International Research Journal of Pure Algebra -4(4), 2014, 477-484
Available online through www.rjpa.info ISSN 2248–9037

A NOVEL APPROACH: FUZZY TOPOLOGICAL CONCEPTS

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(Received on: 30-03-14; Revised & Accepted on: 12-04-14)

ABSTRACT

In this present paper, we study some fuzzy topological concepts and introduce fuzzy open, fuzzy semi-open sets and related notions in a different manner by assigning a topological structure to the universe set X. We construct a fuzzy topology and a fuzzy semi topology over X. Also we investigate some interesting results in this context.

AMS subject Classification: *03E72, 03E75, 54A10, 54A20, 54C08, 54H10.*

Key Words: *Fuzzy set, $L-$Fuzzy set, Fuzzy topology, Fuzzy semi-topology, Fuzzy open set, Fuzzy closed set, Fuzzy semi-open set, Fuzzy semi-closed set, Fuzzy semi-closure, Fuzzy compactness, Fuzzy connectedness.*

INTRODUCTION

The problem of imperfect knowledge has been tackled for a long time by philosophers, logicians and mathematicians. Recently it became also a crucial issue for computer scientists, particularly in the area of artificial Intelligence. There are many approaches to the problem of how to understand and manipulate imperfect knowledge. One of the most successful approaches to tackle this problem is the Fuzzy set theory.

In order to study the control problems of complicate systems and dealing with fuzzy information, American cyberneticist *L. A. Zadeh* introduced fuzzy set theory in his classical paper [8] of 1965. The idea and the concept of fuzzy set were introduced by *Zadeh* used the unit interval [0, 1] to describe and deal with fuzzy phenomena. In 1967, J. *A. Goguen* [3] generalized this concept with $L-$ fuzzy sets. After one year, *C. L. Chang* [1] introduced the definition of fuzzy topology in 1968. Since *Chang* applied fuzzy set theory into topology, many topological notions were introduced by various mathematicians in a fuzzy setting.

In 1976, *R. Lowen* [7] introduced a more natural definition of fuzzy topology which was different from *Chang's* definition. On the other hand, *Norman Levine* [5] introduced the concepts of semi-open sets and semi continuity in 1963. Many mathematicians like *Crossley* [2] were attracted to *Levine's* theory and contributed many interesting topological notions.

In this present work, we introduce some fuzzy topological concepts in a different way by assigning a topological structure to the universe set. The present paper is divided into various sections. We begin each section with some basic notions which are needed in the present study for convenience and we proceed to present our results.

In what follows $\emptyset$ and X stand for the empty set and universe set respectively. Let $\mathcal{P}(X)$ be the collection of all subsets of X (i.e the power set of X).

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1. LATTICES

Definition: 1.1 A relation R on a non-empty set S is said to be a *partially ordered relation* on S if

(a) xRx for all $x \in S$ (*reflexivity*)
(b) xRy and $yRx \Rightarrow x = y$ (*anti-symmetry*)
(c) xRy and $yRz \Rightarrow xRz$ (*transitivity*)

The pair (S, R) is called a partially ordered set.

Definition: 1.2 Let (S, R) be a partially ordered set and $A \subset S$.

(a) An element $u \in S$ is called an *upper bound* of A if $aRu \ \forall \ a \in A$.
(b) An element $u \in S$ is called a *lower bound* of A if $uRa \ \forall \ a \in A$.
(c) An element $u \in S$ is called the least upper bound (*lub*) of A if u is an upper bound of A and uRv for any upper bound v of A. We denote the *lub* of A by the symbol $\vee A$.
(d) An element $u \in S$ is called the greatest lower bound (*glb*) of A if u is a lower bound of A and vRu for any lower bound v of A We denote the *glb* of A by the symbol $\wedge A$.

Definition: 1.3 A partially ordered set (S, R) is said to be a *lattice* if each pair of elements $a, b \in S$ has both *lub* and *glb* in S. We denote the *lub* and *glb* of a and b by the symbols $a \vee b$ and $a \wedge b$ respectively.

Definition: 1.4 A lattice (S, R) is said to be a *complete lattice* if every infinite subset of S has both *lub* and *glb* in S. We denote the *lub* and *glb* of a complete lattice S by the symbols 0 and 1 respectively. They are called the zero element and all element of S.

