International Research Journal of Pure Algebra -3(6), 2013, 242-246
Available online through www.rjpa.info ISSN 2248–9037

UNION FUZZYSOFT N-GROUP

A. Solairaju[1], P. Sarangapani[2] and R. Nagarajan[3*]

[1]AssociateProfessor, PG & Research Department of Mathematics, Jamal Mohamed college, Triuchirappalli-20, Tamilnadu, India

[2]Assistant Professor, Department of Computer Science, Kurinji College of Arts & Science, Tiruchirappalli-02, Tamilnadu, India

[3]Associate Professor, Department of Mathematics, J J College of Engineering & Technology, Tiruchirappalli-09, Tamilnadu, India

(Received on: 08-06-13; Revised & Accepted on: 26-06-13)

ABSTRACT

In this paper, we introduce Union Fuzzy soft N-Group by using Molodtsov's definition of soft sets and investigate their related properties with respect to α-inclusion of soft sets.

Keywords: *Soft set – Fuzzy Soft set – Soft N-group – Union Fuzzy Soft N-Group - α inclusion.*

SECTION - 1:

INTRODUCTION

In 1999, Molodtsov's [26] proposed an approach for Modeling, Vagueness and Uncertainity, called soft set theory, since its inception, works on soft set theory have been progressing rapidly with a wide range applications especially in the mean of Algebraic structures as in [2-12]. The structures of soft sets operations of soft sets and some related concepts have been studied by [14-19]. The theory of soft set continues to experience tremendous growth and diversification in the mean of soft decision making as in the following studies [20-23] as well. Atagun and Sezgin [33] defined soft N-subgroups and soft N-ideals of an N-group, they studied their properties with respect to soft set operators in more detail. In this paper we introduce Union Fuzzy Soft N-Group by using Molodtsov's definition of soft sets and investigate their related properties with respect to α -inclusion of soft sets.

SECTION - 2: PRELIMINARIES

Definition 2.1: Let $(\Gamma, +)$ be a group and μ: N X $\Gamma \to \Gamma(n, \nu) \to n\nu$, (Γ, μ) is called an N-group if x, y$\in$ N and $\forall \nu \in \Gamma$,

(i) $x(y\, \nu) = (xy)\, \nu$ and

(ii) $(x+y)\, \nu = x\, \nu + y\, \nu$. It is denoted by N^{Γ}.

Clearly N itself is an N-group by natural operation. A subgroup H of $\Gamma$with NH$\subseteq$ H is said to be an N-subgroup of $\subseteq\Gamma$. Γandψ be two N- groups then f:$\Gamma\to\psi$ is called an N-homomorphism if $\forall \nu$, H $\in\Gamma$, $\forall$ n $\in$ N

(i) $f(\nu + H) = f(\nu) + f(H)$ and

(ii) $f(n\nu) = nf(\nu)$

For all undefined concepts and notations, we refer to [29]. From now on U refers to initial universe, E is a set of parameters $2^U$is the power set of U and A, B, C $\subseteq$ E

Definition 2.2: Let U be any Universal set, E set of parameters and A$\subseteq$ E, then a pair (F,A) is called soft set over U, where F is a mapping from A to 2^U , the power set of U.

Example 2.1: Let X={c_1,c_2,c_3} be the set of three cars and E={costly(e_1), metallic colour (e_2), cheap(e_3)} be the set of parameters, where A={e_1,e_2} $\subset$ E. then (F,A)={F(e_1)={c_1,c_2,c_3},F(e_2)={c_1,c_2} } is the crisp soft set over X.

Corresponding author: R. Nagarajan[3]

Definition 2.3: Let U be the universal set, E set of parameters and $A \subset E$. Let F(X) denote the set of all fuzzy subsets of U, then a pair (F,A) is called fuzzy soft set over U, where F is a mapping from A to F(U).

Example 2.2: Let $U=\{c_1,c_2,c_3\}$ be the set of three cars and $E=\{costly(e_1),metalliccolor(e_2), cheap(e_3)\}$ be the set of parameters, where $A=\{e_1,e_2\}\subset E$. then $(F,A)=\{F(e_1)=\{c_1/0.6,c_2/0.4,c_3/0.3\}, F(e_2)=\{c_1/0.5,c_2/0.7,c_3/0.8\}$ is the fuzzy soft set over U denoted by F_A.

