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HEXAGONAL SGRAFITTO AUTOMATA

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ABSTRACT

In this paper we come across hexagonal sgrafitto automata. The induced properties on picture languages are considered herewith. It also contains discussion on time complexity.

Subject Classification: *Applied Mathematics.*

Key Words: *Hexagonal Sgrafitto Automaton, Time Complexity, Determinism, Finite state Automaton.*

1. INTRODUCTION

There has been works related to picture languages in the last three decades. Many different mechanisms were considered. In [3] the notion of local picture languages using tiling system were discussed and projection of local picture languages. In [6] the hexagonal tiling system was introduced and defines local as well as the recognizable hexagonal picture languages.

Among several extensions to the two dimensions case of the well-known notion of recognizability it can be defined by other equivalent ways as for example by online tessellation automata [5]. In [11] hexagonal tessellation automata and 6NFA and 6DFA and their relation with hexagonal picture languages are discussed. A Wang hexagonal automaton combines features of both online tessellation acceptors computation assigns states to each hexagonal picture positions in 6 way automata. The input head visits the hexagonal picture moving from one pixel to an adjacent one according to some scanning strategy.

The choice of scanning strategy is the central issue. In [12] we introduce a polite scanning strategy that sort all positions on the picture and visits each of them exactly once in such a way that the next position to visit is always adjacent to the previous one and opens only on this information with neighboring positions already visited.

In this paper a new model called Hexagonal Sgrafitto Automata is introduced. The studies have been done on induced properties on picture languages. Their relation on time complexity and order is discussed. The main part is attempted to show that the language generated by a Hexagonal Sgrafitto Automata is same as the language generated by finite state automata. The deterministic cases are also discussed.

2. PRELIMINARIES

Here we recall the notions of hexagonal picture and hexagonal picture languages [6]. Let Σ be a finite alphabet of symbols. A Hexagonal picture p over Σ is a hexagonal array of symbols of Σ and the set of all hexagonal pictures over Σ is denoted byΣ^{**H}.

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Example: 2.1 A Hexagonal picture over the alphabet {a, b, c, d} is shown in figure.

```
      a     a     a
   b     b     a     b
c     a     b     d     c
   d     a     a     d
      c     c     c
```

For $l, m, n \geq 1$, $\Sigma^{l, m, n}$ denote the set of hexagonal pictures of size (l, m, n) refer [10].

Definition: 2.1 Let $p \in \Sigma^{**H}$ when $\# \notin \Sigma$ is added as boundary to picture p we get a bordered version of p say $\hat{p}$. The size of $\hat{p}$ is ($l+1, m+1, n+1$) if the size of p is (l, m, n). That is for every $p \in \Sigma^{l, m, n}$ then $\hat{p} \in \Sigma^{l+1, m+1, n+1}$.

Example: 2.2 A hexagonal picture over the alphabet {a, b, c, d} surrounded by # is shown in figure.

```
         #     #     #     #
      #     a     a     a     #
   #     b     b     a     b     #
#     c     a     b     d     c     #
   #     d     a     a     d     #
      #     c     c     c     #
         #     #     #     #
```

Definition: 2.2 A pixel is an element p (i, j, k) of p refer [9]. We call (i, j, k) as the position of the pixel in p. We can use the term picture domain to refer the set of all possible positions in a picture of size (l, m, n) without considering the borders. That is the set $l \times m \times n$ = {1, 2…l}× {1, 2…m}×{1, 2…n}. Each position has six edges. An edge for each position is identified by the adjacent positions namely U, L, LL, LR, UR, and UL. or N, NE, NW, S, SE, SW respectively.

Definition: 2.3 Let $p \in \Sigma^{**H}$ is a hexagonal picture. The projection by mapping π of a picture p is the picture $p' \in \Sigma^{**H}$ such that $p'(i, j, k) = \pi[p(i, j, k)]$ for all $1 \leq i \leq l$, $1 \leq j \leq m$, $1 \leq k \leq n$, where (l, m, n) are the size of the hexagonal pictures.

