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A MATHEMATICAL MODEL FOR ARC ROUTING PROBLEM – AN EMPIRICAL STUDY

Dr. Sandeep Tiwari[1] & Sopnamayee Acharya[2]*

[1]Asst. Professor, S. S. in Mathematics, Vikram University, Ujjain, (M.P.), India.

[2]Research Scholar, S. S. in Mathematics, Vikram University, Ujjain, (M.P.), India.

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ABSTRACT

The objective of this study is to construct an efficient master route – over an extended planning horizon (more than one day). Previously, a deterministic arc-routing problem (DARP) model is used to solve the problem. However, this approach ignores the uncertainty in the street segment presence probability—the probability that a street segment requires (i.e., there is a demand for) a visit on a particular day. We have considered a new model, namely, the probabilistic arc-routing problem (PARP) model which deals with the street segment presence probabilities. PARP attempts to minimize the expected length of the master route. It assumes that the street segment presence probabilities are independent. Our computational results show that PARP may produce more efficient master routes than DARP by taking demand uncertainty into account.

***Keyword(s)**: Arc-routing problem, Deterministic arc-routing model, Probabilistic arc-routing problem, Vehicle routing problem.*

1. INTRODUCTION

Small-package shipping firms rely on daily local delivery and pick-up routes to service their customer base. At the operational level, each service provider (SP) is responsible for a specific delivery area (e.g., a service provider's delivery area may contain street segments from a single zip code). In practice, an SP is encouraged to follow a master route, which defines a sequence of street segments and the direction in which each street segment is to be traversed for his/her delivery area. Street segments are defined by address ranges. For instance, a street segment may contain building numbers 1 to 100 on Pithampur Street. On the street network, a street segment can be either one way or two ways. On any given day, the exact set of customers to be served along a given street segment may vary. Servicing the customers in the same order each day (according to a master route of the delivery area) has various advantages for the SPs, including gaining familiarity with their service routes and arriving at regular customers at about the same time each day. In addition, this practice improves the efficiency of delivery because packages are loaded into the vehicles in accordance with the master routes. For instance, packages with destinations located on the street segments that appear early in the master route are placed in the front portion of the cargo area where the SP can easily reach them. Our overall objective is to construct efficient master routes for the service areas.

The issue of planning daily service where the set of customers may vary each day was first recognized by *Jaillet (1985)*, who proposed the probabilistic travelling salesman problem (PTSP) where each potential customer has a given presence probability on any given day. The problem is to find a master route through all of the nodes that will minimize the total expected (daily) cost of servicing all of the customers. In the context of small-package local operations, the number of possible different street addresses for customer delivery may be really quite large, so the PTSP may not be a practical model. It may be more useful to aggregate the set of possible customers into clusters (*Campbell 2006)*. We propose to partition them into a set of street segments, where each segment has a presence probability (probability of requiring service) on a given day. Given an extended planning horizon (more than one day), if the set of street segments requiring service every day remains unchanged, we only need to solve an un--capacitated arc-routing problem once during the entire time horizon. However, in reality, the street segments that need to be visited can vary on a daily basis.

Currently, the problem is often approached in a deterministic manner over a single day. More specifically, an arc-routing problem is solved. The resulting master route is used over the entire planning horizon. On a particular day, the route is realized following the predesigned sequence while skipping the street segments that do not require service.

****Corresponding author: Sopnamayee Acharya[2]****
E-mail: sopna75@gmail.com

We will refer to this approach as the deterministic arc-routing problem (DARP). Essentially, DARP belongs to a family of problems known as the mixed rural postman problem *(Laporte 1997)*. One major problem with this approach, as pointed out by *(Jaillet 1985, 1988)*, stems from the fact that a good solution when all required street segments are present may not remain a good solution when some street segments are skipped.

The uncertainty is to whether a street segment requires service on a particular day suggests that it may be beneficial to study the problem in a probabilistic context. One approach models the problem of finding a suitable master route for an extended planning horizon as a probabilistic arc-routing problem (PARP) where each street segment has a corresponding presence probability on any given day just as in the PTSP.

2. DESCRIPTION OF ARC-ROUTING PROBLEMS

In this section, we describe two arc-routing problems: DARP, PARP; and some local search-based solution approaches for the PARP. All of these arc-routing models have the following common inputs: the starting and ending locations (in case the two locations coincide, it becomes the depot), a set of arcs (street segments), the length of each street segment, the length of the shortest path between an endpoint of any segment to an endpoint of any other segment, and the length of the shortest path to/from the ending/starting location from/to the endpoint of any street segment.

