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CONTRACTIVE CONDITION OF INTEGRAL TYPE FOR FIXED POINTS OF WEAKLY COMPATIBLE MAPS

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ABSTRACT

In this paper we prove Contractive Condition of Integral Type for Fixed Points of Weakly Compatible Maps.

Key Words: *Fixed Point, Weakly Compatible mapping, Contractive Condition.*

1. INTRODUCTION

El Naschie was Motivated by the potential applicability of fuzzy topology to quantum particle physics particularly in connection with both string and $e^{(\infty)}$, Park introduced and discussed in [21] a notion of intuitionistic fuzzy metric spaces which is based on the idea of intuitionistic fuzzy sets due to Atanassov [2] and the concept of fuzzy metric space given by George and Veeramani [11]. Actually, Park's notion is useful in modelling some phenomena where it is necessary to study the relationship between two probability functions.

Alaca *et al.* [1] using the idea of intuitionistic fuzzy sets, they defined the notion of intuitionistic fuzzy metric space as Park [21] with the help of continuous t-norms and continuous t-conorms as a generalization of fuzzy metric space due to Kramosil and Michalek [15]. Further, they introduced the notion of Cauchy sequences in intuitionistic fuzzy metric spaces and proved the well known fixed point theorems of Banach [3] and Edelstein [5] extended to intuitionistic fuzzy metric spaces with the help of Grabiec [10]. Turkoglu *et al.* [25] introduced the concept of compatible maps and compatible maps of types (α) and (β) in intuitionistic fuzzy metric spaces and gave some relations between the concepts of compatible maps and compatible maps of types (α) and (β). Sharma and Tilwankar [24] and Kutukcu [18] proved fixed point theorems for multivalued mappings in intuitionistic fuzzy metric spaces.

Several authors [12], [13], [15], [23] proved some fixed point theorems for various generalizations of contraction mappings in probabilistic and fuzzy metric space. Branciari [4] obtained a fixed point theorem for a single mapping satisfying an analogue of Banach's contraction principle for an integral type inequality. Sedghi *et al.* [22] established a common fixed point theorem for weakly compatible mappings in intuitionistic fuzzy metric space satisfying a contractive condition of integral type. Muralisankar *et al.* [20] proved a common fixed point theorem in an intuitionistic fuzzy metric space for pointwise R-weakly commuting mappings using contractive condition of integral type and established a situation in which a collection of maps has a fixed point which is a point of discontinuity.

2. PRELIMINARIES

Definition: 2.1 ([23]) A binary operation *: [0; 1] x [0, 1] → [0, 1] is continuous t-norm if * is satisfying the following conditions:
(i) * is commutative and associative,
(ii) * is continuous,
(iii) a* 1 = a for all a∈[0, 1],
(iv) a* b ≤ c * d whenever a ≤ c and b ≤ d, a, b, c, d ∈ [0, 1].

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Definition: 2.2 ([23]) A binary operation *: [0, 1] x [0, 1] → [0, 1] is continuous t-conorm if ◊ is satisfying the following conditions:
(i) ◊ is commutative and associative,
(ii) ◊ is continuous,
(iii) a ◊ 0 = a for all a ∈ [0, 1],
(iv) a ◊ b ≤ c ◊ d whenever a ≤ c and b ≤ d, a, b, c, d ∈ [0, 1].

Definition: 2.3 ([1]) A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric spaces if X is an arbitrary set, * is a continuous t-norm, ◊ is a continuous t-conorm and M , N are fuzzy sets on X^2 x [0, 1) satisfying the following conditions for all x, y, z ∈ X and t, s > 0,
(i) M (x, y, t) + N (x; y; t) ≤ 1,
(ii) M (x, y, 0) = 0,
(iii) M (x, y, t) = 1 for all t > 0 if and only if x = y,
(iv) M (x, y, t) = M (y, x, t),
(v) M (x, y, t) * M (y, z, s) ≤ M (x, z, t + s),
(vi) M (x, y, .) : [0, ∞) → [0, 1] is left continuous,
(vii) $\lim_{t\to\infty}$ M (x, y, t) = 1 for all x, y in X,
(viii) N (x, y, 0) = 1,
(ix) N (x, y, t) = 0 for all t > 0 if and only if x = y,
(x) N (x, y, t) = N (y, x, t),
(xi) N (x, y, t) ◊ N (y, z, s) ≥ N (x, z, t + s),
(xii) N (x, y, ·) : [0, ∞) → [0, 1] is right continuous,
(xiii) $\lim_{t\to\infty}$N (x, y, t) = 0 for all x, y ∈ X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M (x; y; t) and N (x; y; t) denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark: 1 Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1-M, *, ◊) such that t-norm * and t-conorm ◊ are associated, i.e., x ◊ y = 1-((1-x) * (1-y)) for all x, y ∈ X.

