International Research Journal of Pure Algebra -3(6), 2013, 232-236
Available online through www.rjpa.info ISSN 2248–9037

# COMMON FIXED POINTS OF SELF MAPS SATISFYING AN INTEGRAL TYPE IN INTUIONISTIC FUZZY METRIC SPACE

**D. Venugopalam[1], T. Kesava Rao[2*] and M. Vijaya Kumar[3]**

*[1]Govt: Pigle Degree &P.G. College, Warangal, India*
*[2]Govt: City College, Hyderabad, India*
*[3]Department of Mathematics, Vaagdevi College of Engineering, Warangal, (A.P.), India*

*(Received on: 09-06-13; Revised & Accepted on: 15-06-13)*

***ABSTRACT***

*In this paper, we prove common fixed point theorems for weakly compatible self maps of integral type in intuitionistic fuzzy metric space. these results are proved with exploiting the notion of continuity and without imposing any condition of t- norm.*

***Key Words:*** *intuitionistic fuzzy metric space, weakly compatible maps of common fixed point.*

## 1. INTRODUCTION

Atanassove[1] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. In 2004, Park [2] defined the notion of intuitionistic fuzzy metric space with the help of continuous $t$-norms and continuous. Alaca *et al.*[3] using the idea of Intuitionistic fuzzy sets, defined the notion of intuitionistic fuzzy metric space with the help of continuous $t$-norm and continuous as a generalization of fuzzy metric space due to Kramosil and Michalek[4]. In 2006, Turkoglu[5] proved Jungck's[6] common fixed point theorem in the setting of intuitionistic fuzzy metric spaces for commuting mappings. Afterwards, many authors proved common fixed point theorems using different variants in such spaces. We prove common fixed point theorem for two weakly compatible maps satisfying an integral type contractive condition in intuitionistic fuzzy metric space. These results are proved without exploiting the notion of continuity and without imposing any condition of $t$-norm.

## II. PRELIMINARIES

**Definition 2.1[7]:** A binary operation $*$ : $[0,1]\times[0,1]\rightarrow[0,1]$ and $\Diamond$ : $[0,1]\times[0,1]\rightarrow[0,1]$ is continuous $t$-norm if $*$ and $\Diamond$ is satisfies the following conditions:

1. $*$ is commutative and associative;
2. $*$ is continuous;
3. $a * 1 = a$ for all $a\in[0, 1]$;
4. $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in[0, 1]$.
5. $\Diamond$ is commutative and associative;
6. $\Diamond$ is continuous;
7. $a \Diamond 0 = a$ for all $a \in[0, 1]$;
8. $a \Diamond b \ge c \Diamond d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in[0, 1]$.

**Definition 2.2[3]:** A 5-tuple $(X, M, N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if $X$ is an arbitrary set, $*$ is a continuous $t$-norm, $\Diamond$ is a continuous $t$-conorm and $M$, $N$ are fuzzy sets on $X^2\times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \le 1$ for all $x, y$ in $X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y$ in $X$;
(iii) $M(x, y, t) = 1$ for all $x, y$ in $X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y$ in $X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \le M(x, z, t + s)$ for all $x, y, z$ in $X$ and $s, t > 0$;
(vi) for all $x, y$ in $X$, $M(x, y, .) : [0, \infty) \rightarrow [0, 1]$ is left continuous;
(vii) $Lim_{t\rightarrow\infty} M(x, y, t) = 1$ for all $x, y$ in $X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y$ in $X$;
(ix) $N(x, y, t) = 0$ for all $x, y$ in $X$ and $t > 0$ if and only if $x = y$;

***Corresponding author:** *T. Keshava Rao[2*]*
*[2]Govt: City College, Hyderabad, India*

(x) *N(x, y, t) = N(y, x, t)* for all *x, y* in *X* and *t > 0*;
(xi) *N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s)* for all *x, y, z* in *X* and *s, t > 0;*
(xii) for all *x, y* in *X, N(x, y, .) :* [0, ∞)→[0, 1] is right continuous;
(xiii) *Limt→∞N(x, y, t) = 0* for all *x, y* in *X*:

Then (*M, N*) is called an intuitionistic fuzzy metric on *X*. The functions *M(x, y, t)* and *N(x, y, t)* denote the degree of nearness and the degree of non-nearness between *x* and *y* w.r.t. *t* respectively. an intuitionistic fuzzy metric space of the form (*X, M, 1-M, *, ◊*) such that *t*-norm * and *t*-conorm ◊ are associated as *x ◊ y = 1-((1-x) * (1-y))* for all *x, y* in *X*.