Remark: 1.5 A partially ordered set with an all element such that every non-vacuous subset has a greatest lower bound is a complete lattice.

Definition: 1.6 A lattice (S, R) is said to be *distributive* if

$$a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c) \text{ or dually}$$
$$a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c) \ \forall \ a, b, c \in S.$$

Definition: 1.7 A lattice (S, R) with 0 and 1 is said to be *complemented* if for every $a \in S$ there exists an element $a^1 \in S$ such that $a \vee a^1 = 1$ and $a \wedge a^1 = 0$.

Remark: 1.8 Define a relation $\geq$ on $\mathcal{P}(X)$ as follows.

$$A \geq B \Leftrightarrow B \subseteq A \quad for \quad A, B \in \mathcal{P}(X).$$

Then $(\mathcal{P}(X), \geq)$ is a complemented distributive lattice. Also it is a complete lattice with zero element $0 = \emptyset$ and all element $1 = X$.

For $A, B \in \mathcal{P}(X)$, $A \vee B = A \bigcup B$, $A \wedge B = A \bigcap B$ *and* $A^1 = X - A$.

2. FUZZY SETS

In what follows L stands for a complete lattice.

Definition: 2.1 An $L-$*fuzzy subset* on X is a mapping $\mu : X \to L$. The collection of all $L-$ fuzzy subsets on X is denoted by L^X and it is called the $L-$ fuzzy space.

Definition: 2.2 An $L-$fuzzy subset $\mu \in L^X$ is called a *crisp subset* on X, if there exists a subset $A \subset X$ such that $\mu = \xi_A$, where $\xi_A : X \to L$ is defined by

$$\xi_A(x) = \begin{cases} 1 & if \ x \in A \\ 0 & if \ x \notin A \end{cases}$$

Definition: 2.3 Let $\mu, \nu \in L^X$, where $(L, \leq)$ is a complete lattice. Then we say that

(a) $\mu \leq \nu \Leftrightarrow \mu(x) \leq \nu(x) \ \forall \ x \in X$.

(b) $\mu = \nu \Leftrightarrow \mu \leq \nu \ and \ \nu \leq \mu$.

Definition: 2.4 Let $\mu, \nu \in L^X$ we define the union $\mu \bigcup \nu : X \to L$ and the intersection $\mu \bigcap \nu : X \to L$ of μ and ν as follows:

$$(\mu \bigcup \nu)(x) = \mu(x) \vee \nu(x) \quad \forall \ x \in X$$
$$(\mu \bigcap \nu)(x) = \mu(x) \wedge \nu(x) \quad \forall \ x \in X$$

Definition: 2.5 Let $\mu \in L^X$ and suppose that L is complemented. Then we define the complement of μ, denoted by μ^1 as a mapping $\mu^1 : X \to L$ by

$$\mu^1(x) = (\mu(x))^1 \quad \forall \ x \in X$$

Definition: 2.6 A *null* $L-$fuzzy subset $\mu_0 : X \to L$ is defined by $\mu_0(x) = 0 \ \forall \ x \in X$. An *absolute* $L-$fuzzy subset $\mu_1 : X \to L$ defined by

$$\mu_1(x) = 1 \ \forall \ x \in X.$$

Definition: 2.7 Let $\mu \in L^X$ and $a \in L$. A $a-$*layer* of μ is an $L-$fuzzy subset $\mu_a : X \to L$ is defined by $\mu_a(x) = a \wedge \mu(x) \quad \forall \ x \in X$.

Definition: 2.8 Let $a \in L$. A *constant* $L-$fuzzy subset $\mu_a^* : X \to L$ is defined by $\mu_a^*(x) = a \quad \forall \ x \in X$. Clearly, $\mu_0^* = \mu_0$ and $\mu_1^* = \mu_1$.

