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SYMMETRIC KNOT GRAPH

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ABSTRACT

In this paper we introduce the Symmetric Knot Graph and their generators. We multiply the generators of Symmetric Knot graphs and prove the associative property.

INTRODUCTION

We introduced Symmetric Knot Graphs which have one to one correspondence between generators of Knot Symmetric Algebras. There is a multiplication among brauer diagrams. Brauer's algebras have been introduced by Brauer in connection with the decomposition problem of tensor product representation of O(n) and sp(2n) into irreducible ones. There is a multiplication among brauer diagrams. The multiplication among Brauer graphs motivated us to define multiplication among Symmetric Knot graphs. In this chapter we define multiplication between two symmetric Knot graphs.

3.1 PRELIMINARIES

we define Symmetric Knot graphs using Knot theory. Let S_n denote a symmetric group of order n. Let $\pi \epsilon S_n$ then π can be represented as a graph in which the vertices of π are represented in two rows such that each row contains n vertices. The vertices of each row is indexed with 1,2, … n from left to right in order. Let $E(\pi)$ denote the set of all edges of π

(ie) $E(\pi) = \{e_i = (i, \pi(i)); 1 \le i \le n\}$

Define $A_\pi=\{a_{ij}=(e_i, e_j); i<j\}$

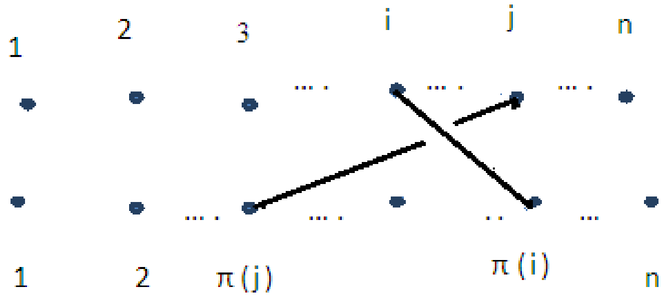
$B_\pi = \{b_{ij}=a_{ij}; \pi(j) < \pi(i)\}$.

3.2 SYMMETRIC KNOT GRAPH

3.2.1 Definition: Let $a_{ij} \epsilon A_\pi$ if $a_{ij} \notin B_\pi$, we draw the edges as in S_n. If $a_{ij} = b_{ij} \epsilon B_\pi$, then we introduce upper edge and lower edge as follows.

$b_{ij}=(e_i, e_j)$ where $e_i=(i, \pi(i))$ and $e_j=(j, \pi(j))$

Case (i): we draw the edges e_i and e_j as follows:



In this case we say e_i is upper than e_j as well as e_j is lower than e_i

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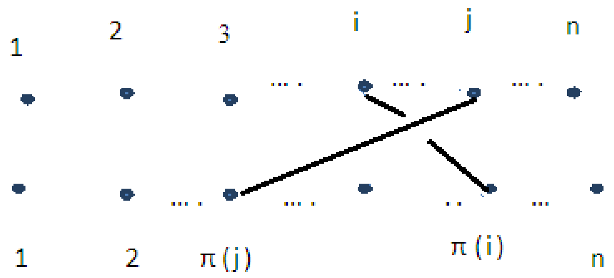
Case (ii): We draw the edges e_i and e_j as follows:



In this case e_i is lower than e_j as well as e_j is upper than e_i

The resulting graph is known as the Symmetric Knot graph of order n derived from π.

Let K_π denote the collection of all Symmetric Knot graphs of order n derived from π.

Let $K_n = \bigcup_{\pi \in S_n} K_\pi$

Let x be an indeterminate. Now $K_n(x)$ is defined by

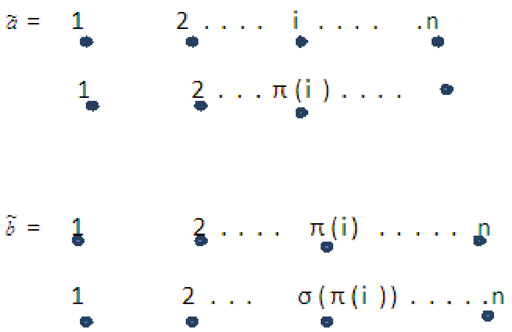
$K_n(x) = \{(x^m, \tilde{a}); \tilde{a} \in K_n, m \in Z\}$

For any element in K_n, can be considered as an element in $K_n(x)$ as $(1, \tilde{a})$.

3.2 .2 Multiplication inKn

Dr. M. Kamaraj and R. Selvarani introduced 2-Knot multiplication among Knot graphs in K_n. Now we define a product among elements in K_n .

3.2.3 Definition: Let $\tilde{a}, \tilde{b}$ be the elements in $K_n(x)$. Let $a = (x^{m_1}, \tilde{a}), b = (x^{m_2}, \tilde{b})$ where m_3 is 0, +2, -2. The product of two diagram $\tilde{a}$ and $\tilde{b}$ of n vertices is determined by putting the diagram $\tilde{a}$ in the top and $\tilde{b}$ is drawn below $\tilde{a}$. The vertices of $\tilde{a}$ and $\tilde{b}$ will be as shown below:



Let c=ab

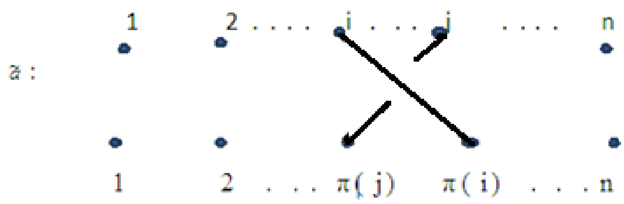
where $\tilde{c} = (x^{m_1+m_2+m_3}, \tilde{a}\tilde{b})$ and m_3 is 0, +2, -2 .

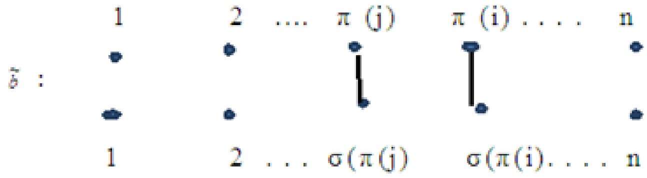
Let $\tilde{a} \in K_\pi$ and $\tilde{b} \in K_\sigma$. Now $\tilde{c}$ is defined as a Symmetric Knot graph of order n derived from σoπ . For each element

$\gamma_i=(i, \sigma o \pi(i)) \in E(\sigma o \pi)$ there are edges $\alpha_i=(i, \pi(i)) \in E(\pi)$, $\beta_i=(\pi(i), \sigma o \pi(i)) \in E(\sigma)$

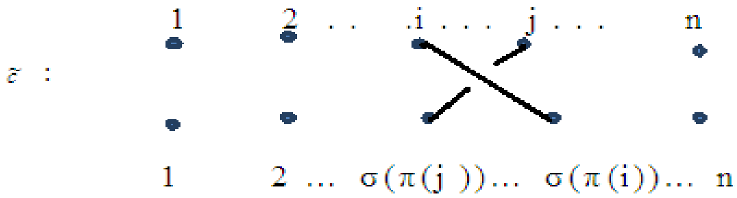
Case 1: Let $(\alpha_i,\alpha_j) \in B_\pi$ and $(\beta_i,\beta_j) \notin B_\pi$ and α_i is upper than α_j

The diagram will be as follows:



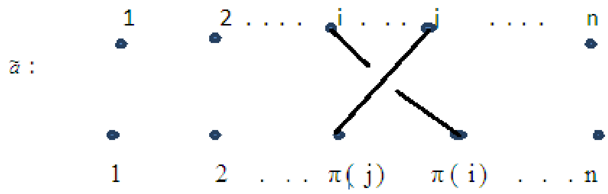


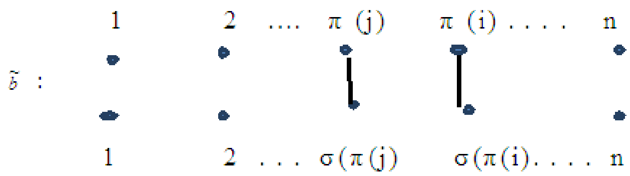
We define $\tilde{c}$ as shown below:



