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# GENERALIZED COMMON FIXED POINT THEOREMS AND INTUITIONISTIC FUZZY METRIC SPACES

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***ABSTRACT***

*In this paper we give some definition and new definition of Common Fixed Point Theorems and Intuitionistic Fuzzy Metric Spaces. We formulate the definition of weakly commuting and R-weakly commuting mappings.*

***KeyWords**: Triangular Norm, Triangular Co-norm, Intuitionistic Fuzzy Metric Space R-Weakly Commuting Mappings, Common Fixed Point.*

***AMS Mathematics Subject Classification:** 47H10, 54H25.*

## 1. INTRODUCTION

Zadeh [32] in 1965 was introduction of the concept of fuzzy sets Grabiec [10] extend two fixed point theorems of Banach and Edelstein to contractive mappings of complete and compact fuzzy metric spaces in the sense of Kramosil and Michalek [17]. George and Veeramani ([8], [9]) modified the concept of fuzzy metric space introduced by Kramosil and Michalek [17] and defined a Hausdorff topology on this fuzzy metric space. obtained common fixed point theorems for weakly commuting maps and R-weakly commuting mappings.

Atanassov [2] introduced and studied the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. Coker [4] introduced the concepts of the so called "intuitionistic fuzzy topological spaces". Park [22], using the idea of intuitionistic fuzzy sets, define the notion of intuitionistic fuzzy metric spaces with the help of continuous t-norm [22] and continuous t-conorms as a generalization of fuzzy metric space due to George and Veeramani ([8], [9]).

Continuous t-conorms as a generalization of fuzzy metric spaces due to Kramosil and Michalek [17]. Further, we introduce the notion of Cauchy sequences in intuitionistic fuzzy metric spaces. We prove a common fixed point theorem for commuting mappings in intuitionistic fuzzy metric spaces. We first formulate the definition of weakly commuting and R-weakly commuting mappings in intuitionistic fuzzy metric spaces and prove the intuitionistic fuzzy version of Pant's theorem [21].

## 2. INTUITIONISTIC FUZZY MERTIC SPACES

**Definition: 1** A binary operation $*: [0, 1] \times [0, 1] \to [0, 1]$ is continuous t-norm if $*$ is satisfying the following conditions:

(i) $*$ is commutative and associative;
(ii) $*$ is continuous;
(iii) $a * 1 = a$ for all $a \in [0, 1]$;
(iv) $a * b \le c * d$ whenever $a \le c$ and $b \le d$ for all $a, b, c, d \in [0, 1]$.

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**Definition: 2** A binary operation ◊: [0, 1] × [0, 1] → [0, 1] is continuous t-conorm if ◊ is satisfying the following conditions:

(i) ◊ is commutative and associative;
(ii) ◊ is continuous;
(iii) a * 0 = a for all a ∈ [0, 1];
(iv) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1].

**Definition: 3** A 5-tuple (X, M, N, *, ◊) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, * is a continuous t-norm, 3 is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) M(x, y, t) + N(x, y, t) ≤ 1 for all x, y ∈ X and t > 0;
(ii) M(x, y, 0) = 0 for all x, y ∈ X;
(iii) M(x, y, t) = 1 for all x, y ∈ X and t > 0 if and only if x = y;
(iv) M(x, y, t) = M(y, x, t) for all x, y ∈ X and t > 0;
(v) M(x, y, t) * M(y, z, s) ≤ M(x, z, t + s) for all x, y, z ∈ X and s, t > 0;
(vi) for all x, y ∈ X, M(x, y, .) : [0,∞) → [0, 1] is left continuous;
(vii) Limt→∞M(x, y, t) = 1 for all x, y ∈ X and t > 0;
(viii) N(x, y, 0) = 1 for all x, y ∈ X;
(ix) N(x, y, t) = 0 for all x, y ∈ X and t > 0 if and only if x = y;
(x) N(x, y, t) = N(y, x, t) for all x, y ∈ X and t > 0;
(xi) N(x, y, t) ◊ N(y, z, s) ≥ N(x, z, t + s) for all x, y, z ∈ X and s, t > 0;
(xii) for all x, y ∈ X, N(x, y, .) : [0,∞) → [0, 1] is right continuous;
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all x, y in X.