Definition: 2.9 Let L^X and L^Y be $L-$fuzzy spaces, $f : X \to Y$ an ordinary mapping. Then we define $L-$fuzzy mappings $f^{\to} : L^X \to L^Y$ and $f^{\leftarrow} : L^Y \to L^X$ as follows.

$$f^{\to}(\mu)(y) = \vee\{\mu(x) / x \in X, f(x) = y\}$$
$$f^{\leftarrow}(\mu)(x) = \mu(f(x)) \quad \forall \ x \in X$$

3. SEMI-OPEN SETS AND SEMI-TOPOLOGY

Definition: 3.1 A collection $\mathcal{T} \subseteq \mathcal{P}(X)$ is called a *topology* on X if

(a) $\emptyset \in \mathcal{T}$ and $X \in \mathcal{T}$.

(b) the union of any collection of sets in $\mathcal{T}$ is a set in $\mathcal{T}$.

(c) any intersection of a finite collection of sets in $\mathcal{T}$ is a set in $\mathcal{T}$.

The pair $(X, \mathcal{T})$ is called a topological space.

Definition: 3.2 A subset G of X is said to be

(a) *open* in $\left(X,\mathcal{T}\right)$, if $G \in \mathcal{T}$.

(b) *closed* in $\left(X,\mathcal{T}\right)$, if $X - G \in \mathcal{T}$.

(c) *a neighborhood* of a point $x \in X$, if $x \in G$ and $G \in \mathcal{T}$.

Definition: 3.3 Let $A \in \mathcal{P}(X)$. Then

(a) the *interior* of A is the largest open subset of A and it is denoted by $Int\ (A)$.

(b) the *closure* of A is the smallest closed superset of A and it is denoted by $Cl\ (A)$.

Definition: 3.4 Let A be a subset of X. A is said to be

(i) *semi-open* in $\left(X,\mathcal{T}\right)$ if there exists an open set $O \in \mathcal{T}$ such that $O \subseteq A \subseteq Cl(O)$ or equivalently, $A \subseteq Cl(Int(A))$.

(ii) *semi-closed* if X - A is semi-open in $\left(X,\mathcal{T}\right)$.

(iii) *semi-neighborhood* of a point $x \in X$ if $x \in A$ and A is semi-open in $\left(X,\mathcal{T}\right)$.

Definition: 3.5 The *semi-closure* of a set A in $\left(X,\mathcal{T}\right)$ denoted by $scl(A)$, is the intersection of all semi-closed supersets of A.

Proposition: 3.6 Let $\left(X,\mathcal{T}\right)$ be a topological space. In $\left(X,\mathcal{T}\right)$,

(a) every open set is semi-open.

(b) if $\left\{G_\alpha / \alpha \in \Delta\right\}$ is any collection of semi-open sets, then $\bigcup_{\alpha\in\Delta} G_\alpha$ is semi-open.

(c) if A is open and B is semi-open then $A \bigcap B$ is semi-open.

(d) if A is semi-open and $A \subset B \subset Cl\,(A)$ then B is semi-open.

(e) if $A \text{ and } B$ are semi-open then it is not necessary that $A \bigcap B$ is semi-open.

Proposition: 3.7 Let $S\left(\mathcal{T}\right)$ be the collection of all semi-open sets in $\left(X,\mathcal{T}\right)$. Then $\left(S\left(\mathcal{T}\right),\le\right)$ is a complete lattice, where $\le$ is defined on $S\left(\mathcal{T}\right)$ by

$$A \le B \Leftrightarrow B \subseteq A.$$

Proof: Clearly, $\left(S\left(\mathcal{T}\right),\le\right)$ is a partially ordered set. For $A, B \in S\left(\mathcal{T}\right)$ put $A \vee B$ = Union of all semi-open sets contained in $A \bigcap B$ and $A \wedge B = A \bigcup B$. It can be easily verified that $A \vee B$ and $A \wedge B$ are the *lub* and *glb* of A and B in $\left(S\left(\mathcal{T}\right),\le\right)$ respectively. Hence $\left(S\left(\mathcal{T}\right),\le\right)$ is a lattice. Clearly ∅ is the all element and X is the zero element in $\left(S\left(\mathcal{T}\right),\le\right)$. Let { $A_\alpha / \alpha \in \Delta$ } be a non-vacuous set in $S\left(\mathcal{T}\right)$. Then $\bigwedge_{\alpha\in\Delta} A_\alpha = \bigcup_{\alpha\in\Delta} A_\alpha$ is the *glb* of { $A_\alpha / \alpha \in \Delta$ } in $\left(S\left(\mathcal{T}\right),\le\right)$. Thus $\left(S\left(\mathcal{T}\right),\le\right)$ is a partially ordered set that has an all element in which every non-vacuous subset has a *glb*. Hence $\left(S\left(\mathcal{T}\right),\le\right)$ is a complete lattice.

