International Research Journal of Pure Algebra -3(10), 2013, 307-310
Available online through www.rjpa.info ISSN 2248–9037

EQUITABLE TOTAL MINIMAL DOMINATING GRAPH

B. Basavanagoud*, V. R. Kulli† and Vijay V. Teli**

**Department of Mathematics, Karnatak University, Dharwad - 580 003, India.*

†Department of Mathematics, Gulbarga University, Gulbarga - 585 106, India.

(Received on: 05-10-13; Revised & Accepted on: 21-10-13)

ABSTRACT

The equitable total minimal dominating graph $E_q^t MD(G)$ of a graph G is defined to be the intersection graph on the minimal equitable total dominating sets of vertices in G. In this paper, we obtain some properties of equitable total minimal dominating graph. Finally, we establish the characterizations of graphs whose equitable total minimal dominating graphs are complete and eulerian.

Mathematics Subject Classification: *05C69.*

Keywords: *total domination number, equitable total dominating set, equitable total domination number, equitable total minimal dominating graph.*

1. INTRODUCTION

All graphs $G = (V, E)$ we considered here are finite, undirected without loops and multiple edges. Terms not defined here are used in the sense of Harary [3] and Haynes *et. al* [4].

A subset D of V is called a *dominating set* of G if every vertex in $V - D$ is adjacent to some vertex in D. A dominating set D of G is minimal if for every vertex $v \in D$, $D - \{v\}$ is not a dominating set of G. The *domination number* $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set.

Cockayne *et. al* [2] introduced the concept of total domination in graphs. A dominating set D of G is called a *total dominating set* if $\langle D \rangle$ has no isolated vertices. The minimum cardinality of a total dominating set of G is called the *total domination number* of G and is denoted by $\gamma_t(G)$.

A subset D of V is called an *equitable dominating set* if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|deg(u) - deg(v)| \leq 1$. The minimum cardinality of such a dominating set is called the *equitable domination* number and is denoted by $\gamma^e(G)$. This concept was introduced by Swaminathan *et. al* [7].

A subset D of V is called an *equitable total dominating set* of G, if D is an equitable dominating set and $\langle D \rangle$ has no isolated vertices. The minimum cardinality taken over all equitable total dominating sets is the *equitable total domination number* [1] and is denoted by $\gamma_t^e(G)$.

In this paper, we use this idea to introduce a new graph valued function in the field of domination theory in graphs.

Let $F = \{S_1, S_2, \cdots, S_n\}$ be a partition of S. Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \phi$.

Definition 1: The *equitable total minimal dominating graph* $E_q^t MD(G)$ of a graph G is defined to be the intersection graph on the family of all minimal equitable total dominating sets of vertices in G.

In Figure 1, a graph G and its equitable total minimal dominating graph $E_q^t MD(G)$ are shown:

Corresponding author: B. Basavanagoud
**Department of Mathematics, Karnatak University, Dharwad - 580 003, India.*

Here the minimal equitable total dominating sets of a graph G are: $D_1 = \{1,2\}, D_2 = \{1,3\}$, $D_3 = \{2,3\}$, $D_4 = \{2,4\}$ and $D_5 = \{3,4\}$.

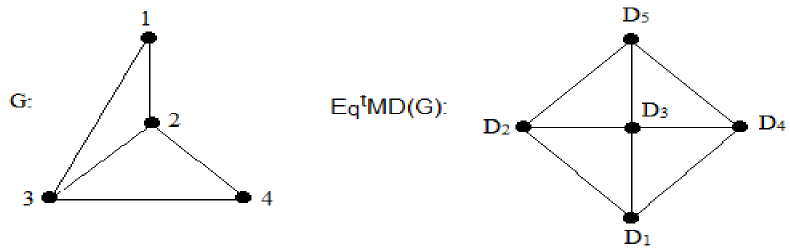

Figure - 1

The following results will be useful in the proof of our results.

Theorem A [3]: *A connected graph G is eulerian if and only if every vertex of G has even degree.*

Theorem B [1]: *For any complete graph K_p with $p \geq 2$ vertices, $\gamma_t^e(K_p) = 2$.*

Theorem C [3]: *If G is a (p, q) graph, whose vertices have degree d_i, then $L(G)$ has q vertices and* $q_L = -q + \frac{1}{2}\sum (d_i)^2$.

2. RESULTS

Remark 2.1: *If $G = K_p$; $p \geq 2$ or $K_{m,n}$, then $E_q^t MD(G) \cong L(G)$.*

Remark 2.2: *If $G = C_{4n}$; $n \geq 1$, then $E_q^t MD(G) = C_4$.*

Theorem 2.1: *For any graph G with no isolated vertices, $E_q^t MD(G)$ is connected.*

Proof: *W*e consider the following cases.

Case-1: Suppose G is connected. Let D_1 and D_2 be any two disjoint minimal equitable total dominating sets in G. If every vertex in D_1 is adjacent to every vertex in D_2, then clearly, $E_q^t MD(G)$ is connected.

Suppose there exist two vertices $u \in D_1$ and $v \in D_2$ such that u and v are not adjacent. Then there exists another minimal equitable total dominating set D_3 containing u and v. It follows that, there is a path in $E_q^t MD(G)$ from D_1 to D_3 and D_3 to D_2. Hence $E_q^t MD(G)$ is connected.

Case-2: Suppose D is disconnected and has no isolated vertices. Then G has at least two components say G_1 and G_2. Each minimal equitable total dominating set of G contains at least two adjacent vertices from each component of G. Thus, every vertex in $E_q^t MD(G)$ has at least two common vertices. Hence $E_q^t MD(G)$ is connected.