Example: 1 Let (X, d) be a metric space. Define t-norm a*b = min{a, b} and t-conorm a ◊ b = maxa, b} and for all x, y ∈ X and t >0,

$$(2a)\quad M_d(x,y,t)=\frac{t}{t+d(x,y)},\quad N_d(x,y,t)=\frac{d(x,y)}{t+d(x,y)},$$

Then (X, M, N, *, ◊) is an intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric (M, N) induced by the metric d the standard intuitionistic fuzzy metric. On the other hand, note that there exists no metric on X satisfying (2a).

Remark: 2 In intuitionistic fuzzy metric space (X, M, N, *, ◊), M (x, y, ·) is non-decreasing and N (x, y, ·) is non-increasing for all x, y ∈ X.

Definition: 2.4 ([1]) Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then
(i) A sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X (denoted by $\lim_{n\to\infty} x_n = x$) if, for all t >0,

$$\lim_{n\to\infty} M(x_n, x, t) = 1,\ \lim_{n\to\infty} N(x_n, x, t) = 0.$$

(ii) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$$\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1,\ \lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$$

Remark: 3 Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

Definition: 2.5 ([1]) An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be complete if and only if every Cauchy sequence in X is convergent.

Lemma: 1 ([1]) Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and $\{y_n\}$ be sequence in X. If there exists a number k ∈ (0, 1) such that

$$M (y_{n+2}, y_{n+1}, kt) \geq M (y_{n+1}, yn, t),\ N (y_{n+2}, y_{n+1}, kt) \leq N (y_{n+1}, yn, t)$$

for all t >0 and n = 1, 2, ………, then $\{y_n\}$ is a Cauchy sequence in X.

Lemma: 2 ([1]) Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space and for all x, y $\in$ X, t > 0 and if for a number k $\in$ (0, 1),

M (x, y, kt) ≥ M (x, y, t) and N (x, y, kt) ≤ N (x, y, t), then x = y.

Definition: 6 ([14]) Two self mappings S and T are said to be weakly compatible if they commute at their coincidence points; i.e., if T u = Su for some u $\in$ X, then T Su = ST u.

3 MAIN RESULTS

Theorem 3.1: Let (X.M.N.*.◊) be an Intuitionistic fuzzy Metric Space with continuous t-norm * and continuous t-conorm ◊ defined by t * t ≥ t and (1 – t) ◊ (1 – t) ≤ (1 – t) for all t $\in$ [0, 1]. Let A, B, S, T, P and Q be mappings from x into itself

(a) $A(x) \subset TP(x)$ and $B(x) \subset SQ(x)$

(b) there exists a constant k $\in$ (0, 1) such that

$$\int_0^{M(Ax,By,kt)} \phi(t)\,dt \le \int_0^{m(x,y,t)} \phi(t)\,dt \text{ and } \int_0^{N(Ax,By,kt)} \phi(t)\,dt \ge \int_0^{n(x,y,t)} \phi(t)\,dt$$