Then (M, N) is called an intuitionistic fuzzy metric on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

**Remark: 1** Every fuzzy metric space (X, M, *) is an intuitionistic fuzzy metric space of the form (X, M, 1 − M, *, ◊) such that t-norm * and t-conorm ◊ are associated ([13]), i.e., x ◊ y = 1− ((1 − x) * (1 − y)) for all x, y ∈ X.

**Remark: 2** In intuitionistic fuzzy metric space X, M(x, y, .) is non-decreasing and N(x, y, .) is non-increasing for all x, y ∈ X.

**Definition: 4** Let (X, M, N, *, ◊) be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be Cauchy sequence if, for all t > 0 and p > 0,

$$\lim_{n\to\infty} M\left(x_{n+p}, x_n, t\right) = 1,\ \lim_{n\to\infty} N\left(x_{n+p}, x_n, t\right) = 0$$

(b) A sequence $\{x_n\}$ in X is said to be convergent to a point x ∈ X if, for all t > 0,

$$\lim_{n\to\infty} M\left(x_n, x, t\right) = 1,\ \lim_{n\to\infty} N\left(x_n, x, t\right) = 0$$

Since * and ◊ are continuous, the limit is uniquely determined from (v) and (xi), respectively.

**Definition: 5** An intuitionistic fuzzy metric space (X, M, N, *, ◊) is said to be

(i).complete if and only if every Cauchy sequence in X is convergent.
(ii).compact if every sequence in X contains a convergent subsequence.

**Lemma: 1** Let S be a continuous maping of a complete metric space (x, d) into itself and T: x → X be a mapping satisfying the following conditions.

(1) $T(x) \subseteq S(x)$
(2) T is commutes with s
(3) There exists 0 < k < 1. Such that for all x, y ∈ X

$$d(T(x), T(y)) \le kd(S(x), S(y))$$

Then T and S have a unique common fixed point in x.

**3. MAIN RESULT**

**Theorem: 3.1** Let (X, M, N, *, ◊) be a complete Intuitionistic Fuzzy Metric Space and S.T: x → X be mapping satisfying the following conditions.

(3.1) $T(x) \subseteq S(x)$

(3.2) S is continuous

(3.3) There exists 0 < k < 1. Such that for all x, y $\in$ X

$$M(Tx,Ty,kt) \geq M(Sx,Sy,t)*M(Sx,Sx,t)*M(Sy,Sy,t)*M(Sx,Sy,\alpha t)*M(Sy,Sx,,(2-\alpha)t)$$

and $$N(Tx,Ty,kt) \leq N(Sx,Sy,t)*M(Sx,Sx,t)*N(Sy,Ty,t)*M(Sx,Sy,\alpha t)*M(Sy,Sx,,(2-\alpha)t)$$

Then s and T have a unique common fixed point in X. Provided S and T commute on X.

**Proof:** Let $x_0$ be an arbitrary point of X. By (3.1), we can construct a sequence $\{y_n\}$ in X such that $y_{2n} = Tx_{2n+1} = Sx_{2n}, y_{2n+1} = Sx_{2n+2} = Sx_{2n+1}$ for n = 0, 1, …… Then, by (3.2), for α = 1 → q, q ∈ (0, 1), we have

$M(Sx_{2n}, Sx_{2n+1}, kt) \leq M(Sx_{2n}, Tx_{2n+1}, t) * M(Sx_{2n}, Sx_{2n}, t) * M(Sx_{2n+1}, Sx_{2n+1}, t) * M(Sx_{2n}, Tx_{2n+1}, (1 \to q)t)$
$* M(Sx_{2n+1}, Sx_{2n}, (1 + q)t)$

and

$N(Sx_{2n}, Sx_{2n+1}, kt) \leq N(Sx_{2n}, Tx_{2n+1}, t) \lozenge N(Sx_{2n}, Sx_{2n}, t) \lozenge N(Sx_{2n+1}, Tx_{2n+1}, t) \lozenge N(Sx_{2n}, Tx_{2n+1}, (1 \to q)t)$
$\lozenge N(Sx_{2n+1}, Sx_{2n}, (1 + q)t)$

and so

$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n-1}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n}, y_{2n}, (1 \to q)t) * M(y_{2n+1}, y_{2n-1}, (1 + q)t)$

