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A REVIEW OF ISOMORPHISM THEOREMS OF ANTI-ORDERED SEMIGROUPS WITH APARTNESS

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ABSTRACT.

A review of isomorphism theorems of anti-ordered semigroups with apartness are presented from Bishop's constructive mathematics point of view.

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1. INTRODUCTION:

Our setting is Bishop's constructive mathematics [2]-[4], [7], [19], mathematics developed with Constructive Logic (or Intuitionistic Logic) – logic without the Law of Excluded Middle $P \vee \neg P$. We have to note that 'the crazy axiom' $\neg P \Rightarrow (P \Rightarrow Q)$ is included in the Constructive Logic. (Precisely, in Constructive Logic the 'Double Negation Law' $P \Leftrightarrow \neg\neg P$ does not hold, but the following implication $P \Rightarrow \neg\neg P$ holds even in Minimal Logic. In Constructive Logic 'Weak Law of Excluded Middle' $\neg P \vee \neg\neg P$ does not hold. It is interesting, in Constructive logic the following deduction principle $A \vee B, \neg A \vdash B$ holds, but this is impossible to prove without 'the crazy axiom'.) One advantage of working in this manner is that proofs and results have more interpretations. On the one hand, Bishop's constructive mathematics is consistent with the traditional mathematics. If we are working constructively, the first problem is to obtain appropriate substitutes of the classical definitions. The classical theory of partially ordered sets is based on the negative concept of partial order. Unlike the classical case, an affirmative concept, introduced in the author's papers [11], [12] and like to Palmgren's [8] and Baroni's [1] excess relation, will be used as a primary relation.

This investigation is in Bishop's constructive algebra in sense of papers [9]-[16] and books [2]-[4], [7] and [19] (Chapter 8: Algebra). Let $(S, =, \neq)$ be a constructive set (in the sense of Bishop ([2]), Bridges ([3]), Mines et all ([7]), Troelstra and van Dalen ([19])). The relation $\neq$ is a binary relation on S, which satisfies the following properties:

$$\neg(x \neq x),\ x \neq y \Rightarrow y \neq x,\ x \neq z \Rightarrow x \neq y \vee y \neq z,\ x \neq y \wedge y = z \Rightarrow x \neq z\ .$$

It called *apartness* (A. Heyting). Let Y be a subset of S and $x \in S$. The subset Y of S is *strongly extensional* in S if and only if $y \in Y \Rightarrow y \neq x \vee x \in Y$ ([10], [19]). A relation q on S is a *coequality* relation on S if and only if it is consistent, symmetric and cotransitive ([9]-[11]):

$$q \subseteq \neq,\ q^{-1} = q,\ q \subseteq q * q.$$

where "$*$" is the *filled product* between relations ([9], [10]). Let $(S,=,\neq,\cdot)$ be a semigroup with an apartness. As in [10], a relation q on S is *anticongruence* if and only if it is coequality relation on S compatible with semigroup operation in the following sense:

$$(\forall x,y,z \in S)(((xz,yz) \in q \Rightarrow (x,y) \in q) \wedge ((zx,zy) \in q \Rightarrow (x,y) \in q)).$$

We will use a generalization of Palmgren's [8] and Baroni's [1] excess relation. Let S be a nonempty set. The binary relation $\nleqslant$ on S is called an *excess relation* if it satisfies the following axioms:

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$\neg(x \nleqslant x)$,
$x \nleqslant y \Rightarrow (\forall z\in S)(x \nleqslant z \vee z \nleqslant y)$.

Clearly, each linear order is an excess relation. From the excess relation we obtain an apartness relation ≠ and a partial order ≤ on S by the following definitions:

$x \neq y \Leftrightarrow (x \nleqslant y \vee y \nleqslant x)$,
$x \leq y \Leftrightarrow \neg(x \nleqslant y)$.

Note that the statement $\neg(x \leq y) \Rightarrow x \nleqslant y$ does not hold in general.

