



# COMMON FIXED POINT THEOREMS IN FUZZY METRIC SPACE USING INTEGRAL TYPE

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## ABSTRACT

*In this paper, the concept of common fixed point theorem in fuzzy metric space using integral type.*

**Key Words:** contraction condition, fuzzy metric space, weakly compatible mapping, common fixed point

**AMS subject classification:** 47H10, 54H25.

## 1. INTRODUCTION

Zadseh in 1965 was introduced fuzzy set. This concept in topology and analysis many authors have developed the theory of fuzzy set and applications. Recently many authors have also studied the fixed point theory in fuzzy metric spaces ([11], [12][15], [22], [28], [29]. Grabiec[15] followed kramsoil and michalek[17] and obtained the banasch's fixed theorem using fuzzy sets. jungck[6] established common fixed point theorems for commuting maps generalizing the banach's fixed point theorems. sessa [25] defined a generalization of commutativity which is called weak commutativity. The study of common fixed point of mappings satisfying contractive type conditions has been a very active field of research during recent years. Several authors [5], [16], [17], [32] proved some fixed point theorems for various generalizations of contraction mapping in probabilistic and fuzzy metric metric space. In this paper, we prove some common fixed point theorems for mappings by using contracting condition of integral type for class of weakly compatible maps in non complete intuitionistic fuzzy metric spaces

## 2. PRELIMINARIES

**Definition 2.1:**[5].A binary operation  $*: [0,1] \times [0,1] \rightarrow [0,1]$  is called a continuous t- norm if  $([0,1],*)$  is an abelian topological monoid with the unit 1 such that  $a*b \leq c*d$  whenever  $a \leq c$  and  $b \leq d$  for all  $a,b,c,d \in [0,1]$ . Examples of t- norms are  $a*b = ab$  and  $a*b = \min\{a, b\}$

**Definition 2.2** ([21]): the 3 –tuples  $(X, M,*)$  is called a fuzzy metric space if  $X$  is an arbitrary set  $*$  is a continuous t- norm, and  $M$  is fuzzy set in  $X^2 \times [0,\infty]$  satisfying the following conditions for all  $x, y, z \in X$  and  $t, s > 0$ .

1.  $M(x, y, 0) = 0$
2.  $M(x, y, t) = 1 \quad \forall t > 0 \Leftrightarrow x = y$
3.  $M(x, y, t) = M(y, x, t)$
4.  $M(x, y, t) * M(y, z, s) \leq M(x, z, t+s)$  and
5.  $M(x, y, *): [0, \infty) \rightarrow [0,1]$  is left continuous.
6.  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$

In what follow  $(X, M,*)$  will denote a fuzzy metric space. Note that  $M(x, y, t)$  can be thought as the degree of nearness between  $x$  and  $y$  with respect to  $t$ . We identify  $x = y$  with  $M(x, y, t) = 1$  for all  $t > 0$  and  $M(x, y, t) = 0$  with  $\infty$  and we can find some topological properties and examples of fuzzy metric spaces in George and veeramani [3].

In the following example, we know that every metric induces a fuzzy metric.

$$M(x, y, t) = \frac{t}{t + d(x, y)}, \quad (A)$$

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Then  $M(x, y, *)$  is a fuzzy metric space. We call the fuzzy metric  $M$  induced by the metric  $d$  the standard fuzzy metric. On the other hand note that there exists no metric on  $x$  satisfying (A)

**Lemma 2.1** ([15]): for all  $x, y \in X$ ,  $M(x, y, *)$  is non decreasing.

**Definition 2.3:** ([15]) Let  $M(x, y, *)$  is a fuzzy metric space:

- (i). A sequence  $\{X_n\}$  in  $X$  is said to be convergent to a point  $x, \in X$  (denoted by  $\lim_{t \rightarrow \infty} X_n = x$ ) if for  $t > 0$  and  $\lim_{t \rightarrow \infty} M(x, y, t) = 1$
- (ii) A sequence  $\{X_n\}$  in  $X$  is said to be Cauchy sequence if, for all  $t > 0$  and  $p > 0$ ,  $\lim_{t \rightarrow \infty} M(x_{n+p}, x_n, t) = 1$
- (iii). A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