$\geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, qt)$

and

$M(y_{2n}, y_{2n+1,} kt) \leq N(y_{2n-1}, y_{2n}, t) \lozenge N(y_{2n}, y_{2n-1}, t) \lozenge N(y_{2n+1}, y_{2n}, t) \lozenge N(y_{2n}, y_{2n}, (1 - q)t) \lozenge N(y_{2n+1}, y_{2n-1}, (1 + q)t)$

$\leq N(y_{2n-1}, y_{2n}, t) \lozenge N(y_{2n}, y_{2n+1}, t) \lozenge N(y_{2n+1}, y_{2n}, qt)$.

Thus it follows that

$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n+1}, y_{2n}, t) * M(y_{2n+1}, y_{2n}, qt)$

and

$N(y_{2n}, y_{2n+1}, kt) \leq N(y_{2n-1}, y_{2n}, t) * N(y_{2n+1}, y_{2n}, t) * N(y_{2n+1}, y_{2n}, qt)$

Since t-norm and t-conorm * and ◊ are continuous and M(x, y, ·) and N(x, y, ·) are continuous, letting q → 1, we have

$M(y_{2n}, y_{2n+1}, kt) \geq M(y_{2n-1}, y_{2n}, t) * M(y_{2n+1}, y_{2n}, t)$

and

$N(y_{2n}, y_{2n+1}, kt) \leq N(y_{2n-1}, y_{2n}, t) * N(y_{2n+1}, y_{2n}, t)$

Similarly, we also have

$M(y_{2n+1}, y_{2n+2}, kt) \geq M(y_{2n}, y_{2n+1}, t) \lozenge M(y_{2n+2}, y_{2n+1}, t)$

and

$N(y_{2n+1}, y_{2n+2}, kt) \leq N(y_{2n}, y_{2n+1}, t) \lozenge N(y_{2n+2}, y_{2n+1}, t)$

In general, we have, for m = 1, 2, . . .,

$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, t)$

and

$N(y_{m+1}, y_{m+2}, kt) \geq N(y_m, y_{m+1}, t) * N(y_{m+1}, y_{m+2}, t)$

Consequently, it follows that, for m, p = 1, 2, …,

$M(y_{m+1}, y_{m+2}, kt) \geq M(y_m, y_{m+1}, t) * M(y_{m+1}, y_{m+2}, \frac{t}{k^p})$

and

$N(y_{m+1}, y_{m+2}, kt) \geq N(y_m, y_{m+1}, t) * N(y_{m+1}, y_{m+2}, \frac{t}{k^p})$

By noting that M($y_{m+1}$, $y_{m+2}$, $\frac{t}{k^p}$ kp ) → 1 and N($y_{m+1}$, $y_{m+2}$, $\frac{t}{k^p}$ ) → 0 as p → ∞, we have, for m = 1, 2, …,

M($y_{m+1}$, $y_{m+2}$, kt) ≥ M($y_m$, $y_{m+1}$, t)

and

N($y_{m+1}$, $y_{m+2}$, kt) ≤ N($y_m$, $y_{m+1}$, t).

Hence, {$y_n$} is a Cauchy sequence in X. Since (X, M, N, *, ◊) is complete, it converges to a point z in X. Since {$Sx_{2n}$}, {$Sx_{2n+1}$}, {$Sx_{2n+2}$} and {$Tx_{2n+1}$} are subsequence of {$y_n$}. Therefore, $Sx_{2n}$, $Sx_{2n+1}$, $Sx_{2n+2}$, $Tx_{2n+1}$ → z as n → ∞.