Where φ: $R^+ \to R^+$ is a Lebesque – Integrable mapping which is summable non negative and such that

$$\int_0^{\epsilon} \varphi(t)dt > 0 \text{ for each } \varepsilon > 0$$

Where
$$m(x,y,t) = \min\{\alpha_1[\frac{M(TPy,By,t)M(SQx,TPy)}{M(TPx,Ax)+M(By,TPx)}]$$
$$+\alpha_2[M(Ax,TPy,t)+M(SQx,Bx,t)+M(Ay,SQy,t)$$
$$+\alpha_3[M(TPx,Bx,t)+M(SQy,TPx,t)+M(By,TPy,t)]+\alpha_4[M(SQx,TPy,t)+M(TPx,By,t)]\} \tag{3.1}$$

and
$$n(x,y,t) = \max\{\alpha_1[\frac{N(TPy,By,t)N(SQx,TPy)}{N(TPx,Ax)+N(By,TPx)}]$$
$$+\alpha_2[N(Ax,TPy,t)+N(SQx,Bx,t)+N(Ay,SQy,t)$$
$$+\alpha_3[N(TPx,Bx,t)+N(SQy,TPx,t)+N(By,TPy,t)]+\alpha_4[N(SQx,TPy,t)+N(TPx,By,t)]\} \tag{3.2}$$

For all x, y ε X and t > 0

(c) if of A(X), TP(X), Q(X) or B(X) is a complete subspace of x.

Then (i) A and SQ have a coincidence points and
(ii) B and TP have a coincidence points

Further if
(d) TP = PT, BP = PB, BT = TP, AS = SQ, SQ = QX and
(e) the paid {A,SQ} is weakly compatible and {B, TP is weakly compatible}

Then
III A, B, S, T, P and Q have a unique common fixed point in X.

Proof: By (a) since $A(X) \subset TP(X)$ for any point $x_0 \in X$. There exists at point $x_1 \in X$ such that Ax_0= TPx_1. Since $B(X) \subset SQ(X)$ for this point x_1 we can choose a point $x_2 \in X$ such that $Bx_1 = SQx_2$ and so on. Inductively, we can define a sequence $\{y_n\}$ in x such that for n = 0.1…

$y_{2n} = Ax_{2n+1} = TPx_{2n+2}$

$y_{2n+1} = Bx_{2n+1} = SQx_{2n+2}$ for n = 1, 2…

By (0) for all t > 0

Proof: Let x_0 be an arbitrary point of X. We define

$Ax_{2n+1} = y_{2n+2}$, $Tx_n = y_{2n}$
$Bx_{2n} = y_{2n+1}$, $Sx_{2n+1} = y_{2n+1}$, n = 1, 2 …

By putting x = x_{2n} and y = x_{2n+1} in (3.1), we write

$$\int_0^{M(Ax_{2n},Bx_{2n+1},t)}\varphi(t)dt \le \int_0^{\alpha_1\left[\frac{M(Tx_{2n+1},Bx_{2n+1})M(Sx_{2n},Tx_{2n+1})}{M(Tx_{2n},Ax_{2n})+M(Bx_{2n+1},Tx_{2n})}\right]+\alpha_2[M(Ax_{2n},Bx_{2n+1})+M(Sx_{2n},Bx_{2n})+M(Ax_{2n+1},Sx_{2n+1})] +\alpha_3[M(Tx_{2n},Bx_{2n})+M(Sx_{2n+1},Tx_{2n})+M(Bx_{2n+1},Tx_{2n+1})] +\alpha_4[M(Sx_{2n},Tx_{2n+1})+M(Tx_{2n},Bx_{2n+1})]}\varphi(t)dt$$

$$\int_0^{\alpha_1\left[\frac{M(y_{2n+1},y_{2n+2})M(y_{2n},y_{2n+1})}{M(y_{2n},y_{2n+1})+M(y_{2n+1},y_{2n})}\right]+\alpha_2[M(y_{2n+1},y_{2n})+M(y_{2n},y_{2n+1})+M(y_{2n+2},y_{2n+1})]+\alpha_3[M(y_{2n},y_{2n+1}) +M(y_{2n+1},y_{2n})+M(y_{2n+1},y_{2n+1})]+\alpha_4[M(y_{2n},y_{2n+1})+M(y_{2n},y_{2n+2})]}\phi(t)\,dt$$