As in [11], we introduce definition for our notion of antiorder: A relation α on semigroup (S,=, ≠) is *anti-order* ([11], [12]) on S if and only if

$$\alpha \subseteq \neq,\ \alpha \subseteq \alpha * \alpha,\ \neq \subseteq \alpha \cup \alpha^{-1}$$

and

$$(\forall x,y,z\in S)(((xz,yz)\in \alpha \Rightarrow (x,y)\in \alpha) \wedge ((zx,zy)\in \alpha \Rightarrow (x,y)\in \alpha)).$$

Let S be a semigroup with apartness ([9], [10]). Constructive notion of quasi-antiorder relation is parallel notion to classical notion of quasi-order relation. Let (S,=, ≠,·) be a semigroup with apartness, α an antiorder on S. A relation σ on S is a *quasi-antiorder* ([12]) on S if

$$\sigma \subseteq (\alpha \subseteq) \neq,\ \sigma \subseteq \sigma * \sigma,$$

and

$$(\forall x,y,z\in S)(((xz,yz)\in \sigma \Rightarrow (x,y)\in \sigma) \wedge ((zx,zy)\in \sigma \Rightarrow (x,y)\in \sigma)).$$

In this paper and some other papers (for example, in [12], [13]) we try to research properties of quasi-antiorder.

Let x be an element of S and A subset of S. We write $x \rhd\lhd A$ if and only if $(\forall a\in A)(x \neq a)$, and $A^C = \{x\in S : x \rhd\lhd A\}$. If σ is a quasi-antiorder on S, then the relation $q = \sigma\cup\sigma^{-1}$ is an anticongruence on S. As the first, the relation $q^C = \{(x,y)\in S\times S : (x,y) \rhd\lhd q\}$ is a congruence on S compatible with q ([10], Theorem 1), in the following sense

$$q^C \circ q \subseteq q \text{ and } q \circ q^C \subseteq q.$$

We can construct semigroup $S/(q^C,q) = \{aq^C : a\in S\}$.

Lemma 1.1 ([10, Theorem 2]) *If* q is *an anticongruence on a semigroup* S *with apartness, then the set* $S/(q,q^C)$ *is a semigroup with*

$$a q^C =_1 b q^C \Leftrightarrow (a,b) \rhd\lhd q,\ a q^C \neq_1 b q^C \Leftrightarrow (a,b) \in q,\ aq^C \cdot bq^C = (ab)q^C .$$

We can also construct the semigroup $S/q = \{aq : a\in S\}$:

Lemma 1.2 ([10, Theorem 3]) *Let* q *be anticongruence on a semigroup* S *with apartness. Then the set* S/q *is a semigroup with*

$$aq =_1 bq \Leftrightarrow (a,b) \rhd\lhd q,\ aq \neq_1 bq \Leftrightarrow (a,b) \in q,\ aq\cdot bq = (ab)q .$$

It is easy to check that $S/q \cong S/(q^C,q)$.

2. PRELIMINARIES:

Our first proposition gives us an explanation what is antiorder relation:

Lemma 2.1 ([12], Lemma 1) *Let* α *be an anti-order relation on semigroup* (S,=,≠,·). *Then relation* α^C *is a partial order relation on* (S, ¬≠, ≠, ·). *If the apartness* ≠ *is tight, then* α^C *is a partial order relation on the semigroup S.*

Here we study some theorems from [4] and [8] for anti-ordered semigroups. As mentioned above, if σ is a quasi-antiorder on S, then the relation $q = \sigma \cup \sigma^{-1}$ is an anticongruence on S, as the first. As the second, strongly complement σ^C of quasi-antiorder s has well known characteristic:

Lemma 2.2 ([12], Lemma 2) *Is* σ *is a quasi-antiorder on* S, *then relation* $\sigma^C = \{(x,y)\in S\times S : (x,y) \rhd\lhd \sigma\}$ *is an quasi-order on S*.

Lemma 2.3 *Let* $\varphi : (S,=,\neq,\cdot,\alpha) \to (T,=,\neq,\cdot,\beta)$ *be a homomorphism of anti-ordered semigroups. Then:*

(1) φ *is isotone if and only if* $\alpha \subseteq \varphi^{-1}(\beta)$;
(2) φ *is reverse isotone if and only if* $\varphi^{-1}(\beta) \subseteq \alpha$.