Then, since the pair (S, T) is compatible of type (I) and T is continuous, we have

$$M(Tz,z,t)\geq \lim_{n\to\infty} M(STx_{2n+1}z,\lambda t),$$

$$N(Tz,z,t)\geq \lim_{n\to\infty} N(STx_{2n+1}z,\lambda t),\ TTx_{2n+1}\to Tz.$$

Now, for α = 1, setting x = $x_{2n}$ and y = $Tx_{2n+1}$ in (3.2), we obtain

(3.3) M($Sx_{2n}$, $STx_{2n+1}$, kt) ≥ M($Sx_{2n}$, $TTx_{2n+1}$, t) * M($Sx_{2n}$, $Sx_{2n}$, t)

≥ M($STx_{2n+1}$, $TTx_{2n+1}$, t) * M($Sx_{2n}$, $TTx_{2n+1}$, t) * M($STx_{2n+1}$, $Sx_{2n}$, t)

and

N($Sx_{2n}$,$STx_{2n+1}$, kt) ≤ N($Sx_{2n}$, $TTx_{2n+1}$, t) ◊ N($Ax_{2n}$, $Sx_{2n}$, t) ◊ N($STx_{2n+1}$, $TTx_{2n+1}$, t) ◊ N($Sx_{2n}$, $TTx_{2n+1}$, t) ◊ NM($STx_{2n+1}$, $Sx_{2n}$, t):

Thus, by letting the limit inferior on both sides of (3.3), we have

$$\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},kt),\ \geq M(z,Tz,t)*M(z,z,t)*\underline{\lim}_{n\to\infty} M(Tz,STx_{2n+1},t),\ *M(z,Tz,t)*\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},t)$$

and

$$\overline{\lim}_{n\to\infty} M(z,STx_{2n+1},kt)\geq M(z,Tz,t)\Diamond M(z,z,t)\Diamond\overline{\lim}_{n\to\infty} N(Tz,STx_{2n+1},t),\ \Diamond N(z,Tz,t)\Diamond\overline{lim}_{n\to\infty} N(z,STx_{2n+1},t)$$

Therefore, it follows that

$$\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},kt)\geq M(z,Tz,t)*\underline{\lim}_{n\to\infty} M(Tz,STx_{2n+1},t)$$

$$\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},t)\geq M(z,Tz,t)*M\left(z,Tz,\frac{t}{2}\right)*\underline{\lim}_{n\to\infty} M\left(z,STx_{2n+1},\frac{t}{2}\right)*\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},t)$$

$$\geq \underline{\lim}_{n\to\infty} M(z,STx_{2n+1},\lambda t)*\underline{\lim}_{n\to\infty} M\left(Tz,STx_{2n+1},\frac{\lambda t}{2}\right)$$

$$*\underline{\lim}_{n\to\infty} M\left(z,STx_{2n+1},\frac{t}{2}\right)*\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},t)$$

and

$$\overline{\lim}_{n\to\infty} N(z,STx_{2n+1},kt)\leq N(z,Tz,t)\Diamond\overline{\lim}_{n\to\infty} N(Tz,STx_{2n+1},t)\ \Diamond\underline{\lim}_{n\to\infty} M(z,STx_{2n+1},t)$$

$$\leq N(z,Tz,t)\Diamond N\left(z,Tz,\frac{t}{2}\right)\Diamond\overline{lim}_{n\to\infty} N\left(z,STx_{2n+1},\frac{t}{2}\right)\Diamond\overline{\lim}_{n\to\infty} N(z,STx_{2n+1},t)$$

$$\leq\overline{\lim}_{n\to\infty} N(z,STx_{2n+1},\lambda t)\Diamond\overline{\lim}_{n\to\infty} N\left(Tz,STx_{2n+1},\frac{\lambda t}{2}\right)\Diamond\overline{\lim}_{n\to\infty} N\left(z,STx_{2n+1},\frac{t}{2}\right)\Diamond\overline{\lim}_{n\to\infty} N(z,STx_{2n+1},t)$$