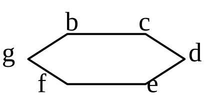
Definition: 2.4 A hexagonal language $L \subseteq \Sigma^{**H}$ is recognizable if there exist a local language L′ over an alphabet Γ and a mapping$\pi: \Gamma \rightarrow \Sigma$ such that $L = \pi(L')$. A hexagonal picture language $L \subseteq \Sigma^{**H}$ is tiling recognizable if there exist a tiling system $T = (\Sigma, \Gamma, \pi, \theta)$ such that $L = \pi[L(\theta)]$.

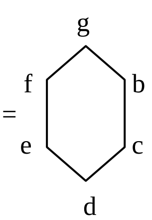
Definition: 2.5 A tiling system T is a quadruple $(\Sigma, \Gamma, \pi, \theta)$ where Σ and I are two finite set of symbols$\pi: \Gamma \rightarrow I$ is a projection and θ is the set of tiles over the alphabet $I \cup \{\#\}$.

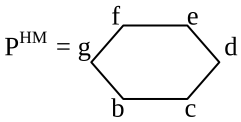
Definition: 2.6 A non deterministic (deterministic) hexagonal on-line tessellation automata referred as 6H0TA (6-HDOTA) is defined by $A = (\Sigma, Q, q_o, F, \delta)$ where

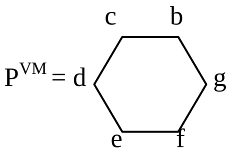
- Σ is the input alphabet.
- Q is a finite set of states.
- $I \subseteq Q(I - \{i\}) \subseteq Q$ is the set of initial states.
- $F \subseteq Q$ is the set of final or accepting states.
- $\delta: Q \times Q \times Q \times \Sigma \rightarrow 2^Q \left(\delta: Q \times Q \times Q \times \Sigma \rightarrow Q\right)$ is the transition function.

In addition to this we introduce clockwise rotation of 120^0say P^R, vertical mirroring P^{VM}, and horizontal mirroring P^{HM} as follows.

P =


P^R =






Remark: 2.1 By using this rotation the vertical mirroring can be expressed as horizontal mirroring and rotation. That is $P^{VM} = ((P^{HM})^R)^R$.

3. HEXAGONAL SGRAFITTO AUTOMATA

H= {E, W, NW, NE, SE, SW, X} is a 7- tuple be the movement of heads in a titled picture. The first 6- tuples denote the directions, east, west, northwest, northeast, southeast, and X denote the zero movement or no movement. That is X $\in$ #. Define a mapping V: S→H such that

$$V(\#) = \begin{cases} \# & i=1, j=1, 1 \le k \le n+1 \\ \# & 1 \le i \le l+1, j=1, k=1 \\ a & 1 \le i \le l, 1 \le j \le m, 1 \le k \le n \\ \# & i=l+1, j=m+1, k=n+1 \end{cases}$$

The above mapping can also be defined in the planes XY, YZ and XZ as follows.

In XY plane	In YZ Plane	In XZ Plane
a (-y) = SE	a(y) = NW	a(z) = E
a (y) = NW	a(-y) = SE	a(-z) = W
a (x) = SW	a(z) = E	a(x) = NE
a (-x) = NE	a(-z) = W	a(-x) = SW

Definition: 3.1 A 3-tape Automaton, 3-TA is defined as follows: A= (Q, ∑, Γ, δ, q_0, Q_F) where,
Q- is the finite set of states.
∑- is the input symbol.
Γ –is the working alphabet.
δ -Transition relation $\delta : Q\times\Gamma^3 \rightarrow Q\times\Gamma^3\times d^3$ is a transition relation
q_0 - Initial state.
Q_F- Final state

For any pair $(q, a_1, a_2, a_3) \in Q\times\Gamma^3$ each element $[q', (a_1', d_1), (a_2', d_2), (a_3', d_3)] \in \delta(q, a_1, a_2, a_3)$ satisfies
$a_i \in S \Rightarrow d_i = V(a_i)$, and $a_i = a_i'$ for all i =1, 2, 3…
$a_i \notin S \Rightarrow a_i' \notin S$.