Remark: 3.8 The set $S\left(\mathcal{T}\right)$ clearly contains $\mathcal{T}$ and is closed under arbitrary unions. However, being not closed under finite intersections, $S\left(\mathcal{T}\right)$ is not a topology on X. However, if $A \in \mathcal{T}$ and $B \in S\left(\mathcal{T}\right)$ then $A \bigcap B \in S\left(\mathcal{T}\right)$.

Definition: 3.9 We define $S_0(\mathcal{T}) = \{A \in S(\mathcal{T}) / A \cap B \in S(\mathcal{T})\ \forall B \in S(\mathcal{T})\}$ and $S_{00}(\mathcal{T}) = \{A \in S(S_0) / A \cap B \in S(S_0)\ \ \forall\ B \in S(S_0)\}$ where $S(S_0)$ is the collection of all semi-open sets in the topological space $(X, S_0(\mathcal{T}))$

Proposition: 3.10

(a) $S_0(\mathcal{T})$ and $S_{00}(\mathcal{T})$ are topologies on X

(b) $T \subseteq S_0(\mathcal{T}) \subseteq S(\mathcal{T})$.

(c) $S(S_0) \subseteq S(\mathcal{T})$

(d) $S_0(\mathcal{T}) = S_{00}(\mathcal{T})$

We call $S_0(\mathcal{T})$, the semi-topology on X .

4. FUZZY TOPOLOGY AND RELATED CONCEPTS

Definition: 4.1 Let L be a complemented, distributive and complete lattice. A collection δ of $L-$fuzzy subsets on X is said to be an $L-$fuzzy topology on X and (L^X, δ) is called an $L-$fuzzy topological space, if δ satisfies the following conditions.

(a) $\mu_0 \in \delta$ *and* $\mu_1 \in \delta$

(b) if $\{\mu_i / i \in \Delta\}$ is any subcollection of δ then $\bigcup_{i\in\Delta} \mu_i \in \delta$

(c) if $\mu, \nu \in \delta$ then $\mu \cap \nu \in \delta$. The members of δ are called open sets in L^X and the complements of members of δ are called closed sets in L^X .

Now we construct an $L-$fuzzy topology and introduce some related concepts in a different manner. Henceforth, we take $L = (\mathcal{P}(X), \geq)$, the lattice which is described in the Remark 1.8.

Definition: 4.2 Let $(X, \mathcal{T})$ be a topological space. We say that, $\mu : X \to L$ is an $L-$fuzzy open subset on X if $\mu(x) \in \mathcal{T}\ \forall\ \ x \in X$ and $\nu : X \to L$ is an $L-$fuzzy closed subset on X if $X - \nu(x) \in \mathcal{T}\ \forall\ \ x \in X$.

Definition: 4.3 Let $(X, \mathcal{T})$ be a topological space. Let $\delta_{\mathcal{T}} \subset L^X$ be the collection of all $L-$fuzzy open subsets on X and $\delta^1_{\mathcal{T}}$ be the collection of all $L-$fuzzy closed subsets on X .

Proposition: 4.4 The collection $\delta_{\mathcal{T}}$ forms an $L-$fuzzy topology on X .

Proof: Since $\mu_0(x) = \varnothing$ and $\mu_1(x) = X$ for all $x \in X$ $\mu_0 \in \delta_{\mathcal{T}}$ and $\mu_1 \in \delta_{\mathcal{T}}$. Let $\{\mu_\alpha / \alpha \in \Delta\}$ be any collection of sets in $\delta_{\mathcal{T}}$, i.e., $\mu_\alpha \in \delta_{\mathcal{T}}\ \ \ \forall\ \ \alpha \in \Delta$.

Put $\mu = \bigcup_{\alpha\in\Delta} \mu_\alpha$ Then $\mu(x) = \bigcup_{\alpha\in\Delta} \mu_\alpha(x)\ \ \ \forall\ \ x \in X$.