Definition 2.4: Let F_A be a fuzzy soft set over U and α be a subset of U then upperα - inclusion of $F_A$denoted by $F^{\alpha}{}_A = \{x\in A /F(x) \geq \alpha \}$. Similarly $F^{\alpha}{}_A = \{x \in A / F(x) \leq \alpha\}$ is called lower α-inclusion of F_A.

Definition 2.5: Let F_A and G_B be fuzzy soft sets over the common universe U and ψ: $A \rightarrow B$ be a function then fuzzy soft image of F_A under ψ over U denoted by $\psi(F_A)$ is a set-valued function, where $\psi(F_A)$: $B\rightarrow 2^U$ defined by

$$\psi(F_A)\ (b)=\{\cup\{F(a) / a\in A \text{ and } \psi\ (a)=b\}$$

If $\psi^{-1}(b)\neq \varphi\}$for all $b\in B$, the soft pre-image of G_B under ψ over U denoted by $\psi^{-1(}G_B)$ is a set-valued function, where $\psi^{-1(}G_B)$: $A \rightarrow\ 2^U$ defined by $\psi^{-1}(G_B)(b) = G(\psi(a))$ for all $a \in A$ then fuzzy soft anti-image of F_A under ψ over U denoted by $\psi(F_A)$ is a set-valued function, where $\psi(F_A)$:$B \rightarrow 2^U$ defined by $\psi^{-1}(F_A)(b)=\{\cap\{F(a) / a\in A \text{ and } \psi\ (a)=b\}$, if $\psi^{-1}(b) \neq \varphi$ for all $b \in B$

Definition 2.6: Let H be an N-subgroup of Γ and F_H be a fuzzy soft over Γ. If for all x, $y\in H$ and $n\in N$,

(i) $F(x-y) \leq F(x) \cup F(y)$ and

(ii) $F(nx) \leq F(x)$ then the fuzzy soft set F_H is called a union fuzzy soft N-subgroup of Γ and denoted by $F_H <_N \Gamma$

Example 2.3: Consider N= {0, 1, 2, 3} be a group with operation +

+	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3	3	0	1	2

If we define a fuzzy soft set G_H over Γ by

$G(x) =\{y \in\Gamma/ 3x=y\}$ for all $x \in H$.

Then G(0)={0} and G(2) ={2} since $G(2-2) = G(0) \neq G(2)$,G_H is not a union soft N-subgroup of Γ

Definition 2.7: The relative complement of the fuzzy soft set F_A over U is denoted by $F^r{}_A$ where $F^r{}_A$: $A\rightarrow 2^U$ is a mapping given as $F^r{}_A(x) = U/F_A(x)$, for all $x \in A$.

SECTION - 3: CHARACTERIZATION'S OF UNION FUZZY SOFT N-GROUP

Proposition 3.1: Let F_A be a fuzzy soft set over Γ and α be a subset of Γ. If F_A is a union fuzzy soft N-subset of Γ, then upper α- inclusion of F_A is an N-subgroup of Γ.

Proof: Since F_A is union fuzzy soft N-subgroup of Γ. Assume x, $y \in F_A{}^{\alpha}$ and $n \in N$, then $F(x)\geq\alpha$ and $F(y) \geq\alpha$, we need to show that $x\text{-}y \in F_A{}^{\alpha}$and $n \in F_A{}^{\alpha}$ since F_A is union fuzzy soft N-subgroup of Γ, it follows that $F(x-y) \leq \max\{F(x), F(y)\}\geq\min\{\alpha,\alpha\}\geq\alpha$ and $F(nx) \leq\alpha \geq\alpha$ which completes the proof.

Proposition 3.2: Let F_A be a fuzzy soft set over Γthen F_A is a union fuzzy soft N-subgroup of Γ if $F_A{}^r$ is fuzzy soft N-subgroup of Γ.