and for λ = 1,

 

$$\underline{\lim}_{n\to\infty} M\left(z, sTx_{2n+1}, kt\right) \geq \underline{\lim}_{n\to\infty} M\left(z, sTx_{xn+1}, \frac{t}{2}\right)$$

and

$$\overline{\lim}_{n\to\infty} N\left(z, sTx_{2n+1}, kt\right) \leq \overline{\lim}_{n\to\infty} N\left(z, sTx_{2n+1}, \frac{t}{2}\right)$$

it follows that $\underline{\lim}_{n\to\infty} sTx_{2n+1} = z$. Now using the compatibility, we have

$$M\left(Tz, z, t\right) \geq \underline{\lim}_{n\to\infty} M\left(z, sTx_{2n+1}, \lambda t\right) = 1$$

$N\left(Tz, z, t\right) \leq \overline{\lim}_{n\to\infty} N\left(z, sTx_{2n+1}, \lambda t\right) = 0$ and so it follows Tz = z.

Again, replacing x by x2n and y by z in (3.2) for α = 1, we have

$$M\left(Ax_{2n}, sz, kt\right) \geq M\left(Sx_{2n}, z, t\right) * M\left(Ax_{2n}, Sx_{2n}, t\right) * M\left(sz, z, t\right) * M\left(Ax_{2n}, z, t\right) * M\left(sz, Sx_{2n}, t\right)$$

and

$$N\left(Ax_{2n}, sz, kt\right) \leq N\left(Sx_{2n}, z, t\right) \lozenge N\left(Ax_{2n}, Sx_{2n}, t\right) \lozenge N\left(sz, z, t\right) \lozenge N\left(Ax_{2n}, z, t\right) \lozenge N\left(sz, Sx_{2n}, t\right)$$

and so letting n → ∞, we have
M(sz, z, kt) ≥ M(sz, z, t) and N(sz, z, kt) ≤ N(sz, z, t), sz = z.

Since B(X) ⊆ S(X), there exists a point $u \in X$ such that Su = z. By (3.2) for α = 1, we have

$$M\left(Au, z, kt\right) \geq M\left(Su, z, t\right) * M\left(Au, Su, t\right) * M\left(z, z, t\right) * M\left(Au, z, t\right) * M\left(z, Su, t\right)$$

and

$$N\left(Au, z, kt\right) \leq N\left(Su, z, t\right) \lozenge N\left(Au, Su, t\right) \lozenge N\left(z, z, t\right) \lozenge N\left(Au, z, t\right) \lozenge N\left(z, Su, t\right)$$

and also
M(Au, z, kt) ≥ M(Au, z, t) and N(Au, z, kt) ≤ N(Au, z, t), Au = z.

Since the pair (A, S) is compatible of type (I) and Au = Su = z, we have

M(Au, SSz, t) ≥ M(Au, ASz, t) and N(Au, SSz, t) ≤ N(Au, ASz, t)
and so
M(z, Sz, t) ≥ M(z, Az, t) and N(s, Sz, t) ≤ N(z, Az, t)

Again by (3.2), for α = 1, we have

M(Az, z, kt) ≥ M(Sz, z, t) * M(Az, Sz, t) *M(z, z, t) * M(Az, z, t) * M(z, Sz, t)
and
N(Az, z, kt) ≤ N(Sz, z, t) ◊ N(Az, Sz, t) ◊N(z, z, t) ◊ N(Az, z, t) ◊ N(z, Sz, t)

Thus it follows that

$$M(Az, z, kt) \geq M(Sz, z, t) * M(Az, Sz, t) * M(Az, z, t) \geq M(Az, z, \frac{t}{2})$$

and

$$N(Az, z, kt) \leq N(Sz, z, t) \lozenge N(Az, Sz, t) \lozenge N(Az, z, t) \geq N(Az, z, \frac{t}{2})$$

and so, by Lemma 2, Az = z. Therefore, Az = Sz = z and z is a common fixed point of A, S. The uniqueness of a common fixed point can be easily verified by using (3.2).

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