Since each $\mu_\alpha(x)$ is open in $(X, \mathcal{T})$, $\mu(x) \in \mathcal{T}\ \ \forall\ \ x \in X$.

Hence $\mu \in \delta_{\mathcal{T}}$. Thus $\delta_{\mathcal{T}}$ is closed under arbitrary unions.

Let $\nu, \eta \in \delta_{\mathcal{T}}$ Then $(\nu \cap \eta)(x) = \nu(x) \cap \eta(x) \ \ \forall \ x \in X$.

Since $\nu(x) \in \mathcal{T}$ and $\eta(x) \in \mathcal{T}$, $(\nu \cap \eta)(x) \in \mathcal{T} \ \ \forall \ x \in X \Rightarrow \ \nu \cap \eta \in \delta_{\mathcal{T}}$. Thus $\delta_{\mathcal{T}}$ forms an $L-$fuzzy topology on X and $(L^X, \delta_{\mathcal{T}})$ is an $L-$Fuzzy topological space.

Example: 4.5 If $\delta = \{\mu_0, \mu_1\}$ then δ is an $L-$fuzzy topology on X and this δ is called the $L-$fuzzy indiscrete topology on X .

Example: 4.6 Let $\mathcal{T} = \{\emptyset, X\}$. Then $\delta_{\mathcal{T}}$ contains μ_0, μ_1 and some crisp sets of the form

$$\xi_A(x) = \begin{cases} 0 & if \quad x \in A \\ 1 & if \quad x \notin A \end{cases} \quad \text{and} \quad \xi_B(x) = \begin{cases} 1 & if \quad x \in B \\ 0 & if \quad x \notin B \end{cases}$$

where $A, B \in \mathcal{P}(X)$. Here $0 = \emptyset$ and $1 = X$ .In this example, it can be observed that $\delta_{\mathcal{T}}$ is not an $L-$fuzzy indiscrete topology, eventhough $\mathcal{T}$ is indiscrete.

Example: 4.7 Let $\mathcal{T} = \mathcal{P}(X)$, the discrete topology on X . Then $\delta_{\mathcal{T}} = L^X$, which we call, the $L-$fuzzy discrete topology.

Definition: 4.8 Let $(X, \mathcal{T})$ be a topological space. We say that $\mu : X \to L$ is an $L-$fuzzy semi-open subset on X , if $\mu(x) \in S(\mathcal{T}) \ \ \forall \ \ x \in X$. The collection of all $L-$fuzzy semi-open subsets on X in $(L^X, \delta_{\mathcal{T}})$ is denoted by the symbol $S(\delta_{\mathcal{T}})$.

Proposition: 4.9

(a) If $\mu \in \delta_{\mathcal{T}}$ then $\mu \in S(\delta_{\mathcal{T}})$.

(b) If $\mu \in \delta_{\mathcal{T}}$ and $\nu \in S(\delta_{\mathcal{T}})$ then $\mu \cap \nu \in S(\delta_{\mathcal{T}})$.

(c) If $\{\mu_i \,/\, i \in \Delta\} \subset S(\delta_{\mathcal{T}})$ then $\bigcup_{i \in \Delta} \mu_i \in S(\delta_{\mathcal{T}})$.

(d) If $G \in \mathcal{T}$ and $\mu \in S(\delta_{\mathcal{T}})$ then the $G-$layer μ_G of μ is $L-$fuzzy semi-open.

(e) If $A \in S(\mathcal{T})$ and $\mu \in \delta_{\mathcal{T}}$ then the $A-$layer μ_A of μ is $L-$fuzzy semi-open.

Definition: 4.10 Let $(X, \mathcal{T})$ be a topological space. We say that $\mu : X \to L$ is an $L-$fuzzy semi-closed subset on X , if $X - \mu(x) \in S(\mathcal{T}) \ \forall \ \ x \in X$. We say that an $L-$fuzzy subset υ on X is an $L-$fuzzy semi-closure of μ , if

(a) υ is $L-$fuzzy semi-closed

(b) $\upsilon \geq \mu$

(c) $\lambda \geq \upsilon$ for every $L-$fuzzy semi-closed set $\lambda \geq \mu$.