Let $p \in \sum^{**H}$ be an input to a bounded 3- TA, then the picture is scanned as follows.
$\hat{P}$ is the bordered picture and q_0 is the initial state from where the head scans it changes to the next position as shown in the figure.

Tape-1 In X- Direction

q_1									

Tape-2 In Y- Direction

	q_2								

Tape-3 In Z- Direction

		q_3							

The head scans the 3 tapes at the same time and read as (a, ε, ε), (ε, b, ε), (ε, ε, c). So we can obtain (a, b, c). Then the machine accepts p if and only if there is a computation starting from the initial configuration and ending in the accepting state.

Definition: 3.2 A time complexity function μ_H is a function t: N×N×N→N defined by t (l, m, n) = max $_{P\in\sum l, m, n}$ t (p). t (p) is the number of steps taken for a longest computation over p.

Definition: 3.3 Hexagonal Sgrafitto Automaton 3HSA is a tuple A= (Q, ∑, Γ, δ, q_o, Q_F, μ) where

- (Q, ∑, Γ, δ, q_o Q_F, μ) is a 3TA.
- μ: Γ→ N is a weight function or scanning function in the domain l×m×n.

The transition function satisfies (q′, a′, b′, c′, d)∈δ (q, a, b, c) for all a ∉ S.

μ (a′) < μ (a)or μ (b′) < μ (b) or μ (c′) < μ (c) for all q, q′∈Q d∈H, a, b, c, a′, b′,c′∈ Γ ∪ S

A is a deterministic 3HSA if the 3TA is deterministic.

Lemma: 3.1 Let A= (Q, ∑, Γ, δ, q_o Q_F, μ) be a 3HSA and let p be anon-empty regular hexagon of size (l, l, l) then the time complexity is equal to the order of the picture.

Proof: Let p be a non-empty picture of size l. While computing over the picture p of size l it passes along z direction towards right. After reaching the end it travels one unit in y direction and scans the picture towards left and one unit in x direction and travels towards right and so on. During this scanning strategy it reduces its weight. Hence at least one of the three consecutive steps decreases weight of some field. There are be 3l (l-1) +1fields in the picture p then the weight can be decreased at most (| Γ | -1) times. Therefore [3l (l-1) +1] (| Γ | -1) can be performed by the scanning strategies. This will be the longest computation over p which is equal to the order of the picture.

Remark: 3.1 For a hexagonal picture of size l, m, n the position visited is [lm+mn+ln-(l+m+n) +1]. The above result can be proved for this result also.

Lemma: 3.2 Let μ_H = (Q, ∑, Γ, δ, q_o Q_F,) be a 3TA. Let i, j, k ∈ N be an integer such that during each computation of M over any picture $\sum^{**H}$ each tape field is visited at most i times along X- axis, j times along Y-axis and k times along Z-axis. Then there is a 3HSA, A such that L (A) = L (μ_H). Moreover μ_H is deterministic, A is also deterministic.

Proof: Define A=(Q,∑,Γ_1,δ_1, q_o Q_F,) where $_1$=∑ ∪Γ×{1,2,…,i}×{1,2,…, j}×{1,2,…,k} and instructions
(q, a, b, c) →{q′,a ′,b ′,c′ , d} from δ is represented in δ_1 by the following set of instructions.
(q, a, b, c) →{q′, a ′, b ′, c′, d}

Along X- Direction
(q (a, i),(b, j),(c, z))→(q′, (a, i+1), (b, j), (c, z), d)

Along Y-Direction
(q (a, i), (b, j),(c, z))→(q′, (a, i), (b, j+1), (c, z), d)

Along Z-Direction
(q (a, i), (b, j), (c, z))→(q′,(a, i), (b, j), (c, z+1), d) for all i=1, 2…l, j=1, 2…m, k=1, 2…n

Finally μ(a) = i μ(b) = j, μ (c) = kfor all a, b, c ∈ ∑.
μ(a, i) = l-1, μ(b, j) = m-1, μ(c, z) = n-1, for all (a, i)∈Γ×(1,2…l), (b, i) ∈ Γ×(1, 2…m), (c, i) ∈ Γ × (1, 2…n)

Therefore it is a 3HSA. Clearly L (A) = L (M). If M is deterministic then δ is a deterministic transition relation. It follows that $\delta_1$is also a deterministic transition relation. Therefore A is also deterministic.