**Note 2.1:** since  $*$  is continuous, it follow from definition 2.2(4) that the limit of the sequence in fuzzy metric space is uniquely determined

**Lemma 2.2** ([23] [33]): Let  $\{y_n\}$  be a sequence in a fuzzy metric space  $M(x, y, *)$  with the condition Definition 2.2(6) . If there exists a number  $k \in (0, 1)$  such that  $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$  for all  $t > 0$  and  $n = 1, 2, \dots$ . Then  $\{y_n\}$  is a Cauchy sequence in  $X$ .

**Lemma 2.3** ([23][33]): if for all  $x, y \in X$ ,  $t > 0$  and for a number  $k \in (0, 1)$

$$M(x, y, kt) \geq M(x, y, t)$$

Then  $x = y$ .

**Definition 2.4** ([23]): Let  $A$  and  $B$  be mappings from a fuzzy metric space  $M(x, y, *)$  into it self. The mapping  $A$  and  $B$  are said to be compatible if  $\lim_{t \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$  For all  $t > 0$ , whenever  $\{X_n\}$  is a sequence in  $X$  such that  $\lim_{t \rightarrow \infty} Ax_n = \lim_{t \rightarrow \infty} Bx_n = Z$  for some  $z \in X$ .

**Definition 2.5** ([33]): Let  $A$  and  $B$  be mappings from a fuzzy metric space  $M(x, y, *)$  into it self. The mapping  $A$  and  $B$  are said to be compatible of type  $(\alpha)$  if for all  $t > 0$

$$\text{if } \lim_{t \rightarrow \infty} M(ABx_n, BBx_n, t) = 1 \text{ and } \lim_{t \rightarrow \infty} M(BAx_n, AAx_n, t) = 1$$

For all  $t > 0$ , whenever  $\{X_n\}$  is a sequence in  $X$  such that  $\lim_{t \rightarrow \infty} Ax_n = \lim_{t \rightarrow \infty} Bx_n = Z$  for some  $z \in X$ .

**Definition 2.6** ([34]): Let  $A$  and  $B$  be mappings from a fuzzy metric space  $M(x, y, *)$  into it self . The mapping  $A$  and  $B$  are said to be compatible of type  $(\beta)$  if for all  $t > 0$

$$\text{if } \lim_{t \rightarrow \infty} M(AAx_n, BBx_n, t) = 1 \text{ when ever For all } t > 0, \text{ whenever } \{X_n\} \text{ is a sequence in } X \text{ such that } \lim_{t \rightarrow \infty} Ax_n = \lim_{t \rightarrow \infty} Bx_n = Z \text{ for some } z \in X.$$

**Definition 2.6** ([9]): two self mappings  $S$  and  $T$  are said to be weakly compatible if they commute at their coincidence points, i.e, if  $Tu = Su$  for  $u \in X$ .  $TSu = Stu$

**Examples 2.2:** Let  $X [0, 2]$  with the metric  $d$  define by  $d(x, y) = |x - y|$ . For each  $t \in (0, \infty)$

$$M(x, y, t) = \frac{t}{t + d(x, y)},$$

And define  $M(x, y, 0) = 0$   $x, y \in X$

Clearly,  $M(x, y, *)$  is a fuzzy metric space on  $X$  where  $*$  is defined by  $a * b = ab$  or  $a * b = \min\{a, b\}$

$$AX = \begin{cases} x, & \text{if } x \in [0, \frac{1}{4}] \\ \frac{1}{4}, & \text{if } x \geq \frac{1}{4} \end{cases} \quad Bx = \frac{x}{1+x}$$