**Lemma 2.3: [3]** In intuitionistic fuzzy metric space (*X, M, N, *, ◊*), *M(x, y, *)* is non-decreasing and *N(x, y, ◊)* is non-increasing for all *x, y* in *X*.

**Definition 2.4[3]:** Let (*X, M, N, *, ◊*) be an intuitionistic fuzzy metric space. Then a sequence {*xn*} in *X* is
(i) said to be Cauchy sequence if, for all *t > 0* and *p > 0*, limn→∞$M(x_{n+p}, x_n, t) = 1$ *and* limn→∞$N(x, x_n, t) = 0$.
(ii) said to be convergent to a point *x* in *X* if$_{n+p}$, for all *t > 0, limn→∞*$M(x_n, x, t) = 1$ *and limn→∞*$N(x_n, x, t) = 0$.
(iii) said to be complete if and only if every Cauchy sequence in *X* is convergent.

**Example 2.5**:[3] X={$\frac{1}{n}$,n∈ N}∪ {0} and let * be the continuous t-norm and commuting if ◊ be the continuous t-norm defined by a*bab and a ◊ b min{1,a+b} Respectively , for all a, b∈[0.1] . for each t∈ $(0, \infty) and\ x, y \in X,$ $defined\ (M, N)$ by

M(x, y, t) $\begin{cases} \frac{t}{t+lx-yl}, & t > o \\ 0, & t = o \end{cases}$ and N(x, y, t) $\begin{cases} \frac{t}{t+lx-yl}, & t > o \\ 0, & t = o \end{cases}$

Clearly, (X, M, N,*, ◊) is complete intuitionistic fuzzy metric space.

**Definition 2.6[5]:** A pair of self mappings (*f, g*) of a intuitionistic fuzzy metric space (*X, M, N, *, ◊*) is said to be *M(fgx , gfx , t ) = 1* and *N(fgx , gfx , t ) = 0* for all *x*∈*X.*

**Definition 2.7[5]:** A pair of self mappings (*f, g*) of a intuitionistic fuzzy metric space (*X, M, N, *,◊* ) is said to be weakly commuting if

*M (fgx , gfx , t) ≥ M(fx, gx, t)* and *N (fgx , gfx , t) ≤ N(fx, gx, t)* for all *x X*∈ and *t > 0.*

**Definition 2.8[5]:** A pair of self mappings (*f, g)* of a intuitionistic fuzzy metric space (*X, M, N, *,◊* ) is said to be compatible if

$\lim_{n\to\infty} M(fgx_n, gfx_n, t)$and $\lim_{n\to\infty} N(fgx_n, gfx_n, t)$

**Definition 2.9 [8]:** Two self maps *f* and *g* on a set *X* are said to be weakly compatible if they commute at the coincidence points i.e., if *fu = gu* for some *u* in *X*, then *fgu = gfu.*

It is easy to see that two compatible maps are weakly compatible but converse is not true.

**Definition 2.13:** A pair of self mappings ( *f, g* ) of a intuitionistic fuzzy metric space (*X, M, N, *, ◊*) is said to be weakly compatible of type (*A*) if

$\lim_{n\to\infty} M(fgx_n, ggx_n, t) = 1$and $\lim_{n\to\infty} N(fgx_n, ggx_n, t) = 0$

$\lim_{n\to\infty} M(gfx_n, fgf, t) = 1$and $\lim_{n\to\infty} N(gfx_n, ffx_n, t) = 0$ for t >0 whenever $\{x_n\}$ is sequence in X such that