And also we define m_3=0

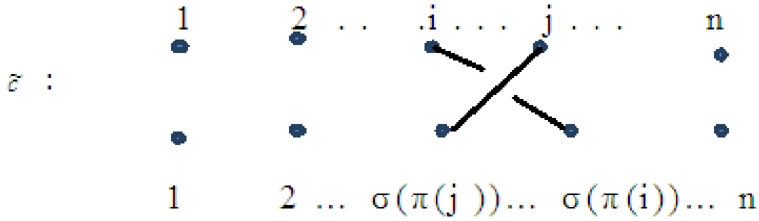
Case 2: .Let $(\alpha_i, \alpha_j)\epsilon B_\pi$ and $(\beta_i, \beta_j) \notin B_\pi$ and α_i is lower than α_j





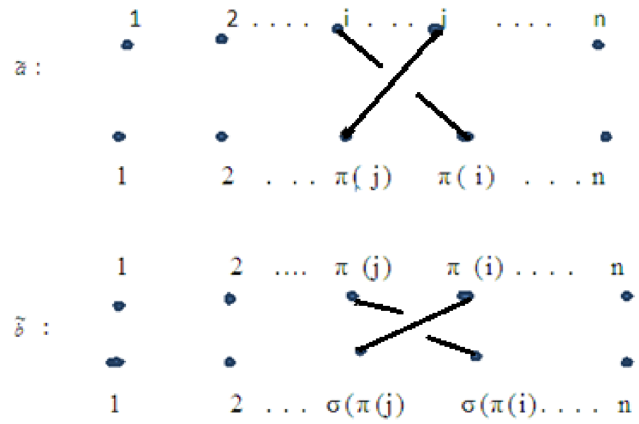
We define $\tilde{c}$ as shown below:

And $(\gamma_i, \gamma_j)\epsilon B_{\sigma o \pi}$ where γ_i= (i, σoπ(i)) and γ_j= (j, σoπ(j))



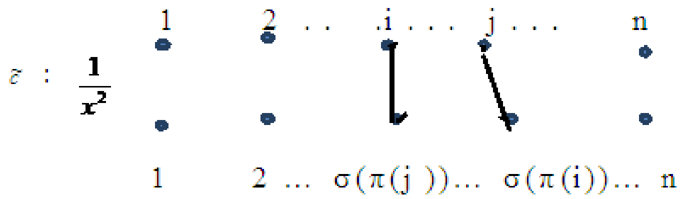
And also we define m3=0

Case 3: .Let $(\alpha_i, \alpha_j)\epsilon B_\pi$ and $(\beta_i, \beta_j)\epsilon\ B_\pi$ and α_i is lower than α_j



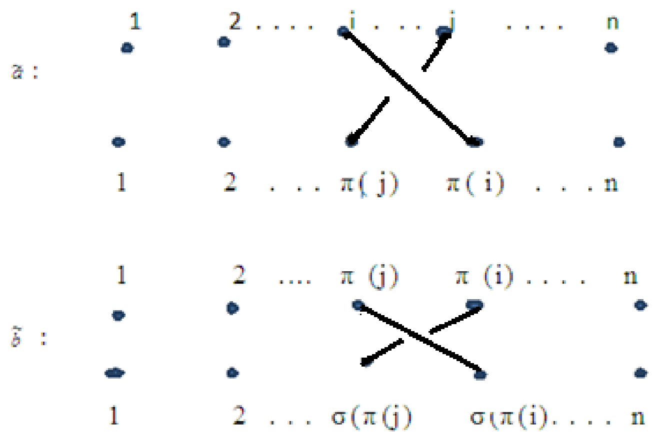
We define $\tilde{c}$ as shown below:

And $(\gamma_i,\gamma_j) \notin B_{\sigma o \pi}$ where γ_i= (i, $\sigma o \pi$(i)) and γ_j= (j, $\sigma o \pi$(j))



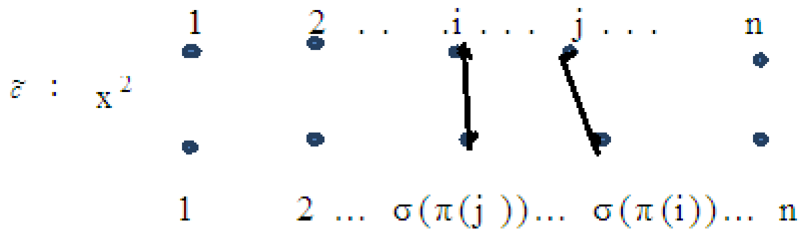
And also we define m_3 = -2

Case 4: .Let $(\alpha_i,\alpha_j)\epsilon B_\pi$ and $(\beta_i,\beta_j)\epsilon\ B_\pi$ and α_i is upper than α_j



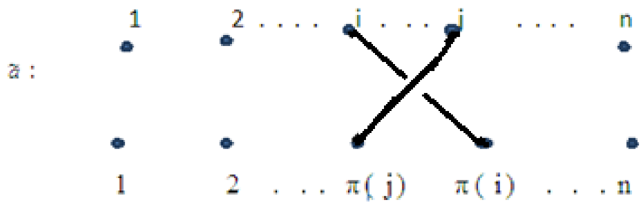
We define $\tilde{c}$ as shown below:

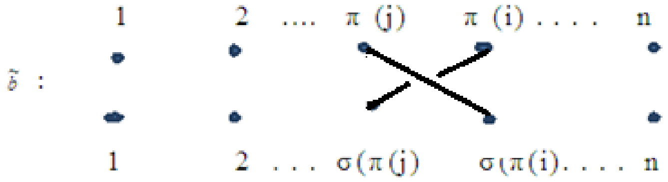
And $(\gamma_i,\gamma_j) \notin B_{\sigma o \pi}$ where γ_i= (i, $\sigma o \pi$(i)) and γ_j= (j , $\sigma o \pi$(j))



And also we define m_3 = 2

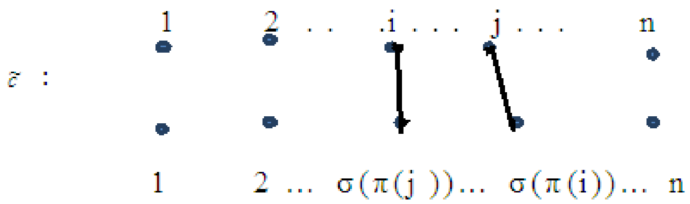
Case 5: Let $(\alpha_i, \alpha_j) \epsilon B_\pi$ and $(\beta_i, \beta_j) \epsilon B_\sigma$ and α_i is lower than α





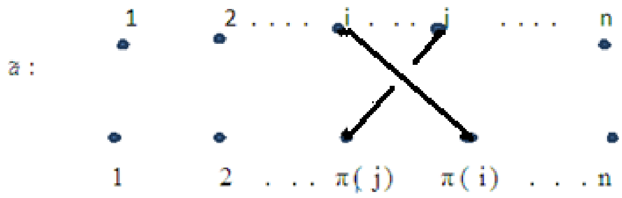
We define $\tilde{c}$ as shown below:

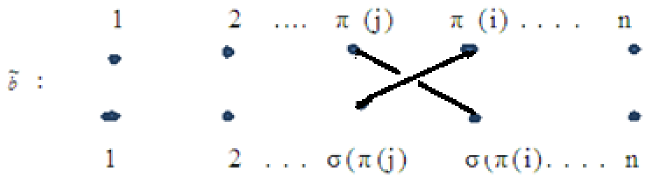
And $(\gamma_i, \gamma_j) \notin B_{\sigma o \pi}$ where $\gamma_i = (i , \sigma o\pi(i))$ and $\gamma_j = (j , \sigma o \pi(j))$



And also we define $m_3 = 0$

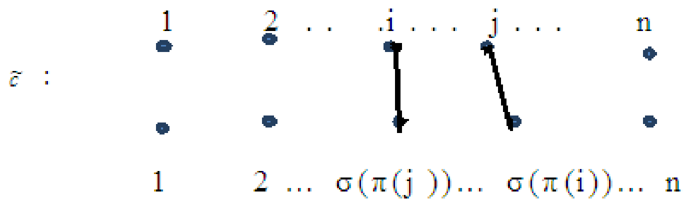
Case 6: Let $(\alpha_i,\alpha_j) \epsilon B_\pi$ and $(\beta_i,\beta_j) \epsilon B_\sigma$ and α_i is upper than α