$$\int_0^{N(Ax_{2n},Bx_{2n+1},t)}\varphi(t)dt \ge \int_0^{\alpha_1\left[\frac{N(Tx_{2n+1},Bx_{2n+1})N(Sx_{2n},Tx_{2n+1})}{N(Tx_{2n},Ax_{2n})+N(Bx_{2n+1},Tx_{2n})}\right]+\alpha_2[N(Ax_{2n},Bx_{2n+1})+N(Sx_{2n},Bx_{2n}) +N(Ax_{2n+1},Sx_{2n+1})]+\alpha_3[N(Tx_{2n},Bx_{2n})+N(Sx_{2n+1},Tx_{2n})+N(Bx_{2n+1},Tx_{2n+1})]+\alpha_4[N(Sx_{2n},Tx_{2n+1})+N(Tx_{2n},Bx_{2n+1})]}\varphi(t)dt$$

$$\int_0^{\alpha_1\left[\frac{N(y_{2n+1},y_{2n+2})N(y_{2n},y_{2n+1})}{N(y_{2n},y_{2n+1})+N(y_{2n+1},y_{2n})}\right]+\alpha_2[N(y_{2n+1},y_{2n}) +N(y_{2n},y_{2n+1})+N(y_{2n+2},y_{2n+1})]+\alpha_3[N(y_{2n},y_{2n+1})+N(y_{2n+1},y_{2n})+N(y_{2n+1},y_{2n+1})] +\alpha_4[N(y_{2n},y_{2n+1})+N(y_{2n},y_{2n+2})]}\varphi(t)dt$$

$$\int_0^{M(y_{2n+1},y_{2n+2},t)}\phi(t)\,dt \le \int_0^{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)M(y_{2n},y_{2n+1})+(\alpha_2+\alpha_3+\alpha_4)M(y_{2n+1},y_{2n+2})}\phi(t)\,dt$$

$$\int_0^{M(y_{2n+1},y_{2n+2})}\phi(t)\,dt \le \int_0^{\frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}M(y_{2n},y_{2n+1})}\phi(t)\,dt$$

$$\int_0^{N(y_{2n+1},y_{2n+2})}\phi(t)\,dt \ge \int_0^{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)N(y_{2n},y_{2n+1})+(\alpha_2+\alpha_3+\alpha_4)N(y_{2n+1},y_{2n+2})}\phi(t)\,dt$$

$$\int_0^{N(y_{2n+1},y_{2n+2})}\phi(t)\,dt \ge \int_0^{\frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}N(y_{2n},y_{2n+1})}\phi(t)\,dt$$

Putting $h=\frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}$ we find h < 1, since $\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 < 1$. Hence

$$M(y_{2n+1},y_{2n+2}) \le hM(y_{2n},y_{2n+1})$$

$$N(y_{2n+1},y_{2n+2}) \ge hN(y_{2n},y_{2n+1})$$

Similarly by putting x = x_{2n-1} and y = x_{2n} in (3.1), we have

$$\int_0^{M(Ax_{2n-1},Bx_{2n})}\phi(t)\,dt \le \int_0^{\alpha_1\left[\frac{M(Tx_{2n},Bx_{2n})M(Sx_{2n-1},Tx_{2n})}{M(Tx_{2n-1},Ax_{2n-1})+M(Bx_{2n},Tx_{2n-1})}\right]+\alpha_2[M(Ax_{2n-1},Tx_{2n-1}) +M(Sx_{2n-1},Bx_{2n-1})+M(Ax_{2n},Sx_{2n})]+\alpha_3[M(Tx_{2n-1},Bx_{2n-1})+M(Sx_{2n},Tx_{2n-1})+M(Bx_{2n},Tx_{2n})] +\alpha_4[M(Sx_{2n-1},Tx_{2n})+M(Tx_{2n-1},Bx_{2n})]}\varphi(t)dt$$

$$=\int_0^{\alpha_1\left[\frac{M(y_{2n},y_{2n+1})M(y_{2n-1},y_{2n})}{M(y_{2n-1},y_{2n})+M(y_{2n+1},y_{2n-1})}\right]+\alpha_2[M(y_{2n},y_{2n-1})+M(y_{2n-1},y_{2n})+M(y_{2n+1},y_{2n})]+\alpha_3[M(y_{2n-1},y_{2n}) +M(y_{2n},y_{2n-1})+M(y_{2n+1},y_{2n})]+\alpha_4[M(y_{2n-1},y_{2n})+M(y_{2n-1},y_{2n+1})]}$$

