

EQUITABLE TOTAL MINIMAL DOMINATING GRAPH

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ABSTRACT

The equitable total minimal dominating graph $E_q^t MD(G)$ of a graph G is defined to be the intersection graph on the minimal equitable total dominating sets of vertices in G . In this paper, we obtain some properties of equitable total minimal dominating graph. Finally, we establish the characterizations of graphs whose equitable total minimal dominating graphs are complete and eulerian.

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Keywords: total domination number, equitable total dominating set, equitable total domination number, equitable total minimal dominating graph.

1. INTRODUCTION

All graphs $G = (V, E)$ we considered here are finite, undirected without loops and multiple edges. Terms not defined here are used in the sense of Harary [3] and Haynes *et. al* [4].

A subset D of V is called a *dominating set* of G if every vertex in $V - D$ is adjacent to some vertex in D . A dominating set D of G is minimal if for every vertex $v \in D$, $D - \{v\}$ is not a dominating set of G . The *domination number* $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set.

Cockayne *et. al* [2] introduced the concept of total domination in graphs. A dominating set D of G is called a *total dominating set* if $\langle D \rangle$ has no isolated vertices. The minimum cardinality of a total dominating set of G is called the *total domination number* of G and is denoted by $\gamma_t(G)$.

A subset D of V is called an *equitable dominating set* if for every vertex $v \in V - D$ there exists a vertex $u \in D$ such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such a dominating set is called the *equitable domination number* and is denoted by $\gamma^e(G)$. This concept was introduced by Swaminathan *et. al* [7].

A subset D of V is called an *equitable total dominating set* of G , if D is an equitable dominating set and $\langle D \rangle$ has no isolated vertices. The minimum cardinality taken over all equitable total dominating sets is the *equitable total domination number* [1] and is denoted by $\gamma_t^e(G)$.

In this paper, we use this idea to introduce a new graph valued function in the field of domination theory in graphs.

Let $F = \{S_1, S_2, \dots, S_n\}$ be a partition of S . Then the intersection graph $\Omega(F)$ of F is the graph whose vertices are the subsets in F and in which two vertices S_i and S_j are adjacent if and only if $S_i \cap S_j \neq \phi$.

Definition 1: The *equitable total minimal dominating graph* $E_q^t MD(G)$ of a graph G is defined to be the intersection graph on the family of all minimal equitable total dominating sets of vertices in G .

In Figure 1, a graph G and its equitable total minimal dominating graph $E_q^t MD(G)$ are shown:

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Here the minimal equitable total dominating sets of a graph G are: $D_1 = \{1,2\}$, $D_2 = \{1,3\}$, $D_3 = \{2,3\}$, $D_4 = \{2,4\}$ and $D_5 = \{3,4\}$.

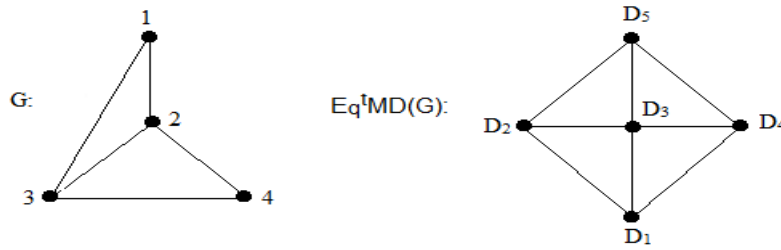


Figure - 1

The following results will be useful in the proof of our results.

Theorem A [3]: A connected graph G is eulerian if and only if every vertex of G has even degree.

Theorem B [1]: For any complete graph K_p with $p \geq 2$ vertices, $\gamma_t^e(K_p) = 2$.

Theorem C [3]: If G is a (p, q) graph, whose vertices have degree d_i , then $L(G)$ has q vertices and $q_L = -q + \frac{1}{2} \sum (d_i)^2$.

2. RESULTS

Remark 2.1: If $G = K_p$; $p \geq 2$ or $K_{m,n}$, then $E_q^t MD(G) \cong L(G)$.

Remark 2.2: If $G = C_{4n}$; $n \geq 1$, then $E_q^t MD(G) = C_4$.

Theorem 2.1: For any graph G with no isolated vertices, $E_q^t MD(G)$ is connected.

Proof: We consider the following cases.

Case-1: Suppose G is connected. Let D_1 and D_2 be any two disjoint minimal equitable total dominating sets in G . If every vertex in D_1 is adjacent to every vertex in D_2 , then clearly, $E_q^t MD(G)$ is connected.

Suppose there exist two vertices $u \in D_1$ and $v \in D_2$ such that u and v are not adjacent. Then there exists another minimal equitable total dominating set D_3 containing u and v . It follows that, there is a path in $E_q^t MD(G)$ from D_1 to D_3 and D_3 to D_2 . Hence $E_q^t MD(G)$ is connected.