3. THE ISOMORPHISM THEOREMS:

Lemma 3.1 ([12], Lemma 1) *Let* $(S,=,\neq,\cdot)$ *be a semigroup with apartness and* σ *be a quasi-antiorder relation on* S. *The relation* $\theta = \pi \circ \sigma \circ \pi^{-1}$ *on* S/q, *where* $q = \sigma \cup \sigma^{-1}$, *is a consistent, cotransitive and linear relation on semigroup* S/q *compatible with the semigroup operation on* S/q.

The following theorem will give opposite assertion to the above theorem.

Theorem 3.2 ([12], Lemma 2) *If* $(S,=,\neq,\cdot)$ *and* $(T,=,\neq,\cdot)$ *are semigroups,* τ *is a quasi-antiorder on* T, *and* $\varphi : S\to T$ *a strongly extensional homomorphism, then the relation*

$$\varphi^{-1}(\tau) = \{(a.b)\in S\times S: (\varphi(a),\varphi(b))\in \tau\}$$

is a quasi-antiorder on S, *the relation* $\mathrm{Coker}\varphi = \{(a,b)\in S\times S : \varphi(a) \neq \varphi(b))\}$ *is anticongruence on* S *compatible with congruence* $\mathrm{Ker}\varphi = \varphi^{-1} \circ \varphi$, *and*

$$\mathrm{Coker}\varphi \supseteq \varphi^{-1}(\tau)\cup(\varphi^{-1}(\tau))^{-1}$$

holds. Also, if the relation τ *is linear we have* $\mathrm{Coker}\varphi = \varphi^{-1}(\tau) \cup (\varphi^{-1}(\tau))^{-1}$.

Let $(S,=,\neq,\cdot)$ and $(T,=,\neq,\cdot,\tau)$ are semigroups, where τ is a quasi-antiorder on T, and $\varphi : S \to T$ is a homomorphism. In the next proposition we will describe condition for decomposition of homomorphism φ:

Theorem 3.3 ((First isomorphism theorem), [18], Theorem 6) *Let* $(S,=,\neq,\cdot,\rho)$ *and* $(T,=,\neq,\cdot,\tau)$ *be semigroups, where* τ *is linear quasi-antiorder on* T, *and* $\varphi : S \to T$ *a strongly extensional homomorphism. Then if* ρ *is a quasi-antiorder in* S *such that* $\rho \supseteq \varphi^{-1}(\tau)$, *and if the apartness in semigroup* T *is tight, then the mapping* f: $S/(\rho\cup\rho^{-1}) \to T$ *is strongly extensional homomorphism of semigroups such that* $f \circ \pi = \varphi$. *Conversely, if* ρ *is a quasi-antiorder on* S *for which there exists a strongly extensional homomorphism* f: $S/(\rho\cup\rho^{-1}) \to T$ *such that* $f \circ \pi = \varphi$, *then* $\rho \supseteq \varphi^{-1}(\tau)$.

For the next proposition we need a lemma in which we will describe the anticongruence α and β on a semigroup such that $\beta \subseteq \alpha$.

Lemma 3.4 ([9], Lemma 2) *Let* α *and* β *be anticongruences on a semigroup* S *with apartness such that* $\beta \subseteq \alpha$. *Then the relation* β/α *on* S/α, *defined by* $\beta/\alpha = \{(x\alpha,y\alpha)\in S/\alpha \times S/\alpha : (x,y)\in \beta\}$, *is an anticongruence on* S/α *and* $(S/\alpha)/(\beta/\alpha) \cong S/\beta$ *holds.*

So, at the end of this article, we are in position to give a description of quasi-antiorder ρ and σ of semigroup S with apartness such that $\sigma \subseteq \rho$.