$\lim_{n\to\infty} fx_n, = \lim_{n\to\infty} gx_n, = u\ for\ some\ u\ in\ X$

## III. MAIN RESULT

**Theorem 3.1:** let f and g be weakly compatible maps of type(A) of a complete intuitionistic fuzzy metric space (X,M,N,*,◊)satisfying the following conditions
1. f(x)⊑ g(x)
2. any one of the mappings f or g is continuous
3. $\int_0^{M(fx,fy,t)} \varphi(t)dt < c\int_0^{M(gx,gy,t)} \varphi(t)dt$

$$\int_0^{N(fx,fy,t)} \varphi(t)dt > c' \int_0^{N(gx,gy,t)} \varphi(t)d$$

For each x, y in X, t >0 c belong to [0.1] where $\varphi: R^+ \to R$ is a lebesgue integrable mapping which is summable non negative and such

4. $\int_0^{\in} \varphi(t)dt$ >0 for each $\in > 0$

Then f and g have a unique common fixed point in X.

**Proof:** Let $x_0$ be an arbitrary point in X, f(x) ⊑g(x)Choose a point $x_1$ in X such that f$x_0 = gx_1$. In general, we cahn choose $x_{n+1}$ such that f$x_n = gx_{n+1}$ for all n≥ $1. Then\ for\ all\ t > 0$

$$\int_0^{M(fx_n,fx_{n+1},t)} \varphi(t)dt \geq c \int_0^{M(fx_n,fx_{n+1},t)} \varphi(t)dt$$

$$= c\int_0^{M(fx_{n,-1},fx_n,t)} \varphi(t)dt \qquad (1)$$

$$> c\int_0^{M(fx_{n,-1},fx_n,t)} \varphi(t)dt$$

$$\int_0^{N(fx_n,fx_{n+1},t} \varphi(t)dt \leq c' \int_0^{N(fx_n,fx_{n+1},t)} \varphi(t)dt$$

$$= c' \int_0^{N(fx_{n,-1},fx_n,t)} \varphi(t)dt \qquad (2)$$

$$< c' \int_0^{N(fx_{n,-1},fx_n,t)} \varphi(t)dt$$

Since c(t)>t and ,$c'$ (t)<t for all 0< t < 1. $\int_0^{M(fx_n,fx_{n+1},t)} \varphi(t)dt$ is an increasing sequence of positive real nymber in [0,1] and $\int_0^{N(fx_n,fx_{n+1},t)} \varphi(t)dt$ is decreasing sequence of positive real number in [0,1] . therefore they converges to limits L≤ $1, and\ L' \geq 0\ respectively.$

Now, we claim that L=1 and $L' = 0$. For, let L <1. letting n→ ∞ in (1), we have L≥ $c(L) > L$, which is a contradiction and so L=1 . Similarly let $L' \geq 0$

letting n→ ∞ in (2), we have $L^1 \leq c(L^1) \leq L^1$ which is a contradiction and so $L'$ =0

Now, for any positive integer p and t >0 we have

$$\int_0^{M(fx_n,fx_{n+p},t)} \varphi(t)dt \geq \int_0^{M\left(fx_n,fx_{n+1},\frac{t}{p}\right)*\ldots\ldots*M\left(fx_{n+p-1},fx_{n+p},\frac{t}{p}\right)} \varphi(t)dt$$

$$\geq \int_0^{M\left(fx_n,fx_{n+1},\frac{t}{p}\right)*\ldots\ldots*M\left(fx_n,fx_{n+1},\frac{t}{p}\right)} \varphi(t)dt$$

$$\int_0^{N(fx_n,fx_{n+p},t)} \varphi(t)dt \leq \int_0^{N\left(fx_n,fx_{n+1},\frac{t}{p}\right)\Diamond\ldots\ldots\Diamond.N\left(fx_{n+p-1},fx_{n+p},\frac{t}{p}\right)} \varphi(t)dt$$

$$\leq \int_0^{N\left(fx_n,fx_{n+1},\frac{t}{p}\right)\Diamond\ldots\ldots\Diamond N\left(fx_n,fx_{n+1},\frac{t}{p}\right)} \varphi(t)dt$$