Proof: Let F_A be a union fuzzy soft N-subgroup of Γ.Then for all x, $y \in A$ and $n \in N$.

$$\begin{aligned} F_A{}^r(x-y) &= \Gamma / F_A(x-y) \\ &\geq \Gamma/ \max\{F_A(x),F_A(y)\} \\ &= \min\{(\Gamma / F_A (x)),\ \Gamma/ F_A(y)\} \\ &= \min\{F_A{}^r (x), F_A{}^r(y)\} \end{aligned}$$

$$\begin{aligned} F_A{}^r(n\,x) &=\Gamma / F_A(n\,x) \\ &\geq \Gamma / F_A(x) \end{aligned}$$

$F_A{}^r(n\,x) = F_A{}^r(x)$, $F_A{}^r$ is fuzzy soft N-subgroup ofΓ.

Proposition3.3: Let F_A: $X \to X^1$ be a soft homomorphism of N-subgroups. If F_A is union fuzzy soft N-subgroups of X^1, then F_A is union fuzzy soft N-subgroups of X^1.

Proof: Suppose F_A is union fuzzy soft N-subgroups of X^1, then
(i) Let $x^1, y^1 \in X^1$, then exists $x, y \in X$ such that

$f(x)= x^1$ and $f(y)=y^1$, we have

$$F_A(x^1-y^1) = F_A(f(x)-(f(y))$$
$$\le \max\{F_A(f(x)),F_A(f(y)\}$$

$$F_A(x^1-y^1) =\max\{F_A{}^1(x),F_A{}^1(y)\}$$

$$\text{(ii) } F_A(n\,x^1) = F_A(nf(x))$$
$$\le F_A{}^f(x)$$

$$F_A(n\,x^1) = F_A{}^f(x)$$

.: F_A is union fuzzy soft N-subgroups X^1

Proposition3.4: Let F_A be union soft N-sub groups of X and $F_A{}^\alpha$ be a fuzzy soft set in X given by $F_A{}^\alpha(x) = F_A(x) +1-F(1)$ for all $x \in X$ then $F_A{}^\alpha$is union fuzzy soft N-subgroups of X and $F_A \subseteq F_A{}^\alpha$.

Proof: Since F_A is union fuzzy soft N-subgroups of X and $F_A{}^\alpha(x) = F_A(x) +1- F_A(1)$ for $x \in X$. For any $x, y \in X$, we have $F_A(1)=F_A(1)+1-F_A(1)=1>F_A{}^\alpha(x)$ and for all $x, y \in X$, we have

$$F_A{}^\alpha(x-y) = F_A(x-y)+1-F_A(1)$$
$$\le \max\{F_A(x),F_A(y)\}+1-F_A(1)$$
$$= \max\{F_A(x)+1-F_A(1),F_A(y)+1-F_A(1)\}$$
$$= \max\{F_A{}^\alpha(x),F_A{}^\alpha(y)\}$$

$$F_A{}^\alpha(nx) = F_A(nx) +1-F_A(1)$$
$$= F_A(x) +1-F_A(1)$$
$$= F_A{}^\alpha(x),$$ $F_A{}^\alpha$ is union fuzzy soft N-subgroup of X.

Proposition3.5: Let F_A and G_B to fuzzy soft gets over Γ, where A and B are N- groups of Γ and $ø: A \to B$ is an N-homomorphism. If F_A is union fuzzy soft N- subgroups of Γ, then so is $ø\,(F_A)$.

Proof: Let $\alpha_1, \alpha_2 \in B$ such ø is surjective, there exists $a_1, a_2 \in A$ such that $Ø\,(a_1) = \alpha_1$ and $Ø\,(a_2) = \alpha_2$ thus

$$(Ø\,F_A)(\alpha_1 -\alpha_2) = \max\{F(a)/A \in A, Ø\,(A)=\alpha_1- \alpha_2\}$$
$$= \max\{F(a)/A\in A,A=Ø^{-1}(\alpha_1 - \alpha_2)\}$$
$$= \max\{F(a)/A\in A,A=Ø^{-1}(ø(a_1 - a_2))= A_1 - A_2\}$$
$$= \max\{F(a_1 - a_2)/\alpha_1,\alpha_2\in B,Ø\,(A_i)= \alpha_i ,i =1,2\}$$
$$= \min\{(\max\{F(a_1)/\alpha_1\in B,Ø\,(a_1)=\alpha_1)\},\{\max(F(a_2)/\alpha_2\in B,Ø\,(a_2)=\alpha_2)\}$$
$$= \min\{Ø\,(F_A)(a_1),Ø\,(F_A)(a_2)\}$$