2.1. Deterministic Arc-Routing Problem

The DARP has the following description. Given the common inputs, and a set of street segments (arcs) that must be serviced (traversed), find the master route of minimum length, which starts at the starting location, traverses all the arcs, and returns to the ending location. DARP belongs to a well-known class of arc-routing problems known as the Mixed Rural Postman Problem (MRPP). Many solution approaches for this class of problems rely on transforming it into traveling salesman problems *(Laporte 1997*). Comprehensive surveys on MRPP, as well as the arc-routing problem in general, can be found in *Eiselt, Gendreau, and Laporte (1995)* and *Assad and Golden (1995)*. In addition, *Corberan, Mart, and Romero (2000)* present an approximate algorithm based on the resolution of some flow and matching problems as well as a tabu search heuristic to solve the MRPP. Note that once we have specified the sequence order for visiting the arcs that must be traversed and the direction in which these arcs are traversed, then it is straightforward to calculate the total route length. After traversing one arc, we always use the shortest path from the end point of the just-traversed arc to the start of the next arc to be traversed. Because these shortest path lengths and the street segment lengths are part of the common input for the arc-routing models, the total route length can be easily computed. For our computational tests, the major small package shipper that we worked with provided a sophisticated state-of-the-art procedure for solving the DARP.

2.2. Probabilistic Arc-Routing Problem

The PARP has the following description. Given common inputs, a set of street segments (arcs) that must be serviced (traversed), and the presence probabilities (probabilities of each segment requiring a visit on a particular day), find the master route of minimum expected length, which starts from the starting location, traverses all the arcs, and returns to the ending location. We now discuss the calculation of the expected length. Our expression is derived from *Bertsimas, Jaillet, and Odoni (1990)*. Without loss of generality, we consider the master route $t = (s, 1, 2... n, e)$, where s is the starting location and e the ending location. Given the presence probability p_i (probability that street segment (arc) i requires a visit on a particular day), we define $q_i = (1 - p_i)$ as the probability that arc i does not require a visit. We use i_0 and i_1 to represent the entry point and the exit point of i, whose length is represented by $l\ (i_0, i_1)$. Also, let $d\ (i_1, j_0)$ be the shortest path from street segment i to street segment j on the street network. We assume that the street segment presence probabilities are independent. This assumption is based on our analysis of real world industrial data where it is often the case that no prevalent correlations among deliveries are found. The expected length of t can be computed with the following expression:

$$E\left[L(\tau)\right] = \sum_{i=1}^{n} d\left(s, i_0\right) p_i \prod_{k=1}^{i-1} q_k \tag{1}$$

$$= \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} d\left(i_1, j_0\right) p_i p_j \prod_{k=i+1}^{j-1} q_k \tag{2}$$

$$= \sum_{i=1}^{n} d\left(i_1, e\right) p_i \prod_{k=i+1}^{n} q_k \tag{3}$$

$$= \sum_{i=1}^{n} l\left(i_0, i_1\right) p_i \tag{4}$$

$$= d\left(s, e\right) \prod_{k-1}^{n} q_k \tag{5}$$

The first component of the equation is the expected cost of using the path from the starting location to a street segment *i*, whereas the third component is that of using the path from a street segment *i* back to the ending location. The second component is the expected cost associated with using the path between street segments *i* and *j*. The fourth component is the expected cost of using street segment *i*. The last component is the cost of traveling from *s* to *e* if no arcs between them are realized. The expected cost of the path is based on the probability that the street segments at both ends of the path are realized, the probability that none of the street segments in between are realized, and the length of the path (i.e., the distance from the exit point of the starting segment to the entry point of the ending segment). The expected cost of using a street segment depends on the probability of it being realized (presence probability) and its distance (length from the entry point to the exit point of the street segment. Although the uncertainty in the street segment presence probability is an important issue, it seems to have been largely ignored by the academic literature on arc routing problems. In our literature search, we have come across only two papers, by *Mohan, Gendreau, and Rousseau (2007, 2008)*, that discuss the issue of uncertainty in presence probability in the context of an Eulerian tour problem.

2.3. Probabilistic Local Search Procedure

For the PARP, we used a solution procedure that adapted local search approaches to our probabilistic context. Our solution heuristic incorporates the presence probabilities using two local search procedures, namely, 1-p-Shift and 2-p-Opt. They act as local improvement techniques for the current solution method, which is essentially an efficient TSP heuristic based on the *Lin-Kernighan (Helsgaun 2000)* algorithm. We now describe 1-p-Shift and 2-p-Opt in the context of arc routing. *Bertsimas and Howell (1993)* provide a clear description of 1-p-Shift and 2-p-Opt, which are designed primarily for the PTSP problem. In the PARP, we consider street segments (arcs) instead of nodes.