$$\int_0^{N(Ax_{2n-1},Bx_{2n})}\phi(t)\,dt \ge \int_0^{\alpha_1\left[\frac{N(Tx_{2n},Bx_{2n})N(Sx_{2n-1},Tx_{2n})}{N(Tx_{2n-1},Ax_{2n-1})+N(Bx_{2n},Tx_{2n-1})}\right]+\alpha_2[N(Ax_{2n-1},Tx_{2n-1}) +N(Sx_{2n-1},Bx_{2n-1})+N(Ax_{2n},Sx_{2n})]+\alpha_3[N(Tx_{2n-1},Bx_{2n-1})+N(Sx_{2n},Tx_{2n-1})+N(Bx_{2n},Tx_{2n})] +\alpha_4[N(Sx_{2n-1},Tx_{2n})+N(Tx_{2n-1},Bx_{2n})]}\varphi(t)dt$$

$$= \int_0^{\alpha_1\left[\frac{N(y_{2n},y_{2n+1})N(y_{2n-1},y_{2n})}{N(y_{2n-1},y_{2n})+N(y_{2n+1},y_{2n-1})}\right]+\alpha_2[N(y_{2n},y_{2n-1})+N(y_{2n-1},y_{2n})+N(y_{2n+1},y_{2n})]+\alpha_3[N(y_{2n-1},y_{2n})+N(y_{2n},y_{2n-1})+N(y_{2n+1},y_{2n})]+\alpha_4[N(y_{2n-1},y_{2n})+N(y_{2n-1},y_{2n+1})]} \varphi(t)dt$$

$$\int_0^{M(y_{2n},y_{2n+1})} \varphi(t)dt \le \int_0^{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)M(y_{2n-1},y_{2n})+(\alpha_2+\alpha_3+\alpha_4)M(y_{2n},y_{2n+1})} \le \int_0^{\frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}M(y_{2n-1},y_{2n})} \phi(t)\,dt$$

$$\int_0^{N(y_{2n},y_{2n+1})} \varphi(t)dt \ge \int_0^{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)N(y_{2n-1},y_{2n})+(\alpha_2+\alpha_3+\alpha_4)N(y_{2n},y_{2n+1})} \le \int_0^{\frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}N(y_{2n-1},y_{2n}.t)} \varphi(t)dt$$

$$\int_0^{M(y_{2n},y_{2n+1})} \phi(t)\,dt \le h\int_0^{M(y_{2n-1},y_{2n})} \phi(t)\,dt \text{, as } h = \frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}$$

$$\int_0^{N(y_{2n},y_{2n+1})} \phi(t)\,dt \ge h\int_0^{N(y_{2n-1},y_{2n})} \phi(t)\,dt \text{, as } h = \frac{(\alpha_1+2\alpha_2+2\alpha_3+2\alpha_4)}{(1-\alpha_2-\alpha_3-\alpha_4)}$$

We find h < 1, since $(\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4) < 1$. Proceeding in this way, we have

$$\int_0^{M(y_{2n},y_{2n+1})} \phi(t)\,dt \le h^{2n-1}\int_0^{M(y_0,y_1)} \phi(t)\,dt$$

$$\int_0^{N(y_{2n},y_{2n+1})} \phi(t)\,dt \ge h^{2n-1}\int_0^{N(y_0,y_1)} \phi(t)\,dt$$

By routine calculations the following inequalities hold for k > n

$$\begin{aligned} M(y_n, y_{n+k}) &\le \sum_{i=1}^{k} M(y_{n+i-1}, y_{n+i}) \\ &\le \sum_{i=1}^{k} h^{n+i-1} M(y_0, y_1) \\ &\le \frac{h^n}{1-h} M(y_0, y_1) \to 0 \quad \text{as } n \to \infty \end{aligned}$$

$$\begin{aligned} N(y_n, y_{n+k}) &\ge \sum_{i=1}^{k} N(y_{n+i-1}, y_{n+i}) \\ &\ge \sum_{i=1}^{k} h^{n+i-1} N(y_0, y_1) \\ &\ge \frac{h^n}{1-h} N(y_0, y_1) \to 0 \quad \text{as } n \to \infty \end{aligned}$$