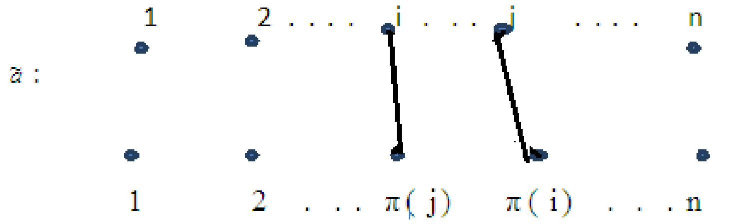
We define $\tilde{c}$ as shown below:

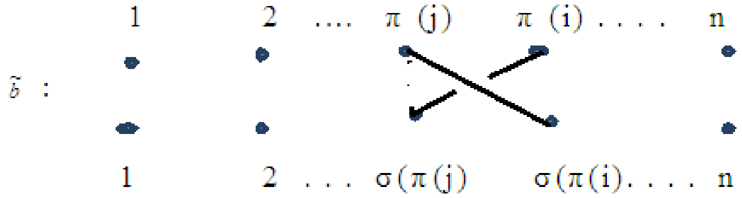
And (γ_i,γ_j)) $\notin B_{\sigma o \pi}$ where $\gamma_i = (i, \sigma o \pi(i))$ and $\gamma_j = (j , \sigma o \pi(j))$



And also we define $m_3 = 0$

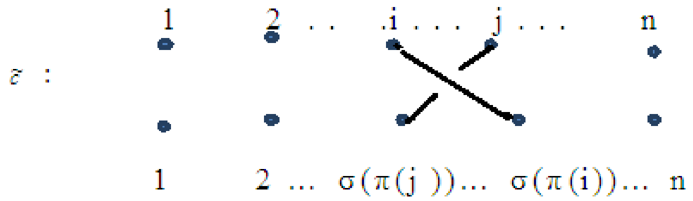
Case 7: Let $(\alpha_i, \alpha_j) \notin B_\pi$ and $(\beta_i, \beta_j) \epsilon B_\pi$





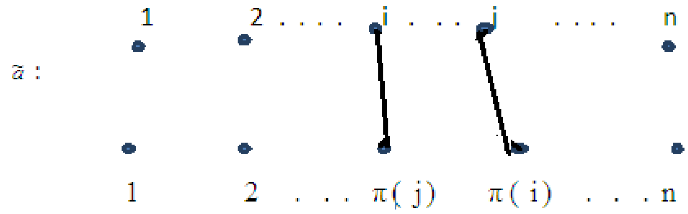
We define $\tilde{c}$ as shown below:

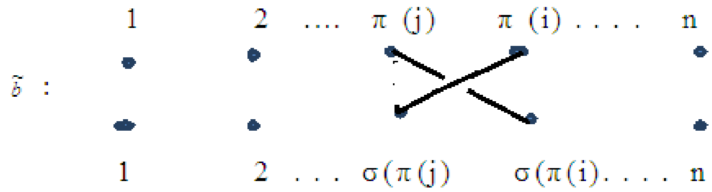
and $(\gamma_i, \gamma_j) \epsilon B_{\sigma o \pi}$ where γ_i= (i , σoπ(i)) and γ_i= (j , σoπ(j))



And also we define m_3 =0

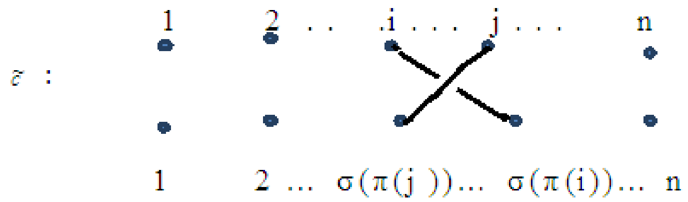
Case 8: Let $(\alpha_i,\alpha_j) \notin B_\pi$ and $(\beta_i,\beta_j) \epsilon B_\pi$





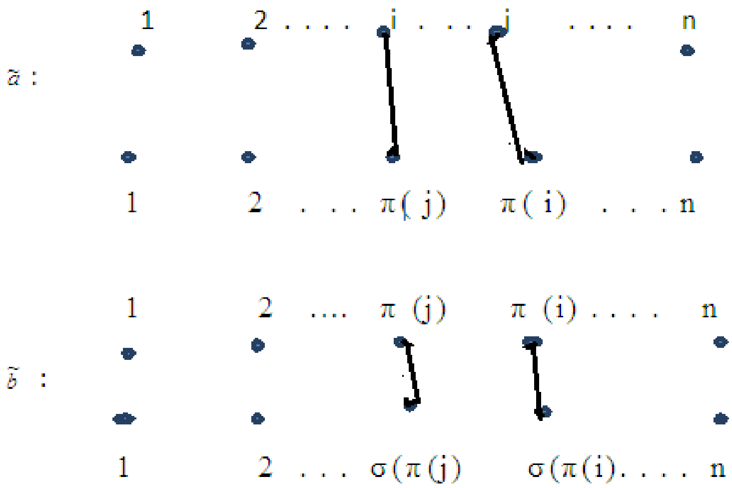
We define $\tilde{c}$ as shown below:

And $(\gamma_i,\gamma_j) \epsilon B_{\sigma o \pi}$ where γ_i= (i , σoπ(i)) and γ_i= (j , σoπ(j))



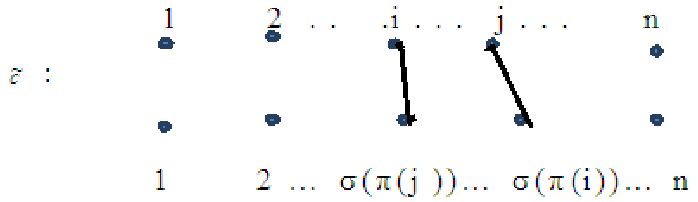
And also we define m_3 =0

Case 9: Let $(\alpha_i, \alpha_j) \notin B_\pi$ and $(\beta_i, \beta_j) \notin B_\pi$



We define $\widetilde{c}$ as shown below:

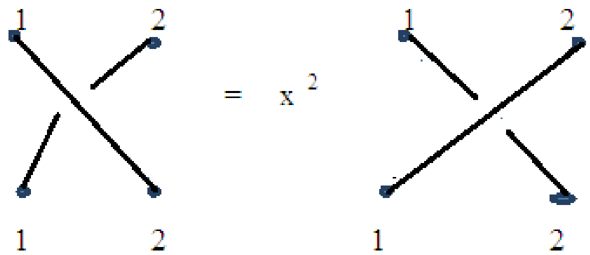
And $(\gamma_i, \gamma_j) \notin B_{\sigma o \pi}$ where $\gamma_i = (i, \sigma o \pi(i))$ and $\gamma_j = (j, \sigma o \pi(j))$



And also we define $m_3 = 0$

3.2.4 Note: To prove the associative property, we need the following definition

3.2.5 Definition we define



(ie) b= x^2 a

3.2.6 Remark: For the edge $\eta_i = (i, \delta o(\sigma o \pi)(i)) \in E(\delta o(\sigma o \pi))$,

There are corresponding edges $\alpha_i = (i, \pi(i)) \in E(\pi)$, $\beta_i = (\pi(i), \sigma o \pi(i)) \in E(\sigma)$

$\gamma_i = (\sigma o \pi(i), \delta o(\sigma o \pi(i)) \in E(\delta)$

Let $\rho_i = (i, \sigma o \pi(i)) \in E(\sigma o \pi)$

$\xi_i = (\pi(i), \delta o(\sigma o \pi)(i)) \in E(\delta o \sigma)$

3.2.7 Theorem If a, b, and c are the elements in $K_n(x)$ Then (ab)c= a(bc)

Proof: Let $a = (x^{m_1}, \widetilde{a})$ $b = (x^{m_2}, \widetilde{b})$ $c = (x^{m_3}, \widetilde{c})$ and $m_i \in Z$, for every $i = 1,2,3$

Let $\tilde{a} \in K_\pi , \tilde{b} \in K_\sigma , \tilde{c} \in K_\delta$ where $\pi,\sigma,\delta \in S_n$.