For all  $x \in [0, 2]$ . Consider the sequence  $\{X_n = 1/3 + 1/n: n \geq 1\}$  in  $X$ . Then

$$\lim_{n \rightarrow \infty} Ax_n = \frac{1}{4}, \lim_{n \rightarrow \infty} Bx_n = \frac{1}{4}$$

$$\lim_{t \rightarrow \infty} M(ABx_n, BAx_n, t) = \frac{1}{t + \frac{1}{4} + \frac{1}{5}} \neq 1$$

and

$$\lim_{t \rightarrow \infty} M(BAx_n, AAx_n, t) = \frac{1}{t + \frac{t-1}{5}} \neq 1. \text{ Thus A and B are}$$

non compatible, but A and B are commuting at their. Coincidencce point, That is weakly compatible at  $x = 0$ . also Thus A and B are not compatible of type  $(\beta)$ . in view of this example, we observe weakly compatible maps need not be compatible. or compatible of type  $(\alpha)$  and type  $(\beta)$

### 3. MAIN THEOREM

**Theorem 3.1:** Let  $(X, M, *)$  be a fuzzy metric space with continuous  $t$  – norm  $*$  defined by  $t*t$  for all  $t \in [0, 1]$ . Let A, B, S, T, P and Q be mappings from X into itself such that

$$(a) P(X) \subset ST(X), Q(X) \subset AB(X)$$

(b) there exists a constant  $k \in (0, 1)$  such that

$$\int_0^{M(Px, Qy, qt)} \varphi(t) dt \geq \int_0^{m(x, y, t)} \varphi(t) dt$$

Where  $\varphi: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is a lebesque – integrable mapping which is summable, non negative, and such that

$$\int_0^\varepsilon \varphi(t) dt > 0 \text{ for each } \varepsilon > 0$$

Where  $m(x, y, t) = \min\{M(ABx, Sty, t), M(Px, ABX, t), M(Qy, Sty, t), M(Px, ABX, t), M(Px, Sty, t)\}$  For all  $x, y \in X$  and  $t > 0$

(c)  $P(x), Q(x), AB(x), ST(x)$  is complete subspace of X, then

- (i). P and AB have a coincidence point, and
- (ii). Q and St have a coincidence point.

$$(d) AB=BA, ST=TS, PB=BP, QT=TQ,$$

(e). the pair  $\{P, AB\}$  is weakly compatible ,

Then A, B, S, T, P and Q have a unique common fixed point in X.

**Proof:** let  $x_0 \in X$ , from (a) there exist  $x_1, x_2 \in X$  such that

$$Px_0 = STx_1 \text{ And } Qx_1 = ABx_2$$

Inductively, we can construct sequence  $\{X_n\}$  and  $\{y_n\}$  in X such that

$$Px_{2n-2} = STx_1 = y_{2n-1} \text{ and}$$

$$Qx_{2n-1} = ABx_{2n} = y_{2n} \text{ for } n=1, 2, 3, \dots$$

**Step - 1:** put  $x=x_{2n}$  and  $y=y_{2n+1}$  in (e), we get

$$\begin{aligned} \int_0^{M(Px_{2n}, Qx_{2n+1}, qt)} \varphi(t) dt &\geq \int_0^{M(ABx_{2n}, STx_{2n+1}, t)M(Px_{2n}, ABx_{2n}, t)M(Qx_{2n+1}, STx_{2n+1}, t)M(Px_{2n}, STx_{2n+1}, t)} \varphi(t) dt \\ &= \int_0^{M(y_{2n}, y_{2n+1}, t)M(y_{2n+1}, y_{2n}, t)M(y_{2n+2}, y_{2n+1}, t)M(y_{2n+1}, y_{2n+2}, t)} \varphi(t) dt \\ &\geq \int_0^{M(y_{2n}, y_{2n+1}, t)M(y_{2n+1}, y_{2n+2}, t)} \varphi(t) dt \end{aligned}$$