Theorem 2.2: *For any (p, q) graph G,*
(i) $|V(E_q^t MD(G))| = |E(G)|$
(ii) $|E(E_q^t MD)| = -q + \frac{1}{2}\sum (d_i)^2$

if and only $G = K_p$; $p \geq 2$ or $K_{m,n}$, where d_i is the degree of each vertex $v_{i \text{ in } G}$.

Proof: By Theorem B, $\gamma_t^e(G) = 2$. Since K_p is complete, every pair of vertices forms a minimal equitable total dominating set. Thus $E_q^t MD(G)$ has $\frac{p(p-1)}{2}$ minimal equitable total dominating sets. Hence $|V(E_q^t MD(G)| = |E(G)|$ and by Theorem C, the number of edges in $L(G)$ are $-q + \frac{1}{2}\sum (d_i)^2$, where d_i is the degree of each vertex $v_{i \text{ in } G}$. Hence $|E(E_q^t MD)| = -q + \frac{1}{2}\sum (d_i)^2$.

Theorem 2.3: *If G is a cycle of order $p \geq 5$ and $p \neq 4n$; $n \geq 1$, then $E_q^t MD(G)$ is complete.*

Proof: Suppose G is a cycle of order $p = 4n$; $n \geq 1$. Then there exist at least two disjoint minimal equitable total dominating sets in G. Thus every vertices in $E_q^t MD(G)$ are not adjacent. Hence $E_q^t MD(G)$ is not complete.

Let G be a cycle C_p with $p \geq 5$ vertices and $p \neq 4n$; $n \geq 1$. Let u and v be any two nonadjacent vertices in G. Then either u or v or uv are in every minimal equitable total dominating sets of G. Thus every pair of vertices in $E_q^t MD(G)$ are adjacent. Hence $E_q^t MD(G)$ is complete.

Theorem 2.4: $E_q^t MD(G) \cong G$ *if and only if* G = C_3 *or* C_4.

Proof: Suppose $E_q^t MD(G) \cong G$. Then by Remark 2.2, $E_q^t MD(G)$ of C_{4n} is C_4 and every two adjacent vertices of $G = C_3$ form a minimal equitable total dominating set. Hence $E_q^t MD(G)$ of C_3 is C_3.

Converse is obvious.

Theorem 2.5: *If* G *is a* $(p-2)$ *- regular graph, then* $L(G) \cong E_q^t MD(G)$.

Proof: Let G be a $(p-2)$ - regular graph and let v be a vertex of G. Then there exists exactly one vertex u which is not adjacent to v. Let w be a vertex of G which is adjacent to both u and v. Thus $D_1 = \{v, w\}$ and $D_2 = \{w, u\}$ are minimal equitable total dominating sets in G. Therefore the corresponding vertices in D_1 and D_2 are adjacent in $E_q^t MD(G)$. Also the corresponding edges vw and wu are adjacent in $L(G)$. Therefore $L(G)$ and $E_q^t MD(G)$ are isomorphic. Hence $L(G) \cong E_q^t MD(G)$.

Next, we prove the necessary and sufficient condition for $E_q D(G)$ to be eulerian.

Theorem 2.6: *If* G *is a complete graph with* $p \geq 3$ *vertices, then* $E_q^t MD(G)$ *is eulerian.*

Proof: Let G is a complete graph with $p \geq 3$ vertices. Then every pair of its vertices forms a minimal equitable total dominating set. For any minimal equitable total dominating set D of G, there exist exactly $(p-2)+1$ minimal equitable total dominating sets containing a vertex of D. Thus D has even degree in $E_q^t MD(G)$. Hence by Theorem A, $E_q^t MD(G)$ is eulerian.

Theorem 2.7: *If* $G = K_{m,n}$, $2 \leq m = n$, *then* $E_q^t MD(G)$ *is eulerian.*

Proof: Let $G = K_{m,n}$, $2 \leq m = n$. Then every two adjacent vertices form a minimal equitable total dominating set. For any minimal equitable total dominating set D of G, there exist exactly $2(m-1)$ minimal equitable total dominating sets containing a vertex of D. Hence by Theorem A, $E_q^t MD(G)$ is eulerian.

Problem 1: Characterize the graphs *G* for which *L(G)* = $E_t^e MD(G)$.

Problem 2: Characterize the equitable total minimal dominating graphs which are eulerian.

Problem 3: Give a necessary and sufficient condition for a given graph *G* is equitable total minimal dominating graph of some graph.

ACKNOWLEDGEMENT

*This research was supported by UGC-SAP DRS-II New Delhi, India: for 2010-2015.

**This research was supported by the University Grants Commission, New Delhi, India. No.F.4-1/2006(BSR)/7-101 / 2007 (BSR) dated: 20th June, 2012.

REFERENCES

[1] B. Basavanagoud, V. R. Kulli and Vijay V. Teli, *Equitable Total Domination in Graphs*, Communicated.

[2] E. J. Cockayne and S. T. Hedetniemi, *Towards a Theory of Domination in Graphs,* Networks, 7, 247-261 (1977).

[3] F. Harary, *Graph Theory*, Addison-Wesley, Reading, Mass, (1969).

[4] T. W. Haynes, S. T. Hedetniemi and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, (1998).

[5] V. R. Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).

[6] V. R. Kulli and Radha R. Iyer, *The Total Minimal Dominating Graph*, Advances in Domination Theory- I, V. R. Kulli, ed. Vishwa International Publications, Gulbarga, India, 121-126 (2012).

[7] V. Swaminathan and K. M. Dharmalingam, *Degree equitable Domination on Graphs*,Kragujevak Journal of Mathematics, 35(1), 191-197 (2011).

Source of Support: University Grants Commission, New Delhi, India, Conflict of interest: None Declared