Now let $n \in N$ and $\alpha \in B$. Since Øsurjective, then exists $\bar{A}\in A$ such that $Ø\,(\bar{A}) = 0$

$$(Ø\,(F_A)\,(n\,\alpha) = \max\{F(A)/A\in A, Ø\,(A)=n\alpha\}$$
$$= \max\{F(A)/A\in A,A=Ø^{1}(n\alpha)\}$$
$$= \max\{F(A)/A\in A,A=Ø^{-1}(nø(\bar{A}))\}$$
$$= \max\{F(A)/A\in A,A=Ø^{-1}(ø(nA))=n\bar{A}\}$$
$$= \max\{F(n\bar{A})/\bar{A}\in A ,Ø\,(\bar{A})=\alpha\}$$
$$= \max\{F(\bar{A})/\bar{A}\in A ,Ø\,(\bar{A})=\alpha\}$$
$$= \max(Ø\,(F_A))(\alpha)$$

$Ø\,(F_A)$ is union fuzzy soft N-subgroup of Γ.

Proposition3.6: Let F_A:$X\to Y$ be a soft homomorphism of N-subgroups. If F_A is union fuzzy soft N-subgroups of Y, then $F_A{}^f$ is union fuzzy soft N-subgroups of X.

Proof: Suppose F_A is union fuzzy soft N-subgroups of Y, then
i) For all x, y € X, we have

$$F_A(x- y) = F_A(f(x- y))$$
$$= F_A(f(x)-f(y))$$
$$= \max\{F_A(f(x),F_A(f(y))\}$$
$$= \max\{F_A^f(x),F_A^f(y)\}$$

ii) $F_A^f(n x) = F_A(f(n x))$
$$\leq F_A(f(x))$$
$$= F_A^f(x)$$

F_A^f is union fuzzy soft N- subgroups of X.

Proposition3.7: Let F_A and G_B be fuzzy soft sets over Γ, where A and B are N-subgroups of Γand Ø be on N-homomorphism from A to B if G_B is a union fuzzy soft N-subgroup of Γ, then so is $Ø^{-1}(G_B)$.

Proof: Let $a_1, a_2 \in A$, then

$$(Ø^{-1}(G_B))(a_1-a_2) = G(Ø(a_1-a_2))$$
$$\geq \max\{G(\{Ø(a_1),Ø(a_2)\}$$
$$= \max\{(Ø^{-1}(G_B)(a_1),Ø^{-1}(G_B(a2))\}$$

Now let $n \in N$ and $A \in A$, then

$$(Ø^{-1}(G_B))(nA) = G(Ø(nA))$$
$$= G(n\,Ø(A))$$
$$= G(Ø(A))$$
$$= (Ø^{-1}(G_B))(A)$$

$Ø^{-1}(G_B)$ is a union fuzzy soft N-subgroups of Γ.

CONCLUSION

This paper summarized the basic concepts of soft sets.By using these concepts we studied the algebraic properties of union fuzzy soft N-groups.This work focused on fuzzy soft pre-image, fuzzy soft image. fuzzy soft anti image.To extend this work one could study theproperties of fuzzy soft N-groups in other algebraic structures such as rings and fields.

