with (x y) z = (x z) y (1)

R is said to be simple if whenever A is an ideal of R then either A=R or A=0.

In a right alternative ring the following identities hold:

$(x, y, z) = -(x, z, y)$ (2)

$(x y.z) y = x (y z.y)$ (3)

and $(w x, y, z) + (w, x, (y, z)) = w (x, y, z) + (w, y, z) x$. (4)

Lemma 1: The set defined b y $T = \{t \in R/ Rt = 0\}$ is an ideal of R.

Proof: Obviously, T is a left ideal, since RT = 0, Let $t \in T$. x, $y \in R$.

Then $x (t y) = x (t y + y t) = (x t) y + (x y) t$, using (2).

But $(x t) y + (x y) t =0$. Thus $x (t y) = 0$ and hence $TR \subset T$.

Thus T is an ideal of R.

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Lemma: 2 If R is a simple right alternative ring with (x y) z = (x z) y, then the square of every element of R is in the nucleus N.

Proof: Let $N_r = \{n \in R / (R, R, n) = 0\}$

By using (2), (1), (3) and (1) in that order, we have

$(w\, x^2)\, y = (w\, x.\, x)\, y = (w\, x.\, y)\, x = w\, (x\, y.\, x) = w\, (x^2 y)$

Thus $(w, x^2, y) = 0$

From (2) $(w, y, x^2) = 0$.this shows that for all $x \in R$ we have $x^2 \in N_r$ (5)

Now for all $x, y \in R$, $x\, y + y\, x = (x + y)^2 - x^2 - y^2 \in N_r$ (6)

From (1) we obtain $(w\, x\, .\, y)\, z = (w\, y.x)\, z = (w\, y.z)\, x$ and $(w\, x)\, (y\, z) = (w.y\, z)\, x$.

Thus by subtraction, $(w\, x, y, z) = (w, y, z)\, x$. (7)

By combining (4), (7) we have $(w, x, (y, z)) = w\, (x, y, z)$ (8)

Let $x = n \in N_r$ in (8) thus $R\, (N_r, R, R) = 0$ (9)

Hence from lemma (1) we have $(N_r, R, R) \subset T$. since T is an ideal of R and

R is simple either T =R or T = 0.

Since we are assuming R to be nonassociative, $T \neq R$.

Thus T = 0. That is $(N_r, R, R) = 0$ (10)

Hence N_r is the nucleus N of R.

From (5) it follows that the square of every element of R is in the nucleus.

Lemma: 3 In a simple 2-divisible right alternative ring $(x, x, y) \in N$ and $N\, (R, R, R) = 0$

Proof: In (8), let $y = n \in N$. Then $(w, x, (n, z)) = 0$, so that $(n, z) \in N$.

From (6) $2nz \in N$ and $2zn \in N$. since R is 2-divisible, $nz \in N$and $zn \in N$.

From (5) it follows that $x^2\, y \in N$ (11)

Then from (1) $(x^2\, y) = (x\, x)\, y = (x\, y)\, x \in N$. we define $a \equiv b$ if and onl y if $a\text{-}b \in N$

Now from (6) implies that $x\, y.\, y + x.\, x\, y \in N$. Hence $-\, x.\, x\, y \equiv x\, y.\, x \in N$

That is $x.\, x\, y \in N$ (12)

From (1) and (2) yields $(x, x, y) \in N$. Let $w = n \in N$ in (8).

We get $0 = (n, x, (y, z)) = n\, (x, y, z)$. Hence $N\, (R, R, R) = 0$

Lemma: 4 The set $S = \{s \in N / s\, (R, R, R) = 0\}$ is an ideal of R.

Proof: For $s \in S$ and $w, x, y, z \in R$, we have

$(s, w, (x, y, z)) = (s\, w).\, (x, y, z) - s.\, (w\, (x, y, z)) = 0$

This implies that $(s\, w).\, (x, y, z) = 0$ and $sw \in S$.

Now $(w, s, (x, y, z)) = w\, s.\, (x, y, z) - w.\, (s\, (x, y, z)) = 0$

⇒ w s. (x, y, z) = w. (s (x, y, z)) = 0

⇒ ws∈ S. Hence S is an ideal of R.

Theorem: If R is a simple right alternative ring with (x y) z = (x z) y, then R is alternative.

Proof: From lemmas (3) and (4) we have all (x, x, y)∈ S.

Since S is an ideal of R and R is simple, either S = R or S = 0. If S = R then R is associative.

But R is not associative.

Hence S = 0 and R is alternative.

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