Since we have

$$Lim_{n\to\infty} \int_0^{M(fx_n,fx_{n+p},\frac{t}{p})} \varphi(t)dt = 1$$

$$Lim_{n\to\infty} \int_0^{N(fx_n,fx_{n+p},\frac{t}{p})} \varphi(t)dt = 0$$

It follows that

$$Lim_{n\to\infty}\int_0^{M(fx_n,fx_{n+p},t)}\varphi(t)dt \geq 1 * \dots\dots\dots 1 \geq 1$$

$$Lim_{n\to\infty}\int_0^{N(fx_n,fx_{n+p},t)}\varphi(t)dt \leq 0 \lozenge \dots\dots\dots \lozenge 0 \leq 0$$

Thus by definition 2.5{f$x_n$} is a Cauchy sequence and by the completeness of X. {f$x_n$} converges to a point z belong to X. Also {g$x_n$} converges to the point z.

Suppose that, by the mapping f is continuous. Then $Lim_{n\to\infty}ffx_n$ = fz, Further, since f and g are weakly commuting,

We have

$$\int_0^{M(fg,gfx_n,t)}\varphi(t)dt \geq \int_0^{M\left(fx_n,gx_n,\frac{t}{R}\right)}\varphi(t)dt$$

$$\int_0^{N(fg,gfx_n,t)}\varphi(t)dt \leq \int_0^{N\left(fx_n,gx_n,\frac{t}{R}\right)}\varphi(t)dt$$

Letting n→ ∞ in the inequality we have $Lim_{n\to\infty}$g$fx_n$ =fz.

Now, we prove that z=fz. Suppose z≠ $fz$. Then there exist t>0 such that

$$\int_0^{M(z,fz,t)}\varphi(t)dt \leq 1\ and, \int_0^{n(z,fz,t)}\varphi(t)dt \geq 0$$

We have

$$\int_0^{M(fx_n,ffx_n,t)}\varphi(t)dt \geq C\int_0^{M(gx_n,gfx_n,t)}\varphi(t)dt$$

$$\int_0^{N(fx_n,ffx_n,t)}\varphi(t)dt \geq C'\int_0^{N(gx_n,gfx_n,t)}\varphi(t)dt$$

Letting n→ ∞ in the above inequalities, we have

$$\int_0^{M(z,fz,t)}\varphi(t)dt \geq c\int_0^{M(z,fz,t)}\varphi(t)dt \geq \int_0^{M(z,fz,t)}\varphi(t)dt$$

$$\int_0^{N(z,fz,t)}\varphi(t)dt \leq c'\int_0^{N(z,fz,t)}\varphi(t)dt \leq \int_0^{N(z,fz,t)}\varphi(t)dt$$

Which are a contradiction, therefore z=fz, since we can find a point $z_1$ ∈ X such z=fz=g$z_1$. now, it follow that

$$\int_0^{M(ffx_n,fz_1,t)}\varphi(t)dt \geq C\int_0^{M(gfx_n,gz_1,t)}\varphi(t)dt$$

$$\int_0^{N(ffx_n,fz_1,t)}\varphi(t)dt \geq c'\int_0^{N(gfx_n,gz_1,t)}\varphi(t)dt$$

Letting n→ ∞ in the above inequalities, we have

$$\int_0^{M(fz,fz_1,t)}\varphi(t)dt \geq c\int_0^{M(fz,gz_1,t)}\varphi(t)dt = 1$$

$$\int_0^{N(fz,fz_1,t)}\varphi(t)dt \leq c'\int_0^{N(fz,gz_1,t)}\varphi(t)dt = 0$$

 

Which implies that $fz_1$=$gz_1$. also, for any t >0

$$\int_0^{M(fz,gz,t)} \varphi(t)dt = \int_0^{M(fgz_1,gfz_1,t)} \varphi(t)dt \geq \int_0^{M\left(fz_1,gz_1,\frac{t}{R}\right)} \varphi(t)dt = 1$$

$$\int_0^{N(fz,gz,t)} \varphi(t)dt = \int_0^{N(fgz_1,gfz_1,t)} \varphi(t)dt \leq \int_0^{N\left(fz_1,gz_1,\frac{t}{R}\right)} \varphi(t)dt = 0$$