Form lemma 2, 1 and 2, 2, we have

Similarly, we have

$$\int_0^{M(y_{2n+2}, y_{2n+3}, qt)} \varphi(t) dt \geq \int_0^{M(y_{2n+1}, y_{2n+2}, t)} \varphi(t) dt$$

Thus, we have

$$\int_0^{M(y_{n+1}, y_{n+3}, qt)} \varphi(t) dt \geq \int_0^{M(y_n, y_{n+1}, t)} \varphi(t) dt \text{ for } n=1, 2, \dots$$

$$\begin{aligned} \int_0^{M(y_n, y_{n+1}, t)} \varphi(t) dt &\geq \int_0^{M(y_{n-1}, y_n, \frac{t}{q})} \varphi(t) dt \\ &\geq \int_0^{M(y_{n-2}, y_{n-1}, \frac{t}{q^2})} \varphi(t) dt \\ &\dots\dots\dots \\ &\geq \int_0^{M(y_1, y_2, \frac{t}{q^N})} \varphi(t) dt \rightarrow 1 \text{ as } n \rightarrow \infty \end{aligned}$$

And hence  $\int_0^{M(y_n, y_{n+1}, t)} \varphi(t) dt \rightarrow 1$  as  $n \rightarrow \infty$  for any  $t > 0$ .

For each  $\varepsilon > 0$  and  $t > 0$ , we choose  $n_0 \in \mathbb{N}$  such that  $\int_0^{M(y_n, y_{n+1}, t)} \varphi(t) dt > 1 - \varepsilon$  for all  $n > n_0$

For  $m, n \in \mathbb{N}$ , we suppose  $m \geq n$ . Then we have

$$\begin{aligned} \int_0^{M(y_n, y_m, t)} \varphi(t) dt &\geq \int_0^{M(y_n, y_{n+1}, \frac{t}{m-n})} \varphi(t) dt \dots\dots\dots \int_0^{M(y_{m-1}, y_m, \frac{t}{m-n})} \varphi(t) dt \\ &\geq (1 - \varepsilon) * (1 - \varepsilon) * \dots * (1 - \varepsilon) (m - n) \text{ times} \\ &\geq (1 - \varepsilon) \end{aligned}$$

And hence  $\{y_n\}$  is a Cauchy sequence in  $X$

Since  $(X, M, *)$  is complete,  $\{y_n\}$  converges to some point  $z \in X$ . Also its subsequence converges to the same point

i.e.  $z \in X$

$$\text{i.e. } \{Qx_{2n+1}\} \rightarrow Z \text{ and } \{STx_{2n+1}\} \rightarrow Z \quad (1)$$

$$\{Px_{2n}\} \rightarrow Z \text{ and } \{ABx_{2n}\} \rightarrow Z \quad (2)$$

**Case - I:** suppose  $AB$  is continuous, we have

$$(AB)^2 x_{2n} \rightarrow ABZ \text{ and}$$

$$ABx_{2n} \rightarrow ABZ$$

As  $(P, AB)$  is compatible pair, we have  $PABx_{2n} \rightarrow ABZ$

**Step -2:** put  $x = ABx_{2n}$  and  $y = x_{2n+1}$  in (e), we get

$$\int_0^{M(PABx_{2n}, Qx_{2n+1}, qt)} \varphi(t) dt \geq \int_0^{M(ABABx_{2n}, STx_{2n+1}, t)M(PABx_{2n}, ABABx_{2n}, t)M(Qx_{2n+1}, STx_{2n+1}, t)M(PABx_{2n}, STx_{2n+1}, t)} \varphi(t) dt$$