Here h < 1. Hence $\{y_n\}$ is a Cauchy sequence and by completeness of X we see that $\{y_n\}$ is converges to a point z in X. Since $\{y_n\}$ is a Cauchy sequence and taking $n \to \infty$, we write

$Ax_{2n} = Tx_{2n+1} \to Z$ and $Bx_{2n+1} = Sx_{2n+2} \to Z$

Now, suppose A is continuous. Since A and S are compatible mappings of type (A), then

AAx_{2n} and $Sax_{2n} \to Az$ as $n \to \infty$

Now putting x = Ax_{2n} and y = x_{2n+1} in (3.1), we write

$$\int_0^{M(AAx_{2n},Bx_{2n+1})}\phi(t)\,dt \le \int_0^{\alpha_1\left[\frac{M(Tx_{2n+1},Bx_{2n+1})M(SAx_{2n},Tx_{2n+1})}{M(TTx_{2n+1},AAx_{2n})+M(Bx_{2n+1},TTx_{2n+1})}\right]+\alpha_2[M(AAx_{2n},TTx_{2n+1})+M(SAx_{2n},BTx_{2n+1}) +M(Ax_{2n+1},Sx_{2n+1})+\alpha_3[M(TTx_{2n+1},BTx_{2n+1})+M(Sx_{2n+1},TTx_{2n+1})+M(Bx_{2n+1},Tx_{2n+1})] +\alpha_4[M(SAx_{2n},Tx_{2n+1})+M(TT_{2n+1},Bx_{2n+1})]}\varphi(t)dt$$

$$\int_0^{N(AAx_{2n},Bx_{2n+1})}\phi(t)\,dt \ge \int_0^{\alpha_1\left[\frac{N(Tx_{2n+1},Bx_{2n+1})N(SAx_{2n},Tx_{2n+1})}{N(TTx_{2n+1},AAx_{2n})+N(Bx_{2n+1},TTx_{2n+1})}\right]+\alpha_2[N(AAx_{2n},TTx_{2n+1})+N(SAx_{2n},BTx_{2n+1}) +N(Ax_{2n+1},Sx_{2n+1})+\alpha_3[N(TTx_{2n+1},BTx_{2n+1})+N(Sx_{2n+1},TTx_{2n+1})+N(Bx_{2n+1},Tx_{2n+1})] +\alpha_4[N(SAx_{2n},Tx_{2n+1})+N(TT_{2n+1},Bx_{2n+1})]}\varphi(t)dt$$

Taking the limit n → ∞, we write

$$M(Az, z) \le \alpha_2 M(Az, z),\ N(Az, z) \ge \alpha_2 N(Az, z)$$

Giving a contradiction $\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 < 1$.

Therefore Az = z.

Similarly by putting x = Sx_{2n} and y = x_{2n+1} in (3.1), we write

$$\int_0^{M(ASx_{2n},Bx_{2n+1})}\phi(t)\,dt \le \int_0^{\alpha_1\left[\frac{M(Tx_{2n+1},Bx_{2n+1})M(SSx_{2n},Tx_{2n+1})}{M(TBx_{2n-1},ASx_{2n})+M(Bx_{2n+1},TBx_{2n-1})}\right]+\alpha_2[M(ASx_{2n},TBx_{2n+1})+M(SSx_{2n},BBx_{2n-1}) +M(Ax_{2n+1},Sx_{2n+1})+\alpha_3[M(TBx_{2n-1},BBx_{2n-1})+M(Sx_{2n+1},TBx_{2n-1})+M(Bx_{2n+1},Tx_{2n+1})] +\alpha_4[M(SSx_{2n},Tx_{2n+1})+M(TB_{2n-1},Bx_{2n+1})]}\varphi(t)dt$$

$$\int_0^{N(ASx_{2n},Bx_{2n+1})}\phi(t)\,dt \ge \int_0^{\alpha_1\left[\frac{N(Tx_{2n+1},Bx_{2n+1})N(SSx_{2n},Tx_{2n+1})}{N(TBx_{2n-1},ASx_{2n})+N(Bx_{2n+1},TBx_{2n-1})}\right]+\alpha_2[N(ASx_{2n},TBx_{2n+1})+N(SSx_{2n},BBx_{2n-1}) +N(Ax_{2n+1},Sx_{2n+1})+\alpha_3[N(TBx_{2n-1},BBx_{2n-1})+N(Sx_{2n+1},TBx_{2n-1})+N(Bx_{2n+1},Tx_{2n+1})] +\alpha_4[N(SSx_{2n},Tx_{2n+1})+N(TB_{2n-1},Bx_{2n+1})]}\varphi(t)dt$$