Proposition: 4.11 Let $S_0(\delta_{\mathcal{T}}) = \{\mu \in S(\delta_{\mathcal{T}}) \,/\, \mu \cap \upsilon \in S(\delta_{\mathcal{T}}) \quad \forall \upsilon \in S(\delta_{\mathcal{T}})\}$. Then

(a) $S_0(\delta_{\mathcal{T}})$ is an $L-$fuzzy topology.

(b) $\delta_{\mathcal{T}} \subseteq S_0(\delta_{\mathcal{T}}) \subseteq S(\delta_{\mathcal{T}})$.

Remark: 4.12 We call the $L-$fuzzy topology $S_0(\delta_{\mathcal{T}})$ by the name, $L-$*fuzzy semi-topology*.

5. FUZZY COMPACTNESS AND FUZZY CONNECTEDNESS

Definition: 5.1 A topological space $\left(X,\mathcal{T}\right)$ is said to be *compact* if for every collection $\left\{G_\alpha \,/\, \alpha\in\Delta\right\}$ of open subsets of X with $X=\bigcup_{\alpha\in\Delta} G_\alpha$ there exists a finite subcollection $\left\{G_{\alpha_1},G_{\alpha_2},\ldots,G_{\alpha_n}\right\}$ of $\left\{G_\alpha \,/\, \alpha\in\Delta\right\}$ such that

$$X=G_{\alpha_1}\bigcup G_{\alpha_2}\bigcup\ldots\bigcup G_{\alpha_n}.$$

Definition: 5.2 A topological space $\left(X,\mathcal{T}\right)$ is said to be *semi-compact* if for every collection $\left\{G_\alpha \,/\, \alpha\in\Delta\right\}$ of semi-open subsets of X with $X=\bigcup_{\alpha\in\Delta} G_\alpha$ there exists a finite subcollection $\left\{G_{\alpha_1},G_{\alpha_2},\ldots,G_{\alpha_n}\right\}$ of $\left\{G_\alpha \,/\, \alpha\in\Delta\right\}$ such that

$$X=G_{\alpha_1}\bigcup G_{\alpha_2}\bigcup\ldots\bigcup G_{\alpha_n}.$$

Definition: 5.3 A topological space $\left(X,\mathcal{T}\right)$ is said to be *disconnected* if there exist two non-empty disjoint open subsets G and H of X such that

$$X=G\bigcup H.$$

Definition: 5.4 An $L-$fuzzy topological space $\left(L^X,\delta_{\mathcal{T}}\right)$ is said to be $L-$*fuzzy compact* if for every collection $\left\{\mu_\alpha \,/\, \alpha\in\Delta\right\}$ of $L-$fuzzy open subsets on X with $\mu_1=\bigcup_{\alpha\in\Delta}\mu_\alpha$ there exists a finite subcollection $\left\{\mu_{\alpha_1},\mu_{\alpha_2},\ldots,\mu_{\alpha_n}\right\}$ of $\left\{\mu_\alpha \,/\, \alpha\in\Delta\right\}$ such that $\mu_1=\mu_{\alpha_1}\bigcup\mu_{\alpha_2}\bigcup\ldots\bigcup\mu_{\alpha_n}$.

Proposition: 5.5 $\left(X,\mathcal{T}\right)$ is compact $\Leftrightarrow$ $\left(L^X,\delta_{\mathcal{T}}\right)$ is $L-$fuzzy compact.

Proof: Let $\left(X,\mathcal{T}\right)$ be a compact space. Let $\left\{\mu_\alpha \,/\, \alpha\in\Delta\right\}\subset\delta_{\mathcal{T}}$ be a collection of $L-$fuzzy open subsets on X such that $\mu_1=\bigcup_{\alpha\in\Delta}\mu_\alpha \quad\Rightarrow \mu_1(x)=\bigcup_{\alpha\in\Delta}\mu_\alpha(x) \quad \forall\, x\in X$.

