International Research Journal of Pure Algebra -4(3), 2014, 437-439
Available online through www.rjpa.info ISSN 2248–9037

LEFT REVERSE DERIVATIONS ON SEMIPRIME RINGS

C. Jaya Subba Reddy*[1] and K. Hemavathi[2]

[1,2]*Department of mathematics, S.V. University, Tirupati – 517502, India.*

(Received on: 18-03-14; Revised & Accepted on: 25-03-14)

ABSTRACT

In this paper some results concerning to left reverse derivations on semi prime rings are presented. A mapping d on a semi prime ring R is a left reverse derivation if and only if it is a central derivation.

Key words: *Prime ring, Semi prime ring, derivation, Reverse derivation, Central derivation.*

INTRODUCTION

Bresar and Vukman [1] have introduced the notion of a reverse derivation. The reverse derivations on semi prime rings have been studied by Samman and Alyamani [2]. Macdonald [3] established some group-theoretic results in terms of inner derivations. Bell and Kappe [4] studied the analogous results for rings in which derivations satisfy certain algebraic conditions. In this paper, we study these results for the rings with left reverse derivations.

PRELIMINARIES

Throughout, R will represent a semi prime ring. We recall that a ring R is called prime if $aRb=0$ implies $a=0$ or $b=0$; and it is called semi prime if $aRa=0$ implies $a=0$. An additive mapping d from R into itself is called a derivation if $d(xy)=d(x)y+xd(y)$ for all $x, y\in R$ and is called a left reverse derivation if $d(xy)=yd(x)+xd(y)$ for all $x, y\in R$. A mapping d from R into itself is called central derivation if $d(x)\in Z$ for all x in R.

MAIN RESULTS

Theorem: 1 A mapping d on a semi prime ring R is a left reverse derivation if and only if it is a central derivation.

Proof: Let R be a semi prime ring and d: R $\rightarrow$ R a mapping on R. It is clear that if d is a central derivation, then d is a left reverse derivation. So, let us suppose that d is a left reverse derivation, then

$d(xy^2) = y^2 d(x) + x d(y^2)$

$= y^2 d(x) + x d(yy)$

$= y^2 d(x) + x(yd(y) + yd(y))$

$d(xy^2) = y^2 d(x) + xy d(y) + xy d(y)$, for all $x, y \in R$ (1)

Also, $d((xy)y) = yd(xy) + xy d(y)$

$= y(yd(x) + x d(y)) + xy d(y)$

$\therefore d(xy^2) = d((xy)y) = y^2 d(x) + yx d(y) + xy d(y)$, for all $x, y \in R$ (2)

Corresponding author: C. Jaya Subba Reddy[1]
[1,2]Department of mathematics, S.V. University, Tirupati – 517502, India.
E-mail: cjsreddysvu@gmail.com

From equ's (1) and (2), we have,

$\Rightarrow y^2d(x) + xy\, d(y) + xy\, d(y) = y^2d(x) + yx\, d(y) + xy\, d(y)$

$\Rightarrow xy\, d(y) = yx\, d(y)$

$\Rightarrow yx\, d(y) - xy\, d(y) = 0$

$\Rightarrow (yx - xy)d(y) = 0$

$\Rightarrow [y, x]d(y) = 0,$ for all $x, y \in R$ (3)

We replace x by zx in equ. (3), and using (3) again, we get,

$\Rightarrow [y, zx]\, d(y) = 0$

$\Rightarrow (z[y, x] + [y, z]x)d(y) = 0$

$\Rightarrow z[y, x]d(y) + [y, z]x\, d(y) = 0$

$\Rightarrow [y, z]x\, d(y) = 0$

By interchanging x and z in the above equation, we get,

$\Rightarrow [y, x]z\, d(y) = 0,$ for all x, y, z in R (4)

On the other hand, a linearization of equation (3) leads to,

$\Rightarrow [y + u, x]d(y +u) = 0$

$\Rightarrow ((y + u)x - x(y + u))(d(y) + d(u)) = 0$

$\Rightarrow (yx + ux - xy - xu)(d(y) + d(u)) = 0$

$\Rightarrow (yx - xy + ux - xu)(d(y) + d(u)) = 0$

$\Rightarrow ([y, x] + [u, x])(d(y) + d(u)) = 0$

$\Rightarrow [y, x]d(y) + [u, x]d(y) + [y, x]d(u) + [u, x]d(u) = 0$

$\Rightarrow [y, x]d(u) + [u, x]d(y) = 0$

$\Rightarrow [u, x]d(y) = - [y, x]d(u)$ (5)

$\Rightarrow [u, x]d(y) = [x, y]d(u)$

We replace z by d(u)z[u, x] in equation (4), then we get,

$\Rightarrow [y, x]\, d(u)z[u, x]d(y) = 0$

$\Rightarrow - [y, x]\, d(u)z[y, x]d(u) = 0$

$\Rightarrow [y, x]\, d(u)z[y, x]d(u) = 0$ (6)

Since R is semi prime, by equation (6) we get, $[y, x]d(u) = 0$, for all $x, y, u \in R$. By [5], $d(u) \in z$ for all $u \in R$. Hence, $d(xy) = yd(x) + xd(y)$. This shows that d is a left reverse derivation on R which maps R into its center.

As a consequence, we get the following

Corollary: 1 Let R be a prime ring. If R admits a non- zero left reverse derivation d, then R is commutative.

REFERENCES

[1] M. Bresar and J. Vukman, on some additive mappings in rings with involution, Aequations math., 38(1989), 178-185.

[2] M. Samman, N. Alyamani, Derivations and reverse derivations in semiprime rings, International Mathematical Forum, 2, No. 39(2007), 1895-1902.

[3] Macdonald. I.D, some groups elements defined by commutators, Math. Scientist, 4(1979), 129-131.

[4] H. E. Bell, and Kappe. L.C, Rings in which derivations satisfy certain algebraic conditions, Acta. Math. Hungar. 53(1989), 339-346.

[5] Herstein. I. N, Rings with involution, Univ. of Chicago press, Chicago, 1976.

Source of Support: Nil, Conflict of interest: None Declared