2.3.1. Solution Procedure

• Input: a list of street segments (arcs) that requires service, presence probabilities for each street segment on the list, an initial master route stating how to traverse the list of street segments, and an iteration limit. Evaluate the expected length of the initial master route. Initialize the total number of iterations to zero.

• Apply a probabilistic 1-Opt or 2-Opt improvement technique to the current master route. Evaluate the expected length of the new master route. If the expected length of the new master route is less than that of the current master route, the new master route becomes the current master route;

• Increase the total number of iterations by one. If the total number of iterations is less than the iteration limit, go back to Step 1. Otherwise, Stop. Each proposed solution change of Step 1 requires that the expected cost of the new route must be calculated. We identified various techniques to reuse the computation of the expected value for the current solution in order to reduce the computation time. However, in our implementation of the probabilistic local search, we did not utilize such computational reduction techniques. Even with these techniques, the computational burden of these expected value computations is quite substantial. For the PTSP, various researchers (for example, see the survey by *Campbell and Thomas 2007*) have noted the substantial burden in computing expected value computations for large-scale problems greater than about 100 nodes even with some proposed possible remedies.

3. AN EMPIRICAL STUDY

As mentioned earlier, arc-routing models are well suited to model the large number of customer locations (about 1,000 on average for the five service routes used in our computational experiments) that can require service over a significant planning horizon (e.g., for a small-package local routing system over a 30-day planning horizon). Instead of having a very large number of customer nodes in the PTSP, we represent the customer locations as a moderate number of street segments to be covered. Finding the optimal solution to a PTSP with hundreds or thousands of nodes is rather challenging with currently available solution techniques (see *Campbell and Thomas 2007*). In our computational results for the arc-routing models, we will see that one of our suggested solution procedures can effectively handle the arc-routing model derived from a 30-day model. In addition, it may be much easier to estimate presence probabilities for street segments instead of individual addresses whose individual service frequencies may be quite small, even when the corresponding street segment service frequency is relatively large. Another issue is the suitability of using arc- routing models for small-package local service operations. Suppose that an arc representing 1 to 100 Maple Street must be traversed. Assume that there are actually three customer locations, at 1, 38, and 70 Maple Street that must be serviced. The description of all the arc-routing models states that the entire arc or street segment must be traversed. In reality, the three customer locations on Maple Street will be covered if the sub-segment from 1–70 Maple Street is traversed. Thus, the arc-routing model may overestimate the mileage and route that must be used to cover the customer locations. However, this overestimation should not affect our analysis in determining the relative suitability of the arc-routing models in evaluating and identifying the preferred master route.

3.1. Computational Results

We implemented the solution procedures for the PARP described in the previous section in VC++ 6.0 and tested them on a computer with Pentium IV 2 GHz and 1.24 GB RAM. As discussed previously, the major small-package shipping firm that we worked with supplied a sophisticated state-of- the-art procedure for obtaining solutions to the DARP.

These solutions are also used as the initial solutions for PARP. This section discusses the results of the computational tests with these solution procedures and their implications concerning the utility of these two arc-routing models for small-package shipping firm local operations. The next subsection describes two sets of test problems— one is drawn from actual industrial data provided by the major small-package shipping firm, whereas the other is computer generated—used for evaluating the performance of these two solution techniques. The second subsection describes the results of the computational tests and some implications of these results.

We first describe the set of test problems drawn from actual industrial data. We collected data on five local service routes used by a major small-package shipping firm. Each local service route encompasses both commercial and residential areas and required a single service provider to handle the daily work. The service routes were located in three different states. The major small-package shipping firm provided the street segments and their lengths, as well as the shortest path length between any two end points of any two street segments, and between the starting/ending location and the end point of any street segment. All of these lengths are derived from the underlying street network of the service territory associated with each route. For each local service route, we also collected daily customer demand data over a historical study interval consisting of 20–30 days. In other words, we solve for a daily master route based on 20–30 observed (historical) realizations. Table 1 gives some summary statistics for these service routes.

Table: 1 (Service Routes)

Routes	Number of days in historical study interval	Number of unique street segments served during historical study interval
Service Route 1	30	230
Service Route 2	30	228
Service Route 3	20	226
Service Route 4	30	179
Service Route 5	30	160

For each local service route, we derived a corresponding test problem, referred to as Service Routes 1 to 5. The list of street segments that must be traversed corresponded to the list of unique street segments serviced during the historical study interval. The presence probability for a street segment is the ratio of the number of days during the historical study interval when the street segment had at least one customer demand to the number of days in the historical study interval. For the number of days and the series of street segment sets corresponding to the set of street segments that must be traversed during each day, we used the number of days in the historical study interval and the daily set of streets segments that must be traversed.