Theorem 3.5 ((Second isomorphism theorem), [11], Theorem 1; [18], Theorem 8) *Let* $(S, =,\neq,\cdot)$ *be a semigroup*, ρ *and* σ *quasi-antiorders on S such that* $\sigma \subseteq \rho$. *Then the relation* σ/ρ, *defined by*

$$\sigma/\rho = \{(x(\rho\cup\rho^{-1}),y(\rho\cup\rho^{-1}))\in S/(\rho\cup\rho^{-1}) \times S/(\rho\cup\rho^{-1}) : (x,y)\in \sigma\},$$

is a quasi-antiorder on $S/(\rho\cup\rho^{-1})$ *and*

$$(S/(\rho\cup\rho^{-1})/((\sigma\cup\sigma^{-1})/(\rho\cup\rho^{-1})) \cong S/(\sigma\cup\sigma^{-1})$$

holds.

In the following theorem we prove that there exists bijective mapping between quasi-antiorder T on S/q and quasi-antiorder t on S included in σ.

Theorem 3.7 ([12], Theorem 3) *Let* $(S,=, \neq,\cdot)$ *be a semigroup with apartness,* σ *a quasi-antiorder on* S. *Let* $\boldsymbol{A} = \{\tau: \tau$ is quasi-antiorder on S such that $\tau \subseteq \sigma\}$. *Let* $\boldsymbol{B}$ *be the set of all quasi-antiorders on* S/q, *where* $q =\sigma\cup\sigma^{-1}$. *For* $\tau\in \boldsymbol{A}$, *we define a relation* $\tau" = \{(aq,bq)\in S/q \times S/q : (a,b)\in \tau \}$. *The mapping* $\psi : \boldsymbol{A} \to \boldsymbol{B}$ *defined by* $\psi(\tau) = \tau"$ *is strongly* *extensional, injective and surjective mapping from **A** onto **B** and for* $\tau_1, \tau_2 \in$ ***A*** *we have* $\tau_1 \subseteq \tau_2$ *if and only if* $\psi(\tau_1) \subseteq \psi(\tau_2)$.

Besides above, we have a special isomorphism theorem on anti-ordered sets.

Theorem 3.8 ([17], Theorem 4): *Let* $(X,=_X,\neq_X,\alpha)$ *and* $(Y,=_Y,\neq_Y,\beta)$ *be ordered sets under antiorders, where the apartness* $\neq_Y$ *is tight. If* $\varphi : X \to Y$ *is reverse isotone strongly extensional function, then there exists a strongly extensional and embedding reverse isotone bijection*

$$((X, =_X, \neq_X, \alpha, c(R))/q, =_1, \neq_1, \gamma) \to (Im(\varphi),=_Y,\neq_Y,\beta)$$

where $c(R)$ *is the biggest quasi-antiorder relation on* X *under* $R = \alpha\cap Coker(\varphi)$, $q = c(R)\cup c(R)^{-1}$ *and* γ *is the antiorder induced by the quasi-antiorder* $c(R)$.

If the condition $\alpha\cap\alpha^{-1} = \varnothing$ *holds, then the there exists the isomorphism*

$$(X, =_X, \neq_X, \alpha, c(R))/q, =_1, \neq_1, \gamma) \cong (Im(\varphi), =_Y, \neq_Y, \beta).$$

4. A SPECIAL ISOMORPHISM THEOREM:

Our first proposition is the following lemma which gives another example of quasi-antiorder relation on semigroup S generated by a strongly extensional subset of S. So, a connection between the family of all strongly extensional subsets of S and the family of all quasy-anatiorders on S is natural.

Lemma 4.0 ([5], Lemma 2.0) *Let* A *be a strongly extensional subset of a semigroup* $(S,=,\neq,\cdot)$. *Then relation* $\Theta_A \subseteq S\times S$, *defined by* $(a,b)\in \Theta_A \Leftrightarrow (\exists x,y\in S^1)(xby\in A \wedge xay \rhd\!\lhd A)$, *is an quasi-antiorder relation on* S.