Which again imply that fz=gz, therefore z is a common fixed point of f and g. Which tp prove the uniqueness of the common fixed point z, let y (y$\neq z$) be another common fixed point of f and g. Then there exists t>o such that

$$\int_0^{M(z,y,,t)} \varphi(t)dt \leq 1. and \int_0^{N(z,y,,t)} \varphi(t)dt \geq 0.$$

$$\int_0^{M(z,y,,t)} \varphi(t)dt = \int_0^{M(fz,fy,,t)} \varphi(t)dt \ \geq c\int_0^{M(gz,gy,,t)} \varphi(t)dt \ = c\int_0^{M(z,y,,t)} \varphi(t)dt \geq \int_0^{M(z,y,,t)} \varphi(t)dt$$

$$\int_0^{N(z,y,,t)} \varphi(t)dt = \int_0^{N(fz,fy,,t)} \varphi(t)dt \ \geq c'\int_0^{N(gz,gy,,t)} \varphi(t)dt \ = c'\int_0^{N(z,y,,t)} \varphi(t)dt \leq \int_0^{N(z,y,,t)} \varphi(t)dt$$

Which is a contradiction since c(t) > t and $c'(t)$<t for any 0<t<1. There fore z=y, i.e z is a unique common fixed point of f and g. this completes the proof.

**Example3.1:**[3] X={$\frac{1}{n}$,n∈ N}∪ {0} and let * be the continuous t-norm and commuting if ◊ be the continuous t-norm defined by a*bab and a ◊ b min{1,a+b} respectively , for all a, b∈[0.1] . for each t∈ $(0,\infty) and\ x,y \in X,$ $defined\ (M,N)$ by

M(x, y, t) $\begin{cases} \frac{t}{t+lx-yl}, & t > o \\ 0, & t = o \end{cases}$ and N(x, y, t) $\begin{cases} \frac{t}{t+lx-yl}, & t > o \\ 0, & t = o \end{cases}$

Clearly, (X, M, N,*, ◊) is complete intuitionistic fuzzy metric space.where * is defined by a*b=ab and a◊$b$ =\min {1, a+b}. Further, (X, M, N,*, ◊) is complete.

Define

$$F(x)=1, g(x)=\begin{cases} 1, & if \quad x \ is \ a \ rational \ number, \\ 0, & if \quad x \ is \ a \ irrational \ number \end{cases}$$

It is evident that f(x)⊑ g(x), f is continuous and g is discontinuous. Define a function C;[0,1]→[0,1] bt c(t) =$\sqrt{t}$ for any 0 <t<1 and C(t) =1 for t=1.

$c'$;[0,1]→[0,1] by c(t) =$t^2$for any 0 <t<1 and C'(t) =0 for t=0. Then$and\ c(t) > t, c'$ (t) < t for any 0 < t < 1 and M(fx, fy, t)≥ $cN(gx,gy,t)$ and N(fx, fy, t)≥ $cN(gx,gy,t)$

For all x, y in X. further, f and g are R-weakly commuting. Thus all the conditions of theorem are satisfied and is a common fixed of f and g.

## REFERENCES

[1].K. Atanassov, Intuitionistic Fuzzy sets, *Fuzzy sets and system*, 20, 1986, pp. 87-96.
[2].J. H. Park, Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, 22, 2004 pp. 1039- 1046.
[3].C. Alaca, D. Turkoglu, and C. Yildiz, Fixed points in Intuitionistic fuzzy metric spaces, *Chaos, Solitons & Fractals*, 29, 2006, pp. 1073-1078.
[4].I. Kramosil and J. Michalek, Fuzzy metric and Statistical metric spaces, *Kybernetica*, 11, 1975, pp. 326-334.
[5].D. Turkoglu, C. Alaca and C. Yildiz, Compatible maps and Compatible maps of type ()α and ()β in intuitionistic fuzzy metric spaces, *Demonstratio Math*., 39, No.3, 2006, pp. 671-684.
[6].G. Jungck, Commuting mappings and fixed points, *Amer. Math. Monthly*, 83, 1976, pp. 261-263.
[7].B. Schweizer and A. Sklar, Probabilistic Metric Spaces, *Morth Holland Amsterdam*, 1983.
[8].G. Jungck, BE Rhoades, Fixed point for set valued functions without continuity, *Indian J Pure Appl Math*, 29.

**Source of support: Nil, Conflict of interest: None Declared**