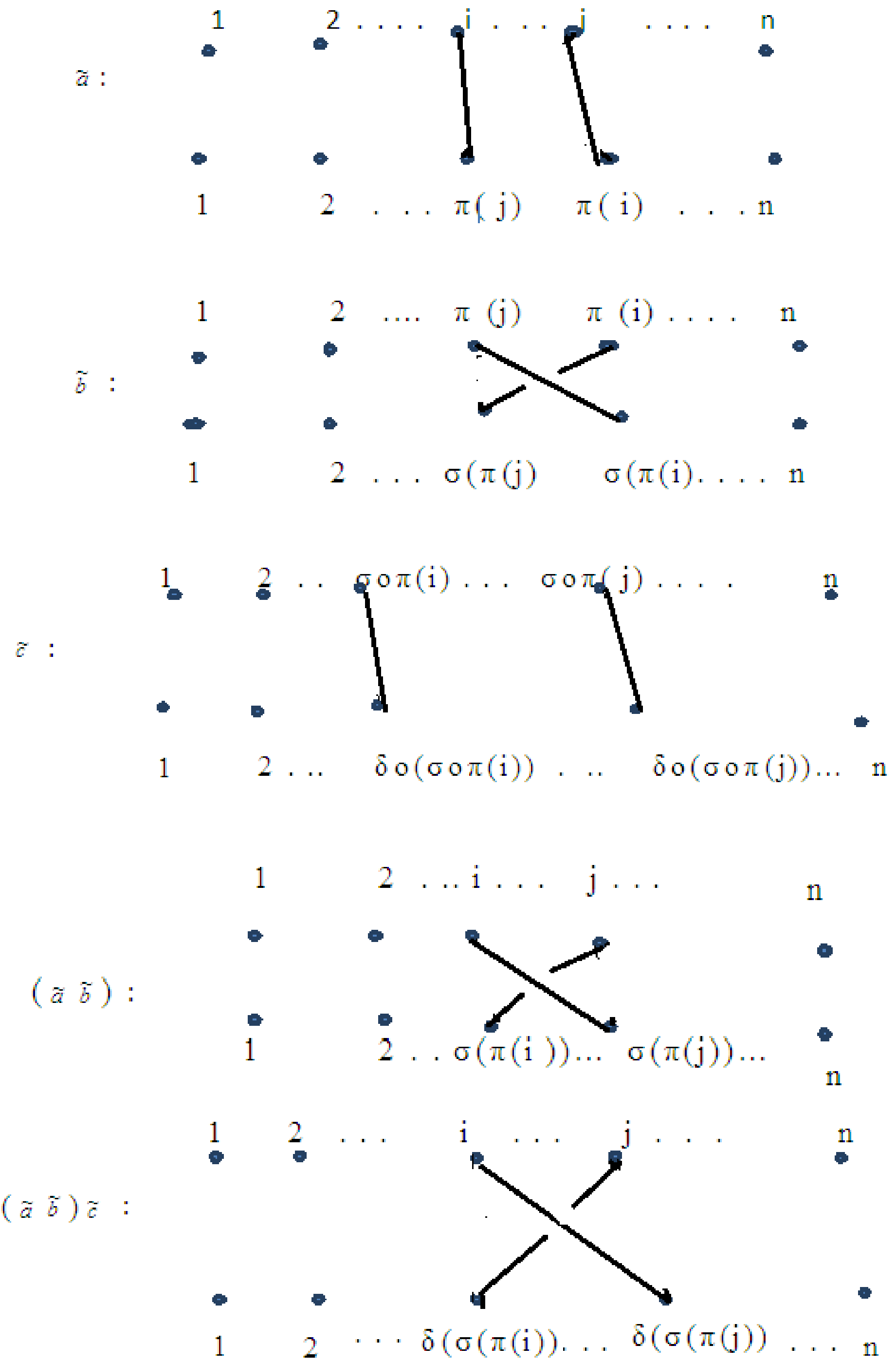
We know that $\delta o(\sigma o\pi)= (\delta o\sigma)o\pi)$

Case 1: a(bc)=(ab)c

To compute LHS= (ab)c

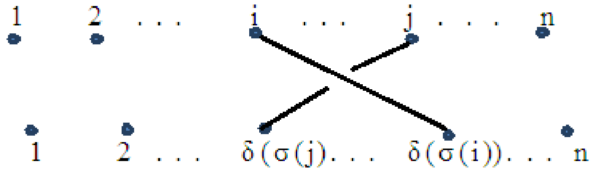
First compute $(\tilde{a}\ \tilde{b})$

Let $(\alpha_i, \alpha_j) \in B_\pi$, $(\beta_i, \beta_j) \in B_\sigma$, $(\gamma_i, \gamma_j) \notin B_\delta$

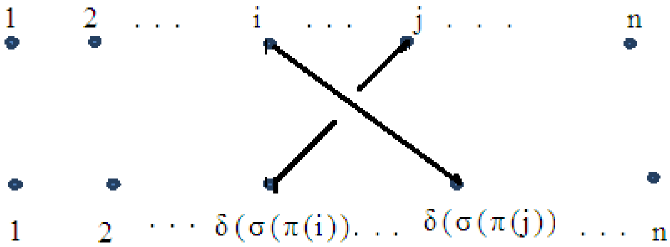


To compute RHS

$\tilde{b}\,\tilde{c}$:

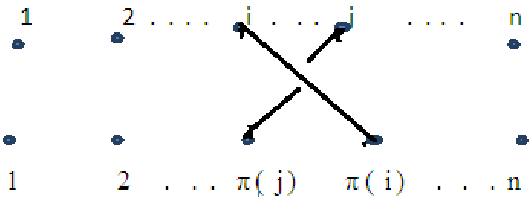


$\tilde{a}\,(\tilde{b}\,\tilde{c})$:

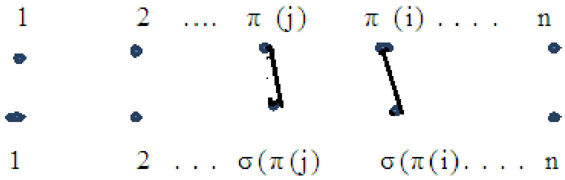


Case 2: Let $(\alpha_i, \alpha_j) \epsilon B_\pi$, $(\beta_i, \beta_j) \notin B_\sigma$, $(\gamma_i, \gamma_j) \epsilon B_\delta$

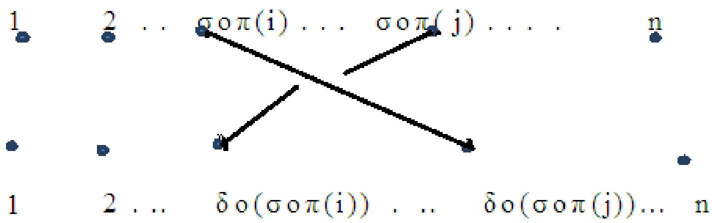
$\tilde{a}$:



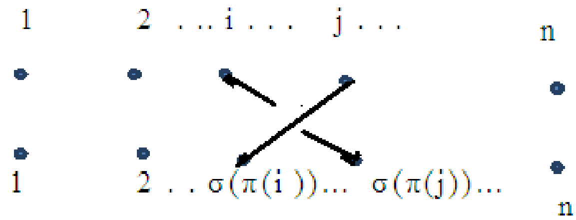
$\tilde{b}$:



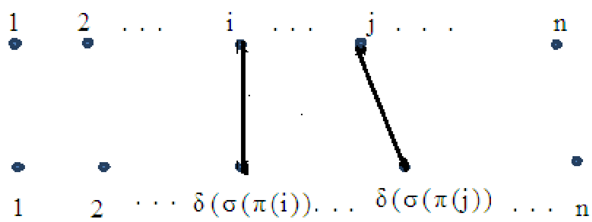
$\tilde{c}$:



$(\tilde{a}\,\tilde{b})$:

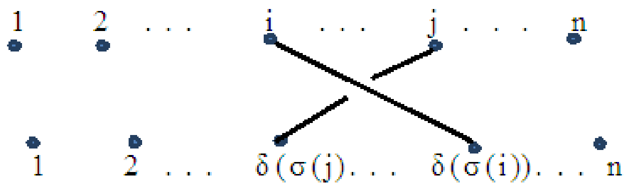


$(\tilde{a}\,\tilde{b})\,\tilde{c}$:

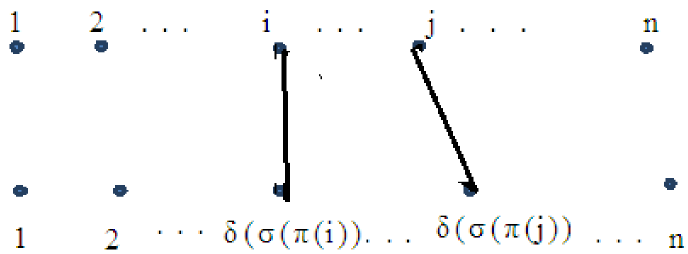


To compute RHS

$\tilde{b}\,\tilde{c}$:

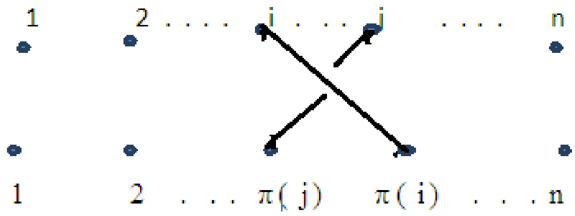


$\tilde{a}\,(\tilde{b}\,\tilde{c})$:

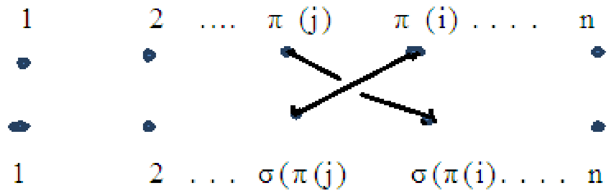


Case 3 : Let $(\alpha_i, \alpha_j) \epsilon B_\pi$, $(\beta_i, \beta_j) \epsilon B_\sigma$, $(\gamma_i, \gamma_j) \notin B_\delta$

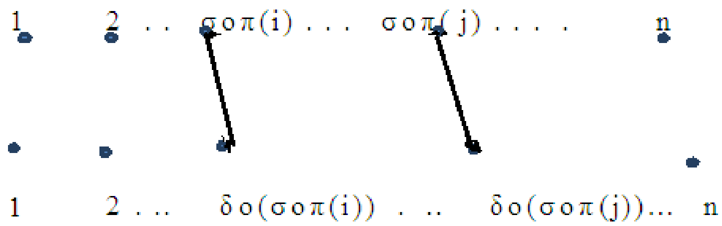
$\tilde{a}$:



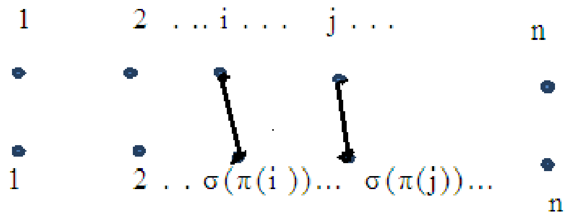
$\tilde{b}$:



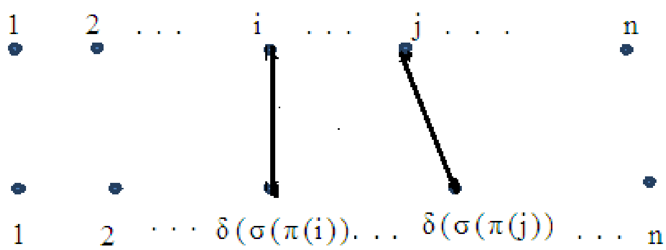
$\tilde{c}$:



$(\tilde{a}\,\tilde{b})$:

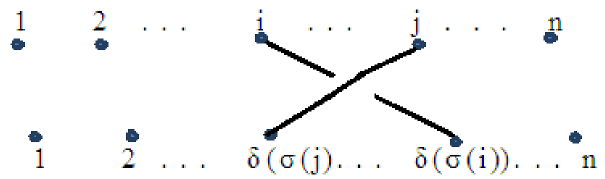


$(\tilde{a}\,\tilde{b})\,\tilde{c}$:

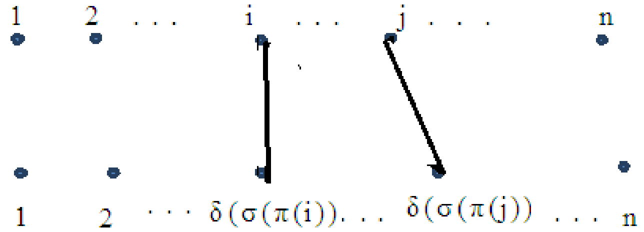


To compute RHS

$\tilde{b}\,\tilde{c}$:

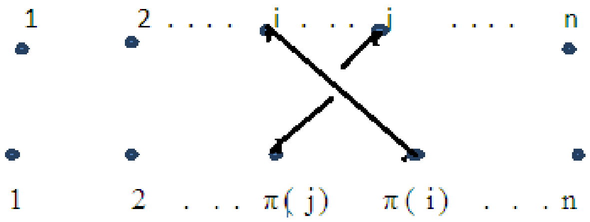


$\tilde{a}(\tilde{b}\,\tilde{c})$:

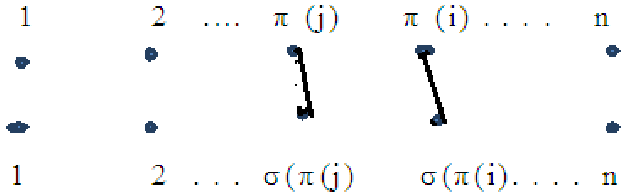


Case 4: Let $(\alpha_i, \alpha_j)\epsilon B_\pi$, $(\beta_i, \beta_j) \notin B_\sigma$, $(\gamma_i, \gamma_j)\, \epsilon B_\delta$

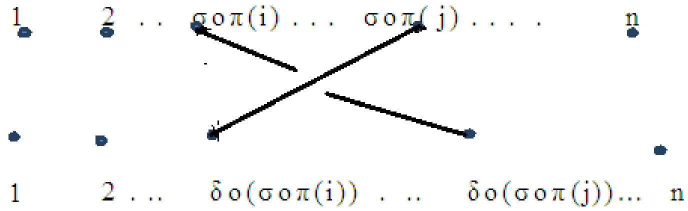
$\tilde{a}$:



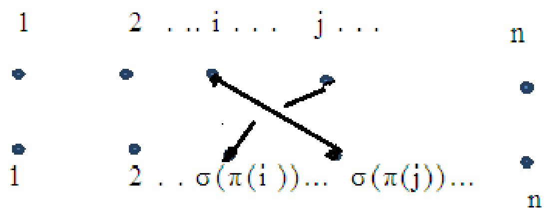
$\tilde{b}$:



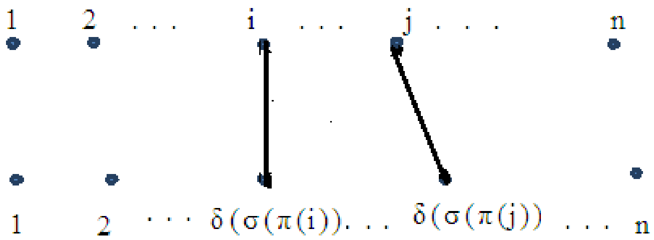
$\tilde{c}$:



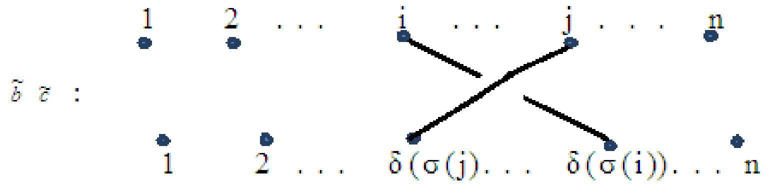
$(\tilde{a}\,\tilde{b})$:

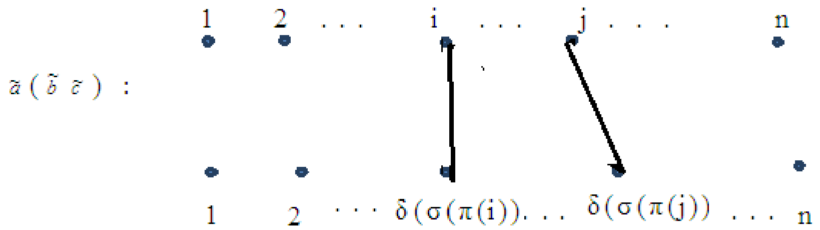


$(\tilde{a}\,\tilde{b})\tilde{c}$:

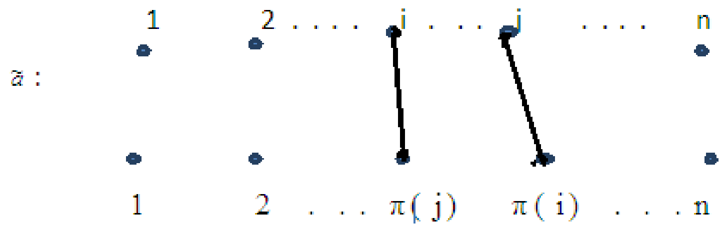


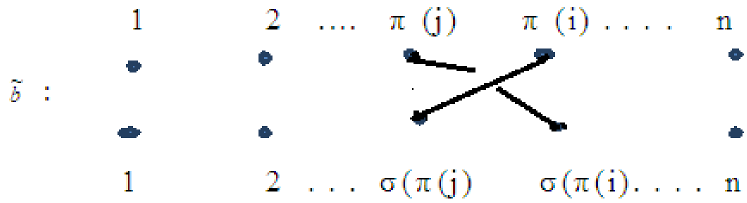
To compute RHS

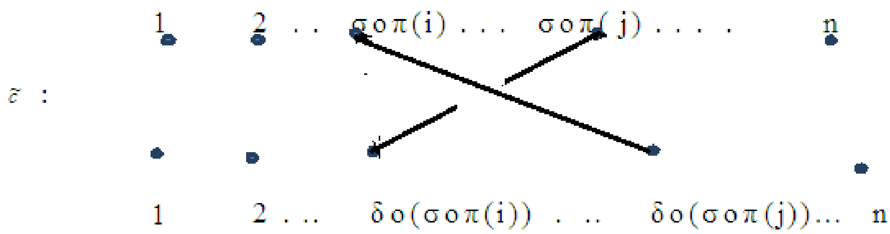


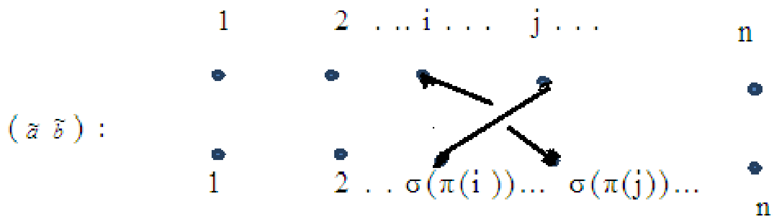


Case 5 : Let $(\alpha_i, \alpha_j) \epsilon B_\pi$, $(\beta_i, \beta_j) \notin B_\sigma$, $(\gamma_i, \gamma_j) \epsilon B_\delta$

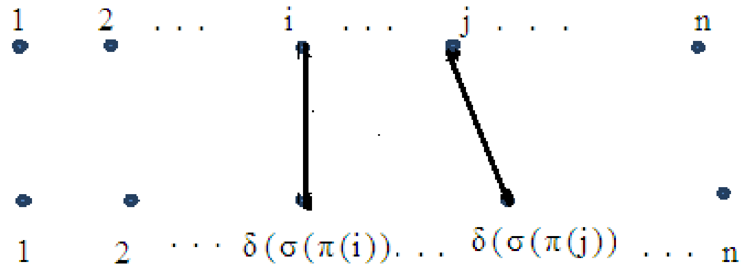






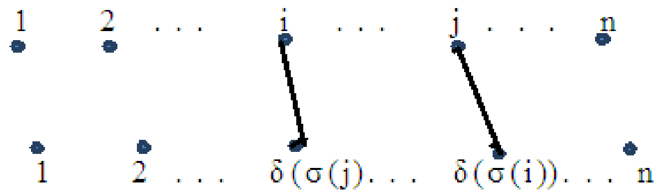


$(\tilde{a}\ \tilde{b})\ \tilde{c}$:

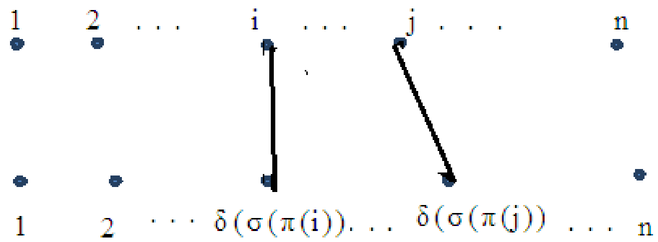


To compute RHS

$\tilde{b}\ \tilde{c}$:

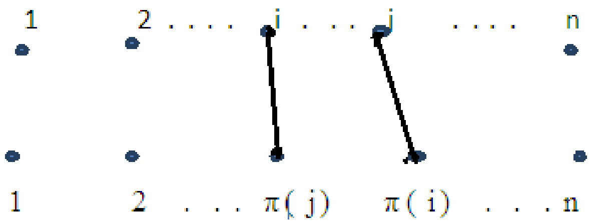


$\tilde{a}\ (\tilde{b}\ \tilde{c})$:

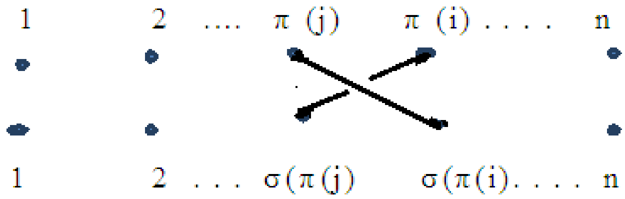


Case 6: Let $(\alpha_i, \alpha_j) \notin B_\pi$, $(\beta_i, \beta_j) \epsilon B_\sigma$, $(\gamma_i, \gamma_j)\ \epsilon B_\delta$

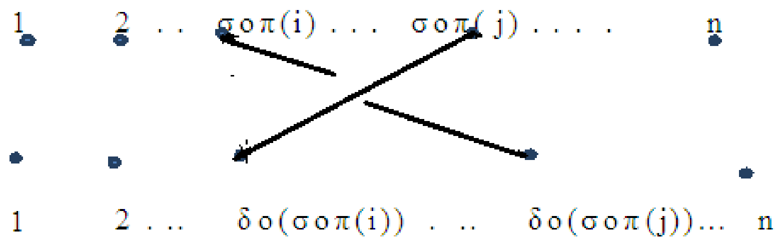
$\tilde{a}$:



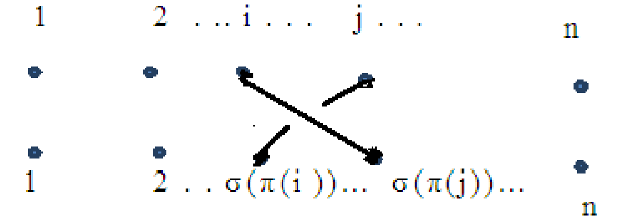
$\tilde{b}$:



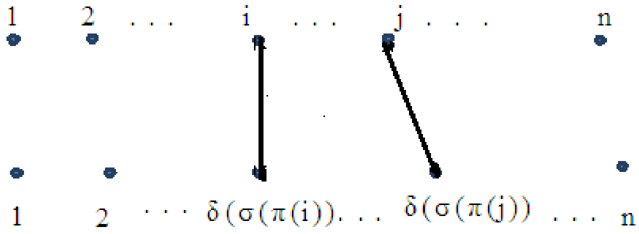
$\tilde{c}$:



$(\tilde{a}\,\tilde{b})$:

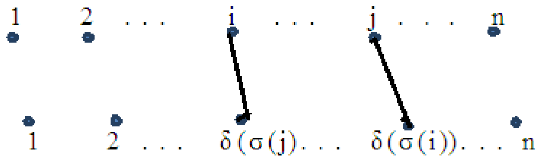


$(\tilde{a}\,\tilde{b})\,\tilde{c}$:

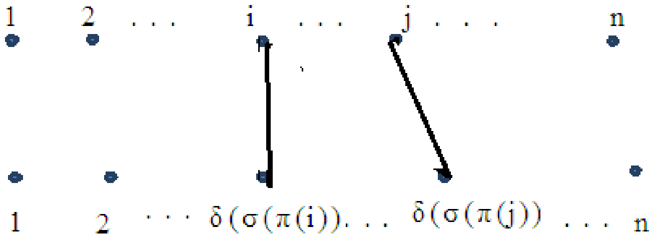


To compute RHS

$\tilde{b}\,\tilde{c}$:

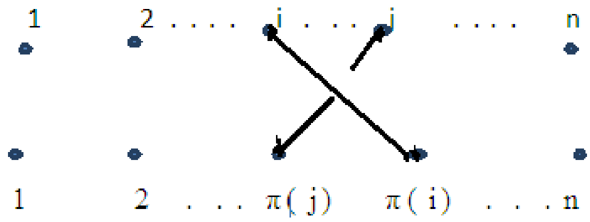


$\tilde{a}\,(\tilde{b}\,\tilde{c})$:

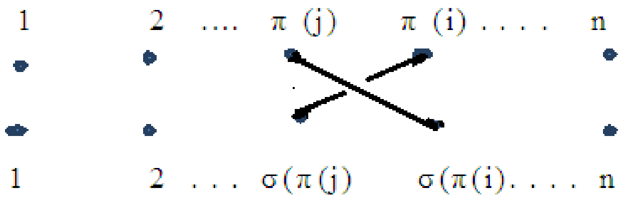


Case 7: Let $(\alpha_i, \alpha_j) \epsilon B_\pi$, $(\beta_i, \beta_j) \epsilon B_\sigma$, $(\gamma_i, \gamma_j) \epsilon B_\delta$

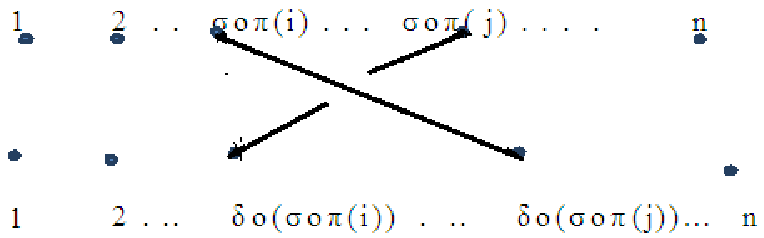
$\tilde{a}$:

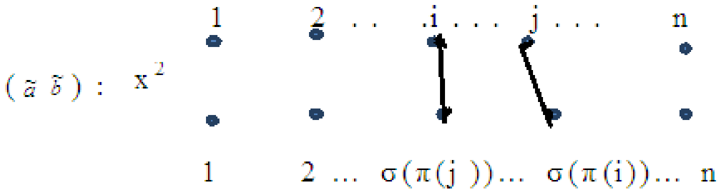


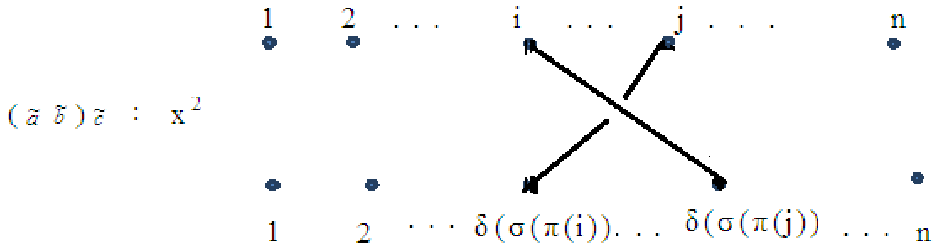
$\tilde{b}$:



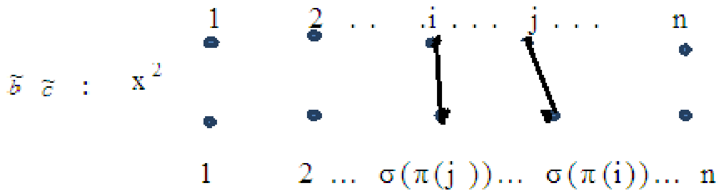
$\tilde{c}$:

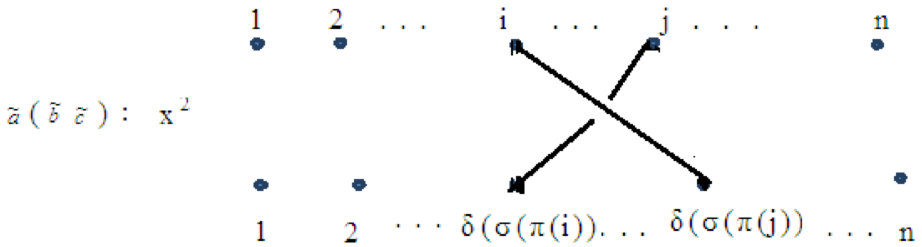




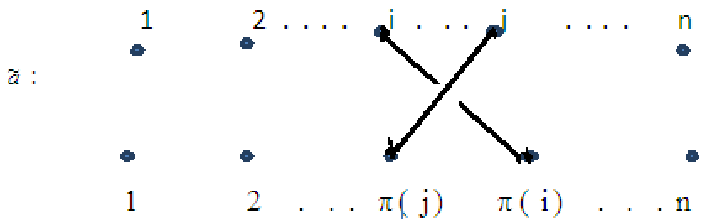


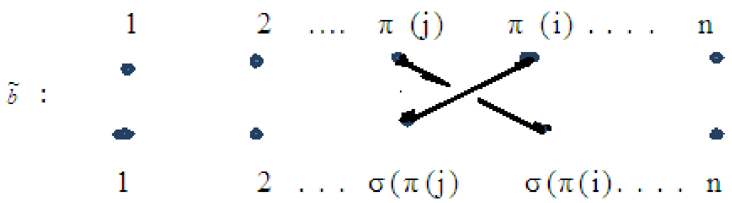
To compute RHS



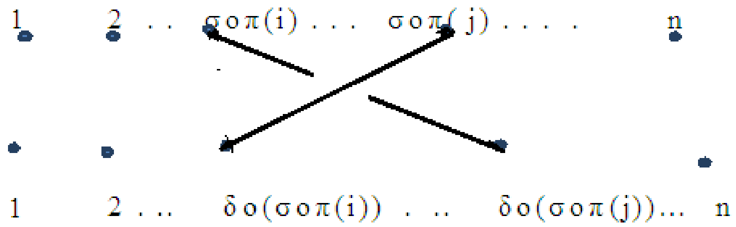


Case 8: Let $(\alpha_i,\alpha_j)\epsilon\, B_\pi$, $(\beta_i,\beta_j)\epsilon B_\sigma$, $(\gamma_i,\gamma_j)\ \epsilon B_\delta$

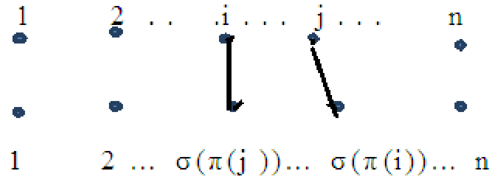




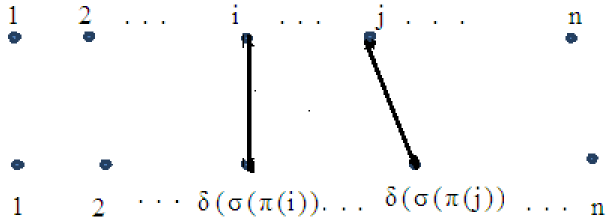
$\tilde{c}$:



$(\tilde{a}\ \tilde{b})$: $\frac{1}{x^2}$

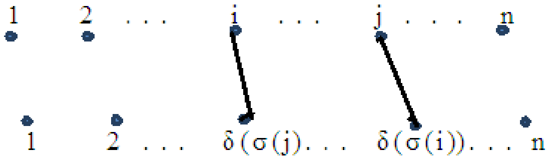


$(\tilde{a}\ \tilde{b})\tilde{c}$: $\frac{1}{x^2}$

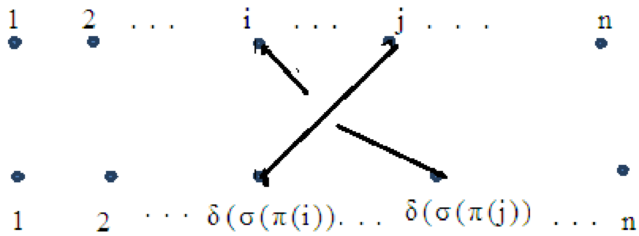


To compute RHS

$\tilde{b}\ \tilde{c}$: $1/x^2$

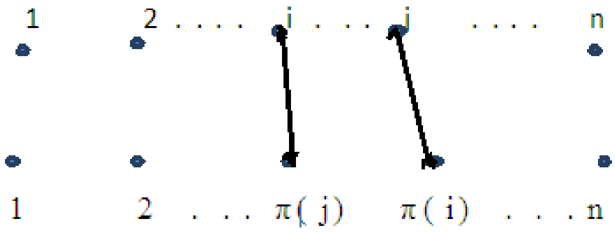


$\tilde{a}\,(\tilde{b}\ \tilde{c})$: $1/x^2$

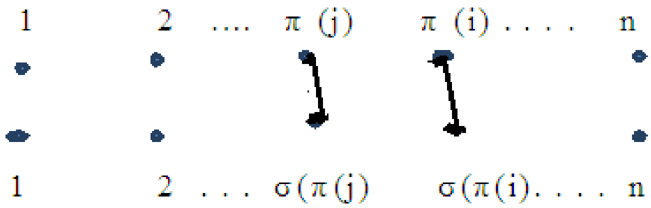


Case 9: Let $(\alpha_i, \alpha_j) \notin B_\pi$, $(\beta_i, \beta_j) \notin B_\sigma$, $(\gamma_i, \gamma_j) \notin B_\delta$

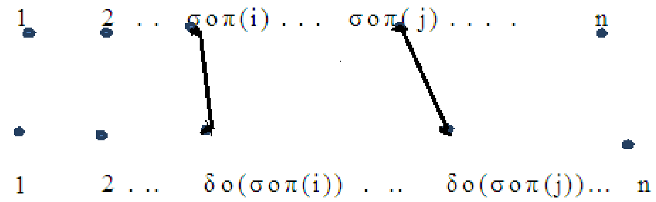
$\tilde{a}$:



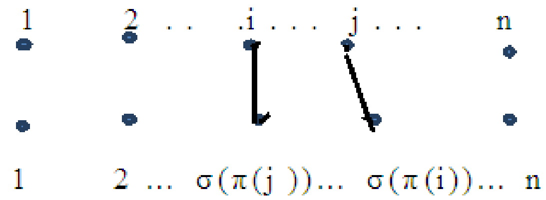
$\tilde{b}$:



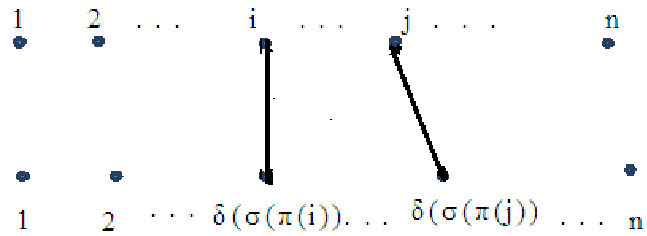
$\tilde{c}$:



$(\tilde{a}\ \tilde{b})$:

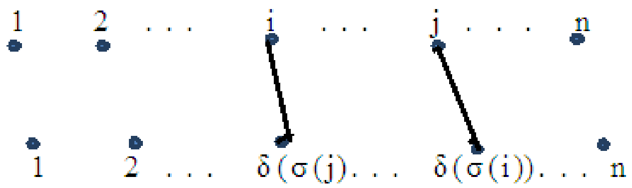


$(\tilde{a}\ \tilde{b})\tilde{c}$:

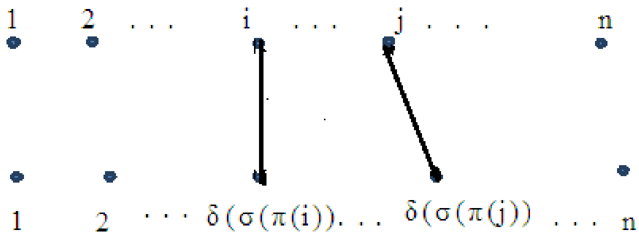


To compute RHS

$\tilde{b}\ \tilde{c}$:

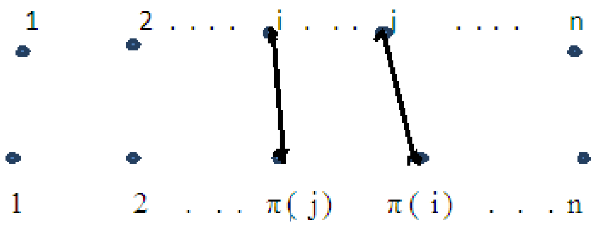


$\tilde{a}(\tilde{b}\ \tilde{c})$:

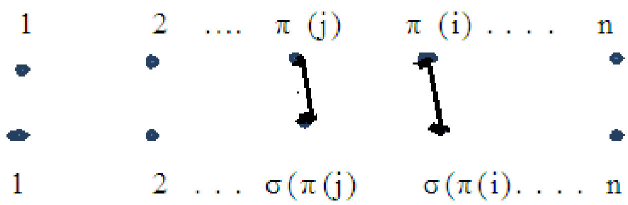


Case 10: Let $(\alpha_i, \alpha_j) \notin B_\pi$, $(\beta_i, \beta_j) \notin B_\sigma$, $(\gamma_i, \gamma_j) \in B_\delta$

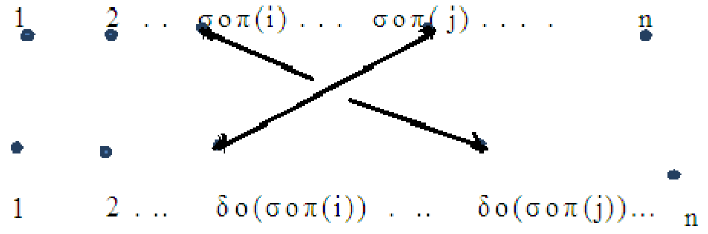
$\tilde{a}$:



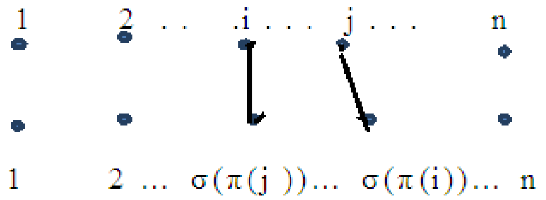
$\tilde{b}$:



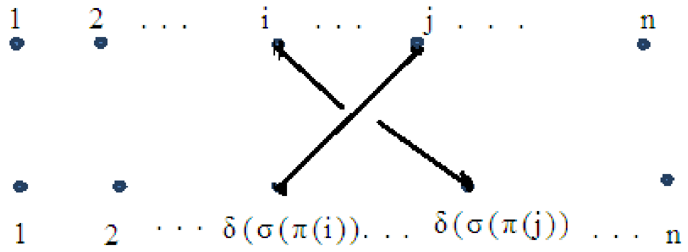
$\tilde{c}$:



$(\tilde{a}\ \tilde{b})$:

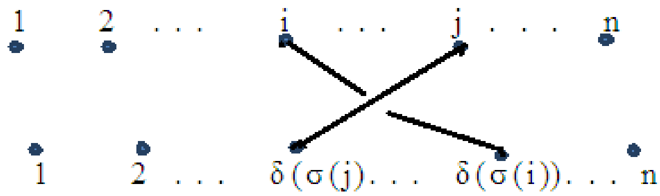


$(\tilde{a}\ \tilde{b})\,\tilde{c}$:

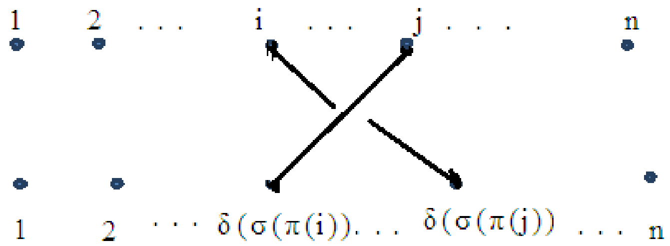


To compute RHS

$\tilde{b}\ \tilde{c}$:



$\tilde{a}\,(\tilde{b}\ \tilde{c})$:



Hence LHS=RHS

In 27 ways we have proved the associative property.Here we have proved in 10 ways and the remaining cases can be proved in similar way.

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