Theorem: 3.1 Let A= (Q,∑, Γ, δ, q $_o$ Q_F, μ) be a one symbol HSA. There is a finite state automaton A_1 such that L (A_1) = L (A).

Proof: Without loss of generality we assume that A moves its head while performing an instruction and it finishes its movement at Q_F. The proof is based on directed multigraph. The edges are labeled by the automaton. Let E be the set of labeled edges in a graph defined by E⊆q×∑×q. Any edge (q_1, a, q_2)∈ E can be written asq_1→ $q_2$along (q_1,a).In initial configuration a 3TA in normal form at any step changes its reading either by a letter of first tape or a letter of second tape or a letter of third tape. A triplet reads all the three tapes if it is accepted. We can describe a successful path using a hexagonal picture as follows. In3TA the edges are as follows: (x, ε, ε), (ε, y, ε), (ε, ε, z).

That is E ⊆ (q×(x, ε, ε) ×q) ∪(q× (ε, y,ε) ×q) ∪(q× (ε, ε, z)×q). Consider hexagonal picture of size (l, m, n). There arise three cases.

Case: 1

For $q_1 \rightarrow q_2$ 3TA goes from (i, j, k) to (i′, j′, k′)

1. k=0 then (i′, j′, k′) = (i+1, j, 0)
2. $k \neq 0$, i =0, $j \neq m$ then (i′, j′, k′) = (0, j+1, k-1)
3. $k \neq 0$, i =0, j = m then (i′, j′, k′) = (1, j, k)
4. $k \neq 0$, $I \neq 0$, then (i′, j′, k′) = (i+1, j, k)

Case: 2

1. j = m then (i′, j′, k′) = (i+1, m, k+1)
2. $j \neq m$ then (i′, j′, k′) = (i, j+1, k)

Case: 3

1. i =0 then (i′, j′, k′) = (0, j, k+1)
2. $i \neq 0$, k=0, $j \neq m$ then (i′, j′, k′) = (i-1, j+1, 0)
3. $i \neq 0$, $k \neq 0$, j = m then (i′, j′, k′) = (i, m, k+1)
4. $k \neq 0$, $I \neq 0$, then (i′, j′, k′) = (i, j, k+1)

By using these cases any string in A can be formed as follows. Consider $x^3y^3z^3$. Let the states be q_0, q_1, q_2, q_3. Then E= $\{(q_0, (a, \varepsilon, \varepsilon), q_1), (q_1, (\varepsilon, b, \varepsilon), q_2), (q_2, (\varepsilon, \varepsilon, c), q_3), (q_3, (a, \varepsilon, \varepsilon), q_1)\}$. The language for $x^3y^3z^3$ is

$(q_0, (a, \varepsilon, \varepsilon), q_1) \rightarrow (q_1, (\varepsilon, b, \varepsilon), q_2) \rightarrow (q_2, (\varepsilon, \varepsilon, c), q_3)$

$\rightarrow (q_3, (a, \varepsilon, \varepsilon), q_1) \rightarrow (q_1, (\varepsilon, b, \varepsilon), q_2) \rightarrow (q_2, (\varepsilon, \varepsilon, c), q_3)$

$\rightarrow (q_3, (a, \varepsilon, \varepsilon), q_1) \rightarrow (q_1, (\varepsilon, b, \varepsilon), q_2) \rightarrow (q_2, (\varepsilon, \varepsilon, c), q_3)$