REFERENCES

1. Acar U and Koyuncu F, Tanay B (2010) Soft sets and soft rings Comput math Appl 59:3458-3463.
2. Ali MI, Feng F, Liu X, Min WK, Shabir M(2009) On some new operations in soft set theory. Comput math Appl 57:1547-1553.
3. Aktas H, Cağman N (2007) Soft sets and Soft groups, Informn Sci 177:2726-2735
4. Atagŭn AO, Sezgin A(2011) Soft structures of rings and fields and modules Comput math Appl 61(3) 592-601
5. Aygŭnoglu A, Aygŭn H (2009) Introduction to fuzzy soft groups Comput math Appl 58:1279-1286.
6. Cogman N Enginoğlu S (2010) Soft set theory and uni-int decision making Eur J Oper Res 207:848-855.
7. Coğman N Enginoğlu S (2010) Soft matrix theory and its decision making Comput math Appl 59:3308-
8. CağmanNCitak F Enginoğlu S (2010) Fuzzy Parameterized fuzzy soft set theory and its applications. Turkish J Fuzzy Syst. 1:21-35.
9. CağmanNCitak F Enginoğlu S (2011) Fuzzy soft set theory and its applications, Iran.J fuzzy syst 8(3):137-147.
10. Feng F, Liu X Y, Leoreanu-Fotea V, Jun Y B (2011) Soft sets and soft rough sets. Inform Sci 181(6): 1125-1137.
11. Feng F, Liu YM Leoreanu- FoteaV (2010) Application of Level soft sets in decision making based on interval-valued Fuzzy soft sets. Comput Math Appl60:1756-1767.
12. Feng F, Li C Davvaz B, Ali MI (2010) soft sets combined with fuzzy sets and rough sets: a tentative approach. Soft Compute 14(9): 899-911.
13. Feng F Jun YB, L iu X, Li L(2010) An adjustable approach to fuzzy soft set based decision making. J Compute Appli Math 234:10-20.
14. Feng F Jun YB Zhao X (2008) Soft semi rings. Compute Math Appli 56:2621-2628.
15. Jun YB (2008) Soft BCK/BCI – Algebras. Comput Math Appli 56:1408-1413.
16. Y.B. Jun, C.H. Park, Applications of soft sets in ideal theory of BCK/BCI-algebras, Inform, Sci. 178 (2008) 2466-2475.
17. Jun YB, Kim H S, Neggers J (2009) Pseudo D – Algebras. Inform Sci 179:1751-1759.
18. Y.B. Jun, Lee K J, Zhan J (2009) Soft P-ideals of soft BCI-algebras, Compute. Math, Appl. 58:2060-2068.

19. Y.B. Jun, Lee K J, Khan A (2010) Soft ordered semi groups. Math Logic Q 56(1): 42-50.
20. Y.B. Jun, Lee K J, Park C H (2010) Fuzzy Soft set theory applied to BCK/BCI algebras. Compute Math appli 59: 3180-3192.
21. Kong Z , Gao L ,Wang L, Li S (2008) The Normal parameter reduction of soft sets and its algorithms. Compute Math. Appli 56:3029-3037.
22. Kovkov D V, Kolbanov V M, D. Molodtsov (2007) Soft sets theory based optimization. J Compute SystSciint 46(6):872-880.
23. P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Compute, Math. Appl. 44 (2002) 1077-1083.
24. P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Compute, Math, Appl. 45 (2003) 555-562.
25. Majumdğar B, Samanta SK (2010) Generalised fuzzy soft sets. Compute math Appli 59:1425-1432.
26. D. Molodtsov, Soft set theory - First results, Compute. Math, Appl. 37 (1999) 19-31.
27. D. Molodtsov, Lionov V Y, Kovkov D V (2006) Soft sets technique and its appli. NechetkieSystemi I MyakieVychisleniya 1(1): 8-39.
28. Mushrif M M, Sengupta S, A. K Roy (2006) Texture classification using a novel, soft set theory based classification algorithm. Lect Notes Compute Science 3851:246-254.
29. C.H. Park, Y.B. Jun, Ozturk M A (2008) Soft WS-Algebras Commun Korean Math Soc 23(3):313-324.
30. P.K. Maji, A.R. Roy, (2007) A fuzzy soft set theoretic approach to decision making problem, J Compute Math. Appli 203: 412-418.
31. R. Nagarajan and N. Rajagopal,(2013) S-Fuzzy version of soft N-group, International Journal of Emerging Technology and Advanced Engineering, Vol.3, No.5, PP 322-327.
32. Sezgin A, AtagunAO (2011) An operations of soft sets. Compute Math Appli 61(5):1457-1467.
33. Sezgin A, Atagun AO Aygun E (2011) A Note on soft near rings and idealistic soft near rings. Filomat 25(1):53-68.
34. Xiao Z, Gong K, Zou Y (2009) A combined forecasting approach based on fuzzy soft sets. J Compute Math Appli 228:326-333.
35. Zhan J, Jun Y B (2010), Soft BL Algebras based on fuzzy sets. Compute math. Appli.59:2037-2046.
36. Zou Y, Xiao Z (2008) Data analysis approaches of soft sets under incomplete information. Knowl-based Syst 21:941-945.

Source of support: Nil, Conflict of interest: None Declared