Case-2: Suppose G is disconnected and has no isolated vertices. Then G has at least two components say G_1 and G_2 . Each minimal equitable total dominating set of G contains at least two adjacent vertices from each component of G . Thus, every vertex in $E_q^t MD(G)$ has at least two common vertices. Hence $E_q^t MD(G)$ is connected.

Theorem 2.2: For any (p, q) graph G ,

(i) $|V(E_q^t MD(G))| = |E(G)|$

(ii) $|E(E_q^t MD)| = -q + \frac{1}{2} \sum (d_i)^2$

if and only $G = K_p$; $p \geq 2$ or $K_{m,n}$, where d_i is the degree of each vertex v_i in G .

Proof: By Theorem B, $\gamma_t^e(G) = 2$. Since K_p is complete, every pair of vertices forms a minimal equitable total dominating set. Thus $E_q^t MD(G)$ has $\frac{p(p-1)}{2}$ minimal equitable total dominating sets. Hence $|V(E_q^t MD(G))| = |E(G)|$ and by Theorem C, the number of edges in $L(G)$ are $-q + \frac{1}{2} \sum (d_i)^2$, where d_i is the degree of each vertex v_i in G . Hence $|E(E_q^t MD)| = -q + \frac{1}{2} \sum (d_i)^2$.

Theorem 2.3: If G is a cycle of order $p \geq 5$ and $p \neq 4n$; $n \geq 1$, then $E_q^t MD(G)$ is complete.

Proof: Suppose G is a cycle of order $p = 4n$; $n \geq 1$. Then there exist at least two disjoint minimal equitable total dominating sets in G . Thus every vertices in $E_q^t MD(G)$ are not adjacent. Hence $E_q^t MD(G)$ is not complete.

Let G be a cycle C_p with $p \geq 5$ vertices and $p \neq 4n$; $n \geq 1$. Let u and v be any two nonadjacent vertices in G . Then either u or v or uv are in every minimal equitable total dominating sets of G . Thus every pair of vertices in $E_q^t MD(G)$ are adjacent. Hence $E_q^t MD(G)$ is complete.

Theorem 2.4: $E_q^t MD(G) \cong G$ if and only if $G = C_3$ or C_4 .

Proof: Suppose $E_q^t MD(G) \cong G$. Then by Remark 2.2, $E_q^t MD(G)$ of C_{4n} is C_4 and every two adjacent vertices of $G = C_3$ form a minimal equitable total dominating set. Hence $E_q^t MD(G)$ of C_3 is C_3 .

Converse is obvious.

Theorem 2.5: If G is a $(p - 2)$ - regular graph, then $L(G) \cong E_q^t MD(G)$.

Proof: Let G be a $(p - 2)$ - regular graph and let v be a vertex of G . Then there exists exactly one vertex u which is not adjacent to v . Let w be a vertex of G which is adjacent to both u and v . Thus $D_1 = \{v, w\}$ and $D_2 = \{w, u\}$ are minimal equitable total dominating sets in G . Therefore the corresponding vertices in D_1 and D_2 are adjacent in $E_q^t MD(G)$. Also the corresponding edges vw and wu are adjacent in $L(G)$. Therefore $L(G)$ and $E_q^t MD(G)$ are isomorphic. Hence $L(G) \cong E_q^t MD(G)$.

Next, we prove the necessary and sufficient condition for $E_q D(G)$ to be eulerian.

Theorem 2.6: If G is a complete graph with $p \geq 3$ vertices, then $E_q^t MD(G)$ is eulerian.

Proof: Let G is a complete graph with $p \geq 3$ vertices. Then every pair of its vertices forms a minimal equitable total dominating set. For any minimal equitable total dominating set D of G , there exist exactly $(p - 2) + 1$ minimal equitable total dominating sets containing a vertex of D . Thus D has even degree in $E_q^t MD(G)$. Hence by Theorem A, $E_q^t MD(G)$ is eulerian.

Theorem 2.7: If $G = K_{m,n}$, $2 \leq m = n$, then $E_q^t MD(G)$ is eulerian.

Proof: Let $G = K_{m,n}$, $2 \leq m = n$. Then every two adjacent vertices form a minimal equitable total dominating set. For any minimal equitable total dominating set D of G , there exist exactly $2(m - 1)$ minimal equitable total dominating sets containing a vertex of D . Hence by Theorem A, $E_q^t MD(G)$ is eulerian.

Problem 1: Characterize the graphs G for which $L(G) = E_r^e MD(G)$.

Problem 2: Characterize the equitable total minimal dominating graphs which are eulerian.

Problem 3: Give a necessary and sufficient condition for a given graph G is equitable total minimal dominating graph of some graph.

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