The graph representation is given below

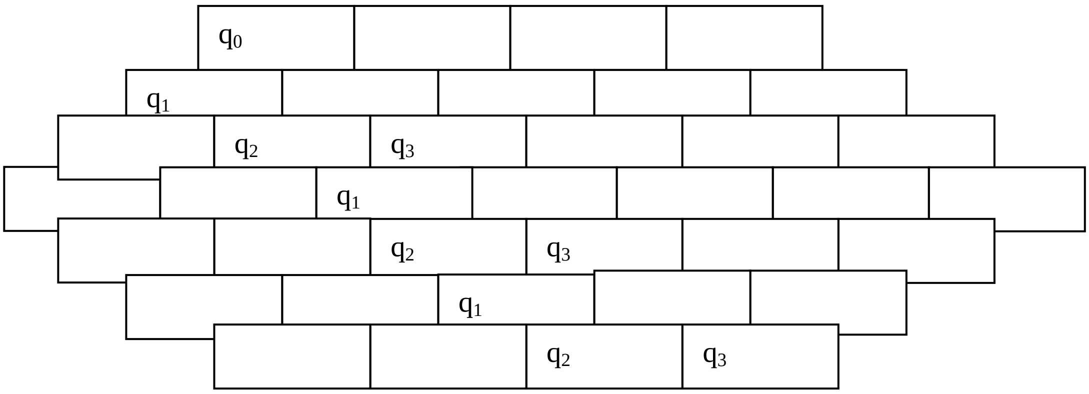


Fig.4.

Here $\{q_0, q_1, q_2, q_3\}$ represents the repeating vertex sets and (q_0, q_1), (q_1, q_2), (q_2, q_3) and (q_3, q_1) represents the repeating edges. Now it remains to check whether A accepts the automaton .If the vertices are renamed as $f_0, f_1, \ldots, f_{n+1}$. Then we have to show that there is a path from $f_1, \ldots, f_n$ exist excluding the beginning and ending node. Consider an edge incoming to f_1 from the X, Y or Z direction. It goes to Y, Z or X direction respectively. Same argument can be shown for outgoing edges also. Let E_i be the sequence of outgoing and incoming edges from and to f_i then for f_k $2 \leq k \leq n-1$. $E_k = \{e_1, e_2 \ldots e_s\}$ then graph is accepted if

1. $S \geq 3$ and it is an odd number for n is odd even number for n is even.
2. If e_j is an incoming edge to f_k then j is odd otherwise j is even.
3. e_j is incoming to f_k from X direction then e_s will be an outgoing edge from f_k Y-direction ,e_j is incoming to f_k from Y direction then e_s will be an outgoing edge from f_k Z-direction also if e_j is incoming to f_k from Z direction then e_s will be an outgoing edge from f_k X-direction.
4. For each j<s e_j will be an outgoing in X, Y or Z- direction then e_{j-1} will be incoming edge from Y, Z, X direction. Similar condition holds for all k. Therefore constructing direct path starting from f_1 we get a path ending in f_n. To show the above constructed path accepts A.

Consider a vertex f_i, and an outgoing edge from f_i of index say j that is ej. The edge labeled $(q_1, a) \rightarrow (q_1, a, d)$.If j=1 that is only one edge then f_i=$f_1$and (q_1, a) reduces to (q_0, a). Otherwise there will be one or more edge which is incoming from the edge index j-1. That is not possible. Therefore for j > 2 there is an outgoing as well as an incoming edge named q_1 and q_3. Proceeding like this for j=n we get f_n as last incoming edge. That is it ends at the accepting state. Therefore A is an accepting automaton.

By using this accepting computation on A we can non-deterministically guess a multigraph representing this accepting computation. This gives us a nondeterministic finite state automaton A_1. As A_1 moves the head to the nodes f_i it fulfills the required properties. Since the incoming and outgoing edge satisfies the properties it is also accepted. Therefore L (A) = L (A_1).

4. CONCLUSION

We have introduced a model of automaton called hexagonal sgrafitto automata. An attempt is made to extend the result in two dimension two three dimensional space. The accepting states of this automaton are also discussed. Further researches can be done on time complexity.

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