Taking  $n \rightarrow \infty$ , we get

$$\int_0^{M(ABZ, Z, qt)} \varphi(t) dt \geq \int_0^{M(ABZ, Z, t)M(ABZ, ABZ, t)M(Z, Z, t)M(ABZ, Z, t)} \varphi(t) dt$$

$$\int_0^{M(ABZ, Z, qt)} \varphi(t) dt \geq \int_0^{M(ABZ, Z, t)M(ABZ, Z, t)} \varphi(t) dt$$

$$\int_0^{M(ABZ, Z, qt)} \varphi(t) dt \geq \int_0^{M(ABZ, Z, t)} \varphi(t) dt$$

Therefore, by using lemma 2.2., we get  $ABZ=Z$

**Step - 3:** put  $x=z$  and  $y=x_{2n+1}$  In (b), we have

$$\int_0^{M(z, z, t)M(Pz, z, t)} \varphi(t) dt \geq \int_0^{M(z, z, t)M(Pz, z, t)M(z, z, t)M(Pz, z, t)} \varphi(t) dt \geq \int_0^{M(Pz, z, t)M(Pz, z, t)} \varphi(t) dt$$

$$\int_0^{M(Pz, z, t)} \varphi(t) dt \geq \int_0^{M(Pz, z, t)} \varphi(t) dt$$

Therefore by using lemma 2.2., we get  $Pz=z$ , therreofre  $ABZ=Pz=z$

**Step - 4:** putting  $x=Bz$  and  $y=x_{2n+1}$  in condition (e) we get

$$\int_0^{M(PBz, Qx_{2n+1}, qt)} \varphi(t) dt \geq \int_0^{M(ABBz, STx_{2n+1}, t)M(Pz, ABBz, t)M(Qx_{2n+1}, STx_{2n+1}, t)M(PBz, STx_{2n+1}, t)} \varphi(t) dt$$

As  $BP=PB$ ,  $AB=BA$ , so we have  $P(Bz)=B(Pz)=Bz$  and

$$(AB)(Bz) = (BA)(Bz) = B(ABZ) = Bz$$

Taking  $n \rightarrow \infty$  and using (1), we get

$$\int_0^{M(Bz, z, qt)} \varphi(t) dt \geq \int_0^{M(Bz, z, t)M(Bz, Bz, t)M(z, z, t)M(Bz, z, t)} \varphi(t) dt \geq \int_0^{M(Bz, z, t)M(Bz, z, t)} \varphi(t) dt$$

$$\int_0^{M(Bz, z, qt)} \varphi(t) dt \geq \int_0^{M(Bz, z, qt)} \varphi(t) dt$$

Therefore, by using lemma 2.2, we get

$$Bz = z$$

And also we have  $ABz=z$

$$\Rightarrow Az=z, \text{ Therefore } Az=Bz=Pz=z$$

(4)

**Step - 5:** As  $P(X) \subseteq ST(X)$ , there exists  $u \in X$  such that

$Z=PZ=STU$ . Putting  $x=x_{2n}$  and  $y=u$  in (e) we get

$$\int_0^{M(Px_{2n}, Qu, qt)} \varphi(t) dt \geq \int_0^{M(ABx_{2n}, STu, t)M(Px_{2n}, ABx_{2n}, t)M(Qu, STu, t)M(Px_{2n}, STu, t)} \varphi(t) dt$$

Taking  $n \rightarrow \infty$  and using (1) and (2), we get

$$\begin{aligned} \int_0^{M(z, Qu, qt)} \varphi(t) dt &\geq \int_0^{M(z, z, t)M(z, z, t)M(Qu, z, t)M(z, z, t)} \varphi(t) dt \\ &\geq \int_0^{M(Qu, z, t)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(z, Qu, qt)} \varphi(t) dt \geq \int_0^{M(z, Qu, t)} \varphi(t) dt$$

Therefore, by using lemma 2.2. we get  $Qu=z$  hence  $Stu=z=Qu$

Thus  $Qz=STz$ ,

**Step - 6:** putting  $x=x_{2n}$  and  $y=z$  in (e)

$$\int_0^{M(Px_{2n}, Qu, qt)} \varphi(t) dt \geq \int_0^{M(ABx_{2n}, STu, t)M(Px_{2n}, ABx_{2n}, t)M(Qu, STu, t)M(Px_{2n}, STu, t)} \varphi(t) dt$$