In what follows, we have the notion of order substructures. We follow the classical Pin's definition of order ideal of ordered semigroup. Here we are dealing with anti-ordered semigroup. An *anti-ideal* of S is a subset K of S such that

$$(\forall x,y)(y\in K \Rightarrow y\Theta x \vee x\in K)$$

Lemma 4.1 ([5], Lemma 2.1) *Let* $\varphi : (S,=,\neq,\cdot,\Theta) \to (T,=,\neq,\cdot,\Omega)$ *be a reverse isotone homomorphism of anti-ordered semigroups. If* W *is an anti-ideal of* T, *then* $\varphi^{-1}(W)$ *is an anti-ideal of* S.

Let $(T,=,\neq,\cdot)$ be an anti-ordered semigroup and let K be an anti-ideal of T. We define on T a relation Q(K) by setting

$$uQ(K)v \Leftrightarrow (\exists x,y\in T^1)((xuy\in K \wedge xvy \rhd\!\lhd K)\vee(xvy\in K \wedge xuy \rhd\!\lhd K)).$$

Theorem 4.1 ([5], Theorem 3.1) *The relation* Q(K) *is an anticongruence on* T.

Note that we are able to construct the quasi-antiorder θ_K ($\subseteq \Theta_T$) on T (see Lemma 2.0) in the following way: $a\theta_K b \Leftrightarrow (\exists u,v\in T^1)(ubv\in K \wedge uav \rhd\!\lhd K)$ such that $Q(K) = \theta_K \cup (\theta_K)^{-1}$. The quasi-antiorder θ_K on T induces an anti-order Θ_K on T/Q(K) by $(aQ(K),bQ(K))\in \Theta_K \Leftrightarrow a\theta_K b$.

Lemma 4.2 ([5], Lemma 3.1) *The map* $\pi(K) : T \to T/Q(K)$ *is a reverse isotone homomorphism of anti-ordered semigroup* $(T,=,\neq,\cdot,\Theta)$ *onto* $(T/Q(K),=_1,\neq_1,\cdot_1,\Theta_K)$.

The following theorem is the main result of this article:

Theorem 4.3 ([5], Theorem 3.2) *Let* $\alpha : R \to S$ *be a reverse isotone surjective homomorphism of anti-ordered semigroups and let* W *be an anti-ideal in* S. *Then, there exists an anti-order reverse isotone isomorphism*

$$\psi : (S/Q(W),=_1,\neq_1,\cdot_1,\Theta_W) \to (R/Q(K),=_2,\neq_2,\cdot_2,\Theta_K)$$

such that $\pi(K) = \psi \circ \pi(W) \circ \alpha$, where $K = \alpha^{-1}(W)$.

5 DEFINITIONS AND BASIC QA-MAPPINGS:

Let $(X, =,\neq)$ be a set with apartness, q be a coequality relation on X and α be an anti-order relation on X. With it is associated the following relative $((X/q,=_1,\neq_1),\Theta)$ where

$$\theta = \pi \circ \alpha \circ \pi^{-1}.$$

In [15] giving an answer on question "When the relation θ, defined above, is an anti-order relation on X/q?" we find necessary and sufficient conditions that the relation $\pi \circ \alpha \circ \pi^{-1}$ is an anti-order relation on X/q.

Lemma 5.1 ([15], Theorem 4) *Let* q *be a coequality relation in anti-ordered set* $(X, =, \neq, \alpha)$. *Then, the relation* $\theta = \pi \circ \alpha \circ \pi^{-1}$ *is an anti-order relation on factor-set* X/q *if and only if the relation* $\tau = \mathrm{Ker}\pi \circ \alpha \circ \mathrm{Ker}\pi$ *is a quasi-antiorder relation on* X *such that* $\tau \cup \tau^{-1} = q$.