Let $x_0\in X$ Then $\mu_1(x_0)=\bigcup_{\alpha\in\Delta}\mu_\alpha(x_0)$

$$\Rightarrow X=\bigcup_{\alpha\in\Delta}\mu_\alpha(x_0).$$

Since X is compact, there exist finitely many indices $\alpha_1,\alpha_2,\ldots,\alpha_n$ in Δ such that

$$X=\mu_{\alpha_1}(x_0)\bigcup\mu_{\alpha_2}(x_0)\bigcup.\ \bigcup\mu_{\alpha_n}(x_0)$$

$$\Rightarrow \mu_1(x_0)=\mu_{\alpha_1}(x_0)\bigcup\mu_{\alpha_2}(x_0)\bigcup.\ \bigcup\mu_{\alpha_n}(x_0)$$

Since $x_0\in X$ is arbitrary, we have

$$\mu_1(x)=\mu_{\alpha_1}(x)\bigcup\mu_{\alpha_2}(x)\bigcup.\ \bigcup.\mu_{\alpha_n}(x) \quad \forall\, x\in X$$

$$\Rightarrow \mu_1=\mu_{\alpha_1}\bigcup\mu_{\alpha_2}\bigcup\ldots\bigcup\mu_{\alpha_n}$$

Hence $\left(L^X,\delta_{\mathcal{T}}\right)$ is $L-$fuzzy compact.

Conversely, suppose that $\left(L^X,\delta_{\mathcal{T}}\right)$ is $L-$fuzzy compact. Let $\left\{G_\alpha \,/\, \alpha\in\Delta\right\}$ be a collection of open sets in $\left(X,\mathcal{T}\right)$ such that $X=\bigcup_{\alpha\in\Delta}G_\alpha$.

Define $\mu_\alpha^*: X\to L$ such that $\mu_\alpha^*(x)=G_\alpha \quad \forall\, x\in X$.

Since $X = \bigcup\limits_{\alpha \in \Delta} G_\alpha \Rightarrow \mu_1(x) = \bigcup\limits_{\alpha \in \Delta} \mu_\alpha^*(x) \;\; \forall\, x \in X$

$\Rightarrow \mu_1 = \bigcup\limits_{\alpha \in \Delta} \mu_\alpha^*$

$\Rightarrow$ there exist finitely many indices $\alpha_1, \alpha_2, \ldots, \alpha_n$ in Δ such that $\mu_1 = \mu_{\alpha_1}^* \cup \mu_{\alpha_2}^* \cup \ldots \cup \mu_{\alpha_n}^*$

$\Rightarrow \mu_1(x) = \mu_{\alpha_1}^*(x) \cup \mu_{\alpha_2}^*(x) \cup .\; \cup . \mu_{\alpha_n}^*(x) \;\; \forall\, x \in X$

$\Rightarrow X = G_{\alpha_1} \cup G_{\alpha_2} \cup G_{\alpha_n}$

$\Rightarrow (X, \mathcal{T})$ is compact.

Definition: 5.6 An $L-$fuzzy topological space $(L^X, \delta_{\mathcal{T}})$ is said to be $L-$*fuzzy semi-compact* if for every collection $\{\mu_\alpha \,/\, \alpha \in \Delta\}$ of $L-$fuzzy semi-open subsets on X with $\mu_1 = \bigcup\limits_{\alpha \in \Delta} \mu_\alpha$ there exists a finite subcollection $\{\mu_{\alpha_1}, \mu_{\alpha_2}, \ldots, \mu_{\alpha_n}\}$ of $\{\mu_\alpha \,/\, \alpha \in \Delta\}$ such that $\mu_1 = \mu_{\alpha_1} \cup \mu_{\alpha_2} \cup \ldots \cup \mu_{\alpha_n}$.

Proposition: 5.7 $(X, \mathcal{T})$ is semi-compact $\Leftrightarrow$ $(L^X, \delta_{\mathcal{T}})$ is $L-$fuzzy semi-compact.

Definition: 5.8 An $L-$fuzzy topological space $(L^X, \delta_{\mathcal{T}})$ is said to be $L-$*fuzzy disconnected* if there exist two $L-$fuzzy open subsets μ and υ such that

(a) $\mu \neq \mu_0$ and $\upsilon \neq \mu_0$ (b) $\mu_1 = \mu \cup \upsilon$ (c) $\mu \neq \mu \cap \upsilon$

Proposition: 5.9 $(X, \mathcal{T})$ is disconnected $\Leftrightarrow$ $(L^X, \delta_{\mathcal{T}})$ is $L-$fuzzy disconnected.

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Source of Support: Nil, Conflict of interest: None Declared