Taking the limit n → ∞, we write

$$M(Sz, z) \le (\alpha_2 + \alpha_4)\, M(Sz, z),\ N(Sz, z) \ge (\alpha_2 + \alpha_4)\, N(Sz, z)$$

Giving a contradiction as $\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 < 1$, therefore Sz = z.

Similarly Bz = Tz = z. Thus z is a common fixed point of A, B, S and T.

For uniqueness let z and w(z ≠ w) be two fixed points in X such that

Az = Bz = Sz = Tz = z and Aw = Bw = Sw = Tw = w,

Then by (3.1), we have

$$\int_0^{M(Az,Bw)}\phi(t)\,dt \le \int_0^{\alpha_1\left[\frac{M(Tw,Bw)M(Sz,Tw)}{M(Tz,Az)+M(Bw,Tz)}\right]+\alpha_2[M(Az,Tz)+M(Sz,Bz)+M(Aw,Sw)+\alpha_3[M(Tz,Bz)+M(Sw,Tz) +M(Bw,Tw)]+\alpha_4[M(Sz,Tw)+M(Tz,Bw)]}\varphi(t)dt$$

$$M(z,w) \le (\alpha_3 + 2\alpha_4)\, M(z,w)$$

$$\int_0^{N(Az,Bw)}\varphi(t)dt \ge \int_0^{\alpha_1\left[\frac{N(Tw,Bw)N(Sz,Tw)}{N(Tz,Az)+N(Bw,Tz)}\right]+\alpha_2[N(Az,Tz)+N(Sz,Bz)+N(Aw,Sw)+\alpha_3[N(Tz,Bz)+N(Sw,Tz) +N(Bw,Tw)]+\alpha_4[N(Sz,Tw)+N(Tz,Bw)]}\varphi(t)dt$$

$$N(z,w) \ge (\alpha_3 + 2\alpha_4)\, N(z,w)$$

Which is a contradiction, since $\alpha_1 + 2\alpha_2 + 2\alpha_3 + 2\alpha_4 < 1$. Hence z = w.

This implies the uniqueness of common fixed point of A, B, S and T.

Theorem: 3.2 Let A, B, S and T be four mappings of complete metric space X into itself and satisfying (3.2), (3.3), (1.4)and

$$\int_0^{M(Ax,By)} \phi(t)\,dt \le \int_0^{\alpha_1\left[\frac{M(Tx,B^2y)M(Ty,Sx)}{M(Sx,A^2y)}\right]+\alpha_2[M(Tx,Ax)+M(Ty,By)+M(Ax,Sx)]+\alpha_3[M(Tx,By)+M(Ty,Sx)+M(Ty,Bx)] +M(Bw,Tw)]+\alpha_4[M(Sz,Tw)+M(Tz,Bw)]+\alpha_4 M(Tx,Ty)+M(Tx,Bx)} \varphi(t)dt$$

$$\int_0^{N(Ax,By)} \phi(t)\,dt \ge \int_0^{\alpha_1\left[\frac{N(Tx,B^2y)N(Ty,Sx)}{N(Sx,A^2y)}\right]+\alpha_2[N(Tx,Ax)+N(Ty,By) +N(Ax,Sx)]+\alpha_3[N(Tx,By)+N(Ty,Sx)+N(Ty,Bx)]+N(Bw,Tw)+\alpha_4[N(Sz,Tw)+N(Tz,Bw)]} \varphi(t)dt$$

Theorem 3.2 can be proved in the similar manner as Theorem 3.1.

Conclusion: This paper is to present some common fixed point theorems by using contractive condition of integral type for class of weakly compatible maps in noncomplete intuitionistic fuzzy metric spaces, without taking any continuous mapping.

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