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NP - COMPLETE IN HAMILTONIAN CIRCUITS

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ABSTRACT

The calculation of Hamiltonian Circuits is an NP-complete task, complete sets of Hamiltonian circuits for the classification of documents, and their properties.

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1. INTRODUCTION

Hamilton circuits are named after the renowned Irish mathematician (and astronomer) Sir William Rowan Hamilton who lived from 1805 to 1865. Hamilton was a child prodigy. He could read English, Hebrew, Greek, and Latin by time he was four years old. In addition, he wrote poetry and maintained close friendships with Wordsworth and Coleridge. Hamilton became Professor of Astronomy at Trinity College in Dublin, Ireland when he was twenty-three years old.

A Hamiltonian path (or traceable path) is a path in an undirected or directed graph that visits each vertex exactly once. A Hamiltonian cycle (or Hamiltonian circuit) is a Hamiltonian path that is a cycle. Determining whether such paths and cycles exist in graphs is the Hamiltonian path problem, which is NP-complete.

2. PRELIMINARIES

Hamilton circuit: 2.1 a circuit that starts at a vertex of a graph, passes through every vertex exactly once, and returns to the starting vertex is a Hamilton circuit.

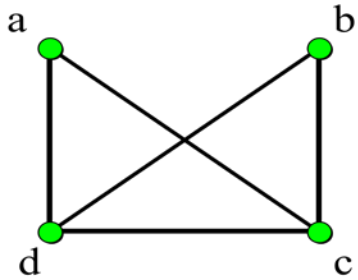
Complete graph: 2.2 a graph in which every vertex is joined to every other vertex by exactly one edge is called a complete graph.

Weighted graph: 2.3 a graph in which each edge is "weighted" is called a weighted graph. a "weight" is a number which might represent a distance, a cost, or some other quantity.

Weight of a path: 2.4 the weight of a path (or circuit) is the sum of the weights of each edge in the path (or circuit).

Optimal Hamilton circuit: 2.5 an optimal Hamilton circuit of a graph is one with the smallest possible weight. *There can be more than one.*

Hamilton Path: 2.6 a path that touches every vertex at most once.



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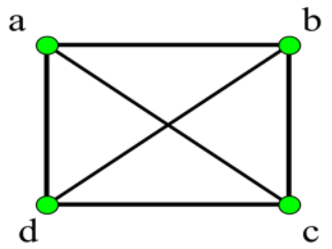
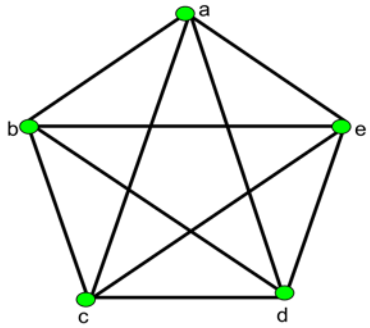
Hamilton Circuit: 2.7 a path that touches every vertex at most once and returns to the starting vertex.



Theorem: 2.8 A connected graph with *n* vertices, *n* >2, has a Hamilton circuit if the degree of each vertex is at least *n*/2



Theorem: 2.9 Let *G* be a connected graph with *n* vertices, and let the vertices be indexed $x_1, x_2, \ldots, x_n$, so that $\deg(x_i) \leq \deg(x_{i+1})$. If for each $k \leq n/2$, either $\deg(x_k) > k$ or $\deg(x_{n+k}) \geq n - k$, then *G* has a Hamilton circuit

Remarks: 2.10 Any Hamiltonian cycle can be converted to a Hamiltonian path by removing one of its edges, but a Hamiltonian path can be extended to Hamiltonian cycle only if its endpoints are adjacent.

All Hamiltonian graphs are biconnected, but a biconnected graph need not be Hamiltonian.

The line graph of a Hamiltonian graph is Hamiltonian. The line graph of an Eulerian graph is Hamiltonian.

3. RESULTS

Theorem: 3.1 If H has a Hamiltonian circuit then the number of circuits in any 2-factor of H is odd.

Proof: Let F = $\{C_1, C_2, .., C_r\}$ be a 2-factor of H with circuits $C_1, C_2, .., C_r$ and let C_0 be a Hamiltonian circuit of H. Then for i = 1, 2, . . . , r, the length $|C_i|$ of C_i is of the form $4k_i + 2$ where k_i is a positive integer. Thus $\sum_{i=1}^{r} 4k_i + 2r = 4k_i + 2r = 4k_0 + 2$. Therefore 2(r - 1) is divisible by 4, i.e., r is odd.

Theorem: 3.2 Let H be a Hamiltonian system and G its graph. Then H has a Hamiltonian circuit if and only if G has a perfect matching M such that the removal from H of M but not of their end vertices results in a connected graph.

Note: 3.3 The Number of circuits in a 2-factor is even: the systems have no Hamiltonian circuit.

Note: 3.4 H has no fixed bonds; The Number of circuits in a 2-factor is odd and the number of internal vertices of H is divisible by 4; but H has no Hamiltonian circuit.

To find a Hamiltonian circuit in a H if there is one works as follows: generate sequentially all perfect matchings of the branching graph of H and check each time if the removal of the bonds in the current perfect matching from H results in a connected graph. If it is the case, then this graph is a Hamiltonian circuit of H.

Cut does not have a polynomial time complexity, but it at least provides a systematic way to generate a Hamiltonian circuit if there is one. Ways to generate all perfect matchings of given in; they are easily extended to the generation of all perfect matchings of a branching graph to have a Hamiltonian path. An almost-perfect matching M ~ of a graph is a set of disjoint edges covering all its vertices but two.

Theorem: 3.5 Let H be a system and G its Graph. Then H has a Hamiltonian path if and only if G has an almost perfect matching m, such that the deletion from H of m^1 but not of their vertices results in a connected graph

Lemma: 3.6 Let G be the graph of a H. Then for each element x of LC(G), the hexagon x(h) must belong to H.

Proof: If x is a leaf of b(G), by the definition of graph x(h) belongs to H. Suppose that x is a cove. If $x(h)$ does not belong to H, then all hexagons adjacent to $x(h)$ must belong to H. This contradicts that x is a cove.

Lemma: 3.7 Let G be the graph of a H. Let h be a hexagon of H such that h does not belong to G and does not contain any element of $LC(G)$. e and e' of H which are in the boundary of H and adjacent to h but do not belong to h are contained in G .

Theorem: 3.8 Let H be a system and G its branching graph. Then H has a Hamiltonian path if and only if G has an almost –perfect matching M' such that the deletion from H of bonds of M' but not of their end vertices results in a connected graph.

Lemma: 3.9 let G be the graph of H. then for each element x of $L(G)$, then hexagon $x(h)$ must belong to H.

Lemma: 3.10 Let G be the graph of H. Let h be a hexagon of H such that h does not belongs to G and does not contain any element of $LC(G)$. Then two bonds e and e' of H which are in the boundary of H and adjacent to h but do not belong to h are contained in G.

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