Taking  $n \rightarrow \infty$  and using (2) and step 5, we get

$$\begin{aligned} \int_0^{M(z, Qz, qt)} \varphi(t) dt &\geq \int_0^{M(z, z, t)M(z, z, t)M(Qz, Qz, t)M(z, qz, t)} \varphi(t) dt \\ &\geq \int_0^{M(z, Qz, qt)M(z, Qz, qt)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(z, Qz, qt)} \varphi(t) dt \geq \int_0^{M(z, Qz, t)} \varphi(t) dt$$

Therefore, by using lemma 2.2, we get  $Qz=z$

**Step - 7:** putting  $x=x_{2n}$  and  $y= Tz$  in (e), we get

$$\int_0^{M(Px_{2n}, QTz, qt)} \varphi(t) dt \geq \int_0^{M(ABx_{2n}, STTz, t)M(Px_{2n}, ABx_{2n}, t)M(QTz, STTz, t)M(Px_{2n}, STTz, t)} \varphi(t) dt$$

As  $QT=TQ$  and  $ST=Ts$ , we have  $QTz=TQz =Tz$  and  $ST(Tz)=T(STz)=TQz=Tz$

Taking  $n \rightarrow \infty$ , we get

$$\begin{aligned} \int_0^{M(Pz, Tz, qt)} \varphi(t) dt &\geq \int_0^{M(z, Tz, t)M(z, z, t)M(Tz, Tz, t)M(z, Tz, t)} \varphi(t) dt \\ &\geq \int_0^{M(z, Tz, t)M(z, Tz, t)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(z, Tz, qt)} \varphi(t) dt \geq \int_0^{M(z, Tz, t)} \varphi(t) dt$$

Therefore, by using lemma 2.2. we get  $Tz=z$  now  $STz=Tz=z$  implies  $Sz=z$  hence  $Sz=Tz=Qz=Z$  (5)

Combining (4) and (5) we get

$$Az = Bz = Pz = Qz = Tz = Sz = z$$

Hence,  $z$  is the common fixed point of  $A, B, S, T, P$  and  $Q$

**Case - II:** suppose P is continuous

As P is continuous

$$P^2x_{2n} \rightarrow Pz \text{ and } P(AB)x_{2n} \rightarrow Pz$$

As (P, AB) is compatible pair.  $(AB)Px_{2n} \rightarrow Pz$

**Step - 8:** putting  $x=Px_{2n}$  and  $y=x_{2n+1}$  in condition (e), we have

$$\int_0^{M(PPx_{2n}, Qx_{2n+1}, qt)} \varphi(t) dt \geq \int_0^{M(ABPx_{2n}, STx_{2n+1}, t)M(PPx_{2n}, ABPx_{2n}, t)M(Qx_{2n+1}, STx_{2n+1}, t)M(PPx_{2n}, STx_{2n+1}, t))} \varphi(t) dt$$

Taking  $n \rightarrow \infty$ , we get

$$\begin{aligned} \int_0^{M(Pz, z, qt)} \varphi(t) dt &\geq \int_0^{M(Pz, z, t)M(Pz, Pz, t)M(z, z, t)M(Pz, z, t)} \varphi(t) dt \\ &\geq \int_0^{M(Pz, z, t)M(Pz, z, t)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(Pz, z, qt)} \varphi(t) dt \geq \int_0^{M(Pz, z, t)} \varphi(t) dt$$

Therefore by using lemma 2.2, we have  $Pz=z$  further, using step 5, 6, 7 we get  $Qz=STz=Sz=Tz=z$