Next, we create the set of computer-generated test problems, referred to as Problem Sets 1 to 5. Each problem has 200 street segments (including the starting and ending locations) and a 30-day study interval. The street segments are randomly placed on a 50 × 50 square grid. The coordinates for the starting and ending locations are (24.5, 0) and (25.5, 0), respectively. Euclidean distances are used as the lengths of the street segments as well as the shortest path length between any two end points of any two street segments and between the starting/ending location and the end point of any street segment. Each street segment presence probability is randomly selected from a uniform distribution on the interval (0, 1). The daily realized street segment data is generated according to the presence probabilities. For example, if the presence probability for a street segment is 0.5, then we randomly select 15 (=30×0.5) days and create a service request for this segment on each of these 15 days.

3.2. Empirical Evaluation of Master Routes

We used the 5 test problems described in the previous subsection to perform various types of evaluations and comparisons. For each test problem, we obtained two master routes (one master route solution obtained by solving each of the two arc-routing models). We obtained the solutions by using the solution procedures described in the previous section. We performed two types of evaluations using these two arc-routing model solutions. First, we evaluated the two solutions using the total route-length criteria of the DARP. Note that, due to proprietary considerations, we use a normalized cost instead of real mileage. As expected, the DARP master route was the best in terms of the total route length criteria. See Table 2 for this evaluation. Next, we evaluated the two solutions using the expected length criteria of the PARP. Table 3 shows that the PARP solutions are better than the DARP solution using expected length criteria. These two evaluations show that using the standard total route length (DARP) criteria can be misleading in terms of evaluating master routes.

The DARP solution is about 10% better in terms of the standard total length criteria than the other solution. However, in terms of the expected route-length (PARP) criteria, the DARP solution is generally about 2% to 5% worse than the PARP solutions. These results confirm that using the standard deterministic single-period criteria of total route length is not a good way to evaluate the master routes because it does not take into account the daily variation in customer demands.

Table: 2 (Master Route Quality Using DARP objective Function)

Routes	Actual Industrial Data		Computer – Generated Data	
	DARP Solution (Normalized Cost)	PARP Solution (Normalized Cost)	DARP Solution (Normalized Cost)	PARP Solution (Normalized Cost)
Service Route 1	1.000	1.1305	15.8673	17.0117
Service Route 2	1.000	1.1873	16.9527	20.0605
Service Route 3	1.000	1.1651	17.3456	19.0176
Service Route 4	1.000	1.0956	16.2723	19.0053
Service Route 5	1.000	1.1335	18.0013	20.1252

Table: 3 (Master Route Quality Using PARP objective Function)

Routes	Actual Industrial Data		Computer – Generated Data	
	DARP Solution (Normalized Length)	PARP Solution (Normalized Length)	DARP Solution (Normalized Length)	PARP Solution (Normalized Length)
Service Route 1	1.000	0.9753	11.4362	10.9843
Service Route 2	1.000	0.9621	11.7853	11.1463
Service Route 3	1.000	0.9883	11.5546	10.8752
Service Route 4	1.000	0.9785	11.3894	10.6723
Service Route 5	1.000	0.9842	11.9364	11.4627

4. CONCLUSIONS

We have considered the local routing problem for small-package shipping where customer demands vary daily. In this context, node-routing problems such as the PTSP may not be appropriate set of customers served over a multiday time horizon may be quite large. Instead, arc-routing problems where a set of arcs instead of nodes must be traversed offer more tractable decision models. We discussed two types of arc-routing problems and described heuristic solution approaches for two of them. Our computational results with test problems based on actual small-package shipping firm data as well as on computer-generated data confirm that the standard deterministic single-period arc-covering model does not produce the most desirable types of master routes. The multiday and the probabilistic arc-routing problems produce better master routes by taking into account daily customer demand variation via multi-time-period or probabilistic models. Our computational results also show that the multiday model (with a moderate number of days) is much simpler to solve than the probabilistic model because it avoids the burden of expected length calculations required by the probabilistic model. because the In the context of small-package shipping firm planning operations, the deterministic arc-routing problem (DARP) is convenient to use in obtaining a master route because it requires only a limited set of input parameters: the starting and ending locations, a set of arcs (street segments), the length of each street segment, the length of the shortest path between an end point of any segment to an end point of any other segment, and the length of the shortest path from/to the starting/ending location to/from the end point of any street segment, and is a relatively simple model. However, our results indicate that noticeable (ranges from 2% to 5%) improvements can be obtained in master route quality by using a somewhat more complex model such as a multiday or probabilistic arc-routing problem that takes into account the daily variation in customer demand instead of the simpler deterministic and single-period arc-routing problem. We intend to pursue this possible new approach to solving the PTSP and related models as an area of future research. We intend to analyze this new approach and determine the number of days required to be a reasonable approximation to the probabilistic model (i.e., PARP) in terms of the master route produced.

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