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THE BEAL CONJECTURE

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ABSTRACT

The field of Algebra and Number Theory has a vast number of applications. The Beal Conjecture has been pondered and written by the Texas billionaire Andrew Beal. It started when he became interested in solving the 350-year old mystery of Fermat's Last Theorem. Andrew Wiles proved this theorem in 1994 that, for all n-positive integers, there are no solutions. For example, if n = 3, Wiles proved that there were no solutions. Now since we have n = 3, what if all three exponents were different integers higher from two, then will the integers a, b, and c have the same prime factor? Conversely, if the integers a, b, and c have the same prime factor, would the integer exponents be higher than two?

INTRODUCTION

In "about 1637, Fermat stated that there are no solutions to the equation

$$a^n + b^n = c^n$$

for all positive integers $n > 2$"[5]. Mathematicians have long been baffled by Pierre Fermat's famous assertion that it is impossible to prove past $n = 2$. There is a statement written in the margin of his book that he had a demonstration illustrating such a finding to then become known as Fermat's Last Theorem. This conjecture to me is indeed part of the field of number theory and that Fermat's Last Theorem was the pinnacle of algebra's turnaround. Beal's conjecture can be looked upon as a modified version of Fermat's Last Theorem.

BODY

Fermat's Last Theorem formally states that, no three positive integers a, b, and c can satisfy the equation

$$a^n + b^n = c^n \qquad [4]$$

if n is an integer greater than two. Let us set n equally to two. Then as we know it, becomes the famous Pythagorean Theorem such that,

$$a^2 + b^2 = c^2$$

This equation as we know is trivial. The Beal Conjecture is derived from Fermat's Last Theorem and it states, there exists a common prime factor to the equation,

$$a^x + b^y = c^z$$

if a, b, and c are co-prime integers, and x, y, and z are all integers larger than two. The problem is to either prove this conjecture correct or disprove it with illustrating a counter-example. Our goal here is to find a counter-example that shows that The Beal Conjecture does not hold. When it comes to a composite number n, "we can factor it in an efficient way if n is a product of two integers that are close to one another. The method defined is called Fermat Factorization."[1].

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THE BEAL CONJECTURE

If

$$a^x + b^y = c^z$$

where *a, b, c, x, y*, and *z* are positive integers and *x, y*, and *z* are all greater than 2, then *a*, *b*, and *c* must have the same prime factor other than 1. Even though this is a conjecture, this must be shown to be true or false. "If we have a composite integer, it is required to part it again and again until the additional factorization is impossible if told to do so. For example,

$$250 = 2*125 = 2*(5*5*5) = 2*5^3.$$

Integers that cannot be factored down any further are called prime numbers"[2]. Let's "assume that integers *a*, *b*, and *c* have the highest common factor of 1. We must show that for,

$$a^x + b^y = c^z$$

the integer exponents *x, y*, and *z* is greater than 2"[3]. We chose *a*, *b*, *c*, *x*, *y*, and *z* to be positive integers of $x = 4, z = 6$, $a = 7, b = 4,$ *and* $c = 5$. Then the equation yields from the chosen numbers,

$$7^4 + 4^y = 5^6.$$

Since we have $7^4 = 2401$, 4^y, and $5^6 = 15625$, the equation yields the enumeration,

$$2401 + 4^y = 15625.$$

We must find out how 4^y breaks down and what y is in order to satisfy the conjecture. So now we subtract 2401 from 15625 to get 13224. Then, we set 4^y equal to 13224 to get $4^y = 13224$. In order to get the factors of the above equation, we must divide 13224 by four to then get 3306. Putting this into our equation would give us,

$$2401 + 4^1*3306 = 15625.$$

We notice that 3306 is not prime and has more factors to it. Let us divide 3306 by 2. The result of that is 1653. So the new enumeration yields,

$$2401 + 4*(2*1653) = 15625.$$

Notice that 1653 is not prime and that the prime factorization of it is 3, 19, and 29. Since $7^4 = 7*7*7*7 = 2401$ and $5^6 = 5*5*5*5*5*5 = 15625$, the equation now yields,

$$2401 + 2^3*1653 = 15625$$

$$7^4 + 2^3*1653 = 5^6.$$

In 1823, a different approach to Fermat's Last Theorem was developed by Legendre to handle a special case of the conjecture in which *a, b, and c* are prime to *n*, where

$$a^n + b^n = c^n.$$

In this case, our *n*-integers are not the same such that we have different integers *x*, *y, and z*. Since seven is prime to 4, two is prime to 3, and five is prime to 6, this case does not correspond with our current example. Notice that *a, b, and c* are all co-prime to each other. When we attempt to find a general, prime part of *a*, *b, and c*, we notice that there is none because as we part the significant numbers all the way down we see that there are no common prime factors. Thus, contradicts the conjecture of there being a common prime factor other than 1.

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