**Step - 9:**  $Q(X) \subseteq AB(X)$ , there exists  $w$  belongs to  $X$  such that

$$Z=Qz=ABw$$

Put  $x=w$  and  $y=x_{2n+1}$  in (e), we have

$$\int_0^{M(Pw, Qx_{2n+1}, qt)} \varphi(t) dt \geq \int_0^{M(ABw, STx_{2n+1}, t)M(Pw, ABw, t)M(Qx_{2n+1}, STx_{2n+1}, t)M(Pw, STx_{2n+1}, t)} \varphi(t) dt$$

Taking  $n \rightarrow \infty$ , we get

$$\begin{aligned} \int_0^{M(Pw, z, qt)} \varphi(t) dt &\geq \int_0^{M(z, z, t)M(Pw, z, t)M(z, z, t)M(Pw, z, t)} \varphi(t) dt \\ &\geq \int_0^{M(Pw, z, t)M(Pw, z, t)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(Pw, z, qt)} \varphi(t) dt \geq \int_0^{M(Pw, z, t)} \varphi(t) dt$$

Therefore, buy using  $ABw=Pw=z$

As (P, AB) is compatible pair. We have  $Pz=ABz$

Also, from step4, we get  $Bz=z$ .

Thus  $Az = Bz = Pz = z$  and we see that  $z$  is the common fixed point in this case also.

**Uniqueness:** Let  $u$  be another common fixed point of  $A, B, S, T, P$  and  $Q$  put  $x=z$  and  $y=uy$  in (e) we get

$$\int_0^{M(Pz, Qu, qt)} \varphi(t) dt \geq \int_0^{M(ABzSTu, t)M(Pz, ABz, t)M(Qu, STu, t)M(Pz, STu, t)} \varphi(t) dt$$

Taking  $n \rightarrow \infty$ , we get

$$\begin{aligned} \int_0^{M(z,u,qt)} \varphi(t) dt &\geq \int_0^{M(z,u,t)M(z,z,t)M(u,u,t)M(z,u,t)} \varphi(t) dt \\ &\geq \int_0^{M(z,u,t)M(z,u,t)} \varphi(t) dt \end{aligned}$$

$$\int_0^{M(z,u,qt)} \varphi(t) dt \geq \int_0^{M(z,u,t)} \varphi(t) dt$$

Therefore by using lemma (2.2), we get  $z=u$

Therefore  $z$  is the unique common fixed point of self maps  $A, B, S, T, P$  and  $Q$ .

**Corollary 3.1:** Let  $(X, M, *)$  be a fuzzy metric space with continuous  $t$  – norm  $*$  defined by  $t*t$  for all  $t \in [0,1]$ . Let  $A, S, P$  and  $Q$  be mappings from  $X$  into itself such that

(a)  $P(X) \subset S(X), Q(X) \subset A(X)$

(b) there exists a constant  $k \in (0,1)$  such that

$$\int_0^{M(Px,Qy,qt)} \varphi(t) dt \geq \int_0^{m(x,y,t)} \varphi(t) dt$$

Where  $\varphi: \mathbb{R}^+ \rightarrow \mathbb{R}^+$  is a lebesgue – integrable mapping which is summable, non negative, and such that

$$\int_0^\varepsilon \varphi(t) dt > 0 \text{ for each } \varepsilon > 0$$

Where  $m(x,y,t) = \min\{M(Px,Qy,qt), M(Ax,Sy,t), M(Px,Ax,t), M(Qy,Sy,t), M(Px,Sy,t)\}$

For all  $x, y \in X$  and  $t > 0$

(c).  $P(x), AB(x)$ , or  $ST(x)$  is complete subspace of  $X$ , then

- (i).  $P$  and  $AB$  have a coincidence point, and
- (ii).  $Q$  and  $St$  have a coincidence point.

(d). the pair  $\{P, A$  and  $(Q, S)$  are compatible, Then  $A, S, P$  and  $Q$  have a unique common fixed point in  $X$ .

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