By definition, for a quasi-antiorder ρ on an anti-ordered set $(X, =, \neq, \alpha)$ holds $\rho \subseteq \alpha$. Opposite inclusion does not hold, but result in Theorem 5.1 is a motive for introducing of the following new notion:

Definition 1 Let $(X, =, \neq, \alpha)$ be an anti-ordered set. A quasi-antiorder ρ on X is called a *quotient quasi-antiorder* (abbreviated to Q-quasi-antiorder) on X if holds

$$\alpha \subseteq \mathrm{Ker}\pi \circ \rho \circ \mathrm{Ker}\pi.$$

Let $\varphi : (X,=,\neq,\alpha) \rightarrow (Y,=,\neq,\beta)$ be a strongly extensional reverse isotone mapping between anti-ordered sets. Then, by Theorem 3.2, the relation $\varphi^{-1}(\beta)$ is a quasi-antiorder on X with $\varphi^{-1}(\beta) \cup (\varphi^{-1}(\beta)) = \mathrm{Coker}\varphi$, and $X/\mathrm{Coker}\varphi \cong Im\varphi$ as anti-ordered sets. Besides, holds $\varphi^{-1}(\beta) \subseteq \alpha$ because φ is a reverse isotone mapping. A little generalization of notion introduced in the definition 1 is the following notion:

Definition 2 Let $(X,=,\neq,\alpha)$ and $(Y,=,\neq,\beta)$ be anti-ordered sets. A reverse isotone strongly extensional mapping $\varphi : X \rightarrow Y$ is called a *quotient anti-ordered mapping* (abbreviated to QA-mapping) of X to Y if holds

$$\alpha \subseteq \mathrm{Ker}\varphi \circ \varphi^{-1}(\beta) \circ \mathrm{Ker}\varphi .$$

In the case when φ is onto, T is called a *quotient anti-ordered set* of X.

In the following theorem a characteristic of Q-quasi-antiorder is present:

Theorem 5.2 *Let* $(X, =, \neq)$ *be an anti-ordered set and* ρ *a* Q-*quasi-antiorder on* X. *Then* $\pi : X \rightarrow X/(\rho\cup\rho^{-1})$ *is a* QA-*mapping from* X *onto* $X/(\rho\cup\rho^{-1})$. *Thus*, $X/(\rho\cup\rho^{-1})$ *is a quotient anti-ordered set of* X.

In the next assertion we give a connection between QA-mappings and Q-quasi-antiorders on anti-ordered sets.

Theorem 5.3 *Let* $(X,=,\neq,\alpha)$ *and* $(Y,=,\neq,\beta)$ *be anti-ordered sets and* $\varphi : X \rightarrow Y$ *a strongly extensional reverse isotone* QA-*mapping. Then,* $\varphi^{-1}(\beta)$ is *a* Q-*quasi-antiorder on* X *with* $\varphi^{-1}(\beta) \cup (\varphi^{-1}(\beta))^{-1} = \mathrm{Coker}\varphi$.

6. ISOMORPHISM THEOREMS FOR QA-MAPPINGS:

In this section we present two isomorphism theorems on QA-mappings and Q-quasi-antiorder.

Theorem 6.1 (First Isomorphism theorem) *Let* $(X, =, \neq, \alpha)$ *and* $(Y,=,\neq,\beta)$ *be anti-ordered sets and* $\varphi : X \rightarrow Y$ *a* QA-*mapping and* ρ *a* Q-*quasi-antiorder on* X. *Then,* $\rho \supseteq \varphi^{-1}(\beta)$ *if and only if there is a unique* QA-*mapping* $\psi : X/(\rho\cup\rho^{-1}) \rightarrow Y$ *such that* $\varphi = \psi \circ \pi$. *Moreover*, $Im\varphi = Im\psi$.

Theorem 6.2 (Second Isomorphism Theorem) *Let* $(X,=,\neq,\alpha)$ *be a set*, ρ *and* σ Q-*quasi-antiorders on* X *such that* $\sigma \subseteq \rho$. *Then the relation* σ/ρ, *defined by*

$$\sigma/\rho = \{(x(\rho\cup\rho^{-1}), y(\rho\cup\rho^{-1})) \in X/(\rho\cup\rho^{-1}) \times X/(\rho\cup\rho^{-1}) : (x,y) \in \sigma\},$$

is a Q-*quasi-antiorder on* $X/(\rho\cup\rho^{-1})$ *and*

$$(X/(\rho\cup\rho^{-1}))/((\sigma/\rho)\cup(\sigma/\rho)^{-1}) \cong X/(\sigma\cup\sigma^{-1})$$

holds as anti-ordered sets.

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