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PO IDEALS IN PARTIALLY ORDERED SEMIGROUPS

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ABSTRACT

In this paper, the terms, partially ordered semigroup, posubsemigroup, posubsemigroup generated by a subset, two sided identity of a posemigroup, left zero, right zero, zero of a posemigroup, poleft ideal,poright ideal, po ideal, po ideal generated bya subset and po ideal generated by an element a in a posemigroup are introduced.It is proved that, if S is a posemigroup and $A \subseteq S$, $B \subseteq S$, then (i) $A \subseteq (A]$, (ii) $((A]] = (A]$, (iii) $(A](B] \subseteq (AB]$ and (iv) $A \subseteq B \Rightarrow A \subseteq (B]$, (v) $A \subseteq B \Rightarrow (A] \subseteq (B]$.It is proved that the nonempty intersection of any family of posubsemigroups of a posemigroup S is a posubsemigroup of S. It is proved that (1) the nonempty intersection of any family of poleft ideals (orporight ideals orpoideals) of a posemigroupS is a poleft ideal (or po right ideals or po ideals) of S, (2) the union of any family of poleft ideals (or po right ideals or po ideals) of a posemigroup S is a poleft ideal (or po right ideals or po ideals) of S. Let S be a posemigroup and A is a nonempty subset of S, then it is proved that
(1) $L(A) = (A \cup SA]$
(2) $R(A) = (A \cup AS]$ and (3) $J(A) = (A \cup SA \cup SAS \cup AS]$.

Mathematical subject classification (2010): *20M07; 20M11; 20M12.*

KeyWords: *partially ordered semigroup, posubsemigroup, posubsemigroup generated by a subset, cyclic posubsemi - group of a posemigroup, two sided identity of a posemigroup, zero of a posemigroup,poleft ideal, poright ideal, poideal, po ideal generated by a subset and po ideal generated by an element a.*

1. INTRODUCTION

The algebraic theory of semigroups was widely studied by CLIFFORD [2], [3], PETRICH [5]. The ideal theory in general semigroups was developed by ANJANEYULU [1].JIAN TANG and XIANG YUN XIE [4] studied on radicals of ideals of ordered semigroups. In this paper we introduce the notions of ordered subsemigroups and characterize ordered subsemigroups and the notions of ordered ideals and characterize partially ordered ideals in posemigroups.

2. PARTIALLY ORDERED SEMIGROUPS

Definition 2.1: A semigroup S is said to be a *partially ordered semigroup* if S is a partially ordered set such that $a \le b \Rightarrow ax \le bx$, $xa \le xb$ for all $a, b, x \in$ S.

Note 2.2: A partially ordered semigroup is also called as po semigroup or ordered semigroup.

Notation 2.3: Let S be a po semigroup and T be a nonempty subset of S. If H is a nonempty subset of T, we denote $\{t \in T : t \le h$ for some $h \in H\}$ by $(H]_T$.

Notation 2.4: Let S be a po semigroup and T be a nonempty subset of S. If H is a nonempty subset of T, we denote $\{t \in T: h \le t$ for some $h \in H\}$ by $[H)_T$.

Note 2.5: $(H]_T$ and $[H)_T$ are simply denoted by (H] and [H) respectively.

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Example 2.6: Let S = {∅, {a}, {b}, {c}, {a, b}, {b, c}, {a, c}, {a, b, c}}. If for all A, B ∈ S, AB = A ∩ B and A ≤ B ⟺ A ⊆ B, then S is a po semigroup.

Note 2.7: In general, let P(X) be the power set of any nonempty set X. If we define for all A, B ∈ S, AB = A ∩ B and A ≤ B ⟺ A ⊆ B, then P(X) is a posemigroup.

Example 2.8: For a, b∈ [0, 1], let M = [0, a]. Then M is a po semigroup under usual multiplication and usual partial order relation.

Theorem 2.9: Let S be a po semigroup and A ⊆S, B ⊆S. Then
(i) A ⊆ (A] (ii) ((A]] = (A] (iii) (A](B] ⊆ (AB] (iv) A ⊆ B ⇒ A ⊆ (B] and(v) A ⊆ B ⇒ (A] ⊆ (B].

***Proof*:**
(i) Let x∈ A. x∈ A, x∈ T and $x \leq x \Rightarrow x$∈ (A].

Therefore A⊆ (A].

(ii) Let x∈ ((A]] ⇒$x \leq y$ for some y∈ (A].

y∈ (A] ⇒$y \leq z$ for some z∈ A.

$x \leq y$, $y \leq z \Rightarrow x \leq z$.

$x \leq z$, z∈ A ⇒x∈ (A].

Therefore ((A]] ⊆ (A] and from (i) (A] ⊆ ((A]] and hence ((A]] = (A].

(iii) Let x∈ (A] (B] ⇒$x = ab$ where a∈ (A], b∈ (B].

a∈ (A] ⇒$a \leq \alpha$ for some α ∈ A ⇒$ab \leq \alpha b$.

b∈ (B] ⇒$b \leq \beta$ for some β ∈ B ⇒ $\alpha b \leq \alpha \beta$.

Now $x = ab \leq \alpha b \leq \alpha \beta$ where $\alpha \beta$ ∈ AB ⇒x∈ (AB].

Therefore (A](B]⊆ (AB].

(iv) From (i) B ⊆ (B] ⇒A⊆ B ⊆ (B].

(v) A ⊆ B ⇒ A ⊆ (B] ⇒ (A] ⊆ ((B]] = (B]. Therefore (A] ⊆ (B].

Definition 2.10: An element a of a posemigroup S is said to be a *left identity*of S provided as=s and $s \leq a$ for all s∈S.

Note 2.11: Left identity element a of a posemigroup S is also called as *left unital element.*

Definition 2.12: An element a of a posemigroup S is said to be a *right identity*of Sprovided sa=s and $s \leq a$ for all s∈S.

Note 2.13: Right identity element a of a posemigroup S is also called as *right unital element.*

Definition 2.14: An element 'a' of a posemigroup S is said to be a *two sided identity* provided it is both a left identity and a right identity of S.

Note 2.15: An element 'a' of a posemigroup S is a *two sided identity* provided $as = sa = s$ and $s \leq a$ for all s∈S.

Note 2.16: Two sided identity element of a posemigroup S is also called as *bi-unital element.*

Theorem 2.17: If a is a left identity element and b is a right identity element of a posemigroup S, then $a = b$.

***Proof*:** Since a is a left identity of S, as=$s$and $s \leq a$ for all s∈S and

Hence $ab = b$ and $b \leq a$.

Since b is a right identity of S, $sb= s$and $s\leq b$ for all $s \in$ S and

Hence $ab = a$ and $a \leq b$.

Now $b \leq a$ and$a \leq b \Rightarrow a = b$.

Theorem 2.18: A posemigroup S has at most one two sided identity.

***Proof*: Let** a, b be two sided identity elements of the posemigroup S.

Now a can be considered as a left identity and b can be considered as a right identity of S.

By theorem 2.17, $a = b$. Then S has at most one two sided identity.

Definition 2.19: An element a of a posemigroup S is said to be a *left zero* of S provided $ab = a$ and $a \leq b$ for all $a, b \in$ S.

Definition 2.20: An element a of a posemigroup S is said to be a *right zero* of S provided $ba = a$ and $a \leq b$ for all $a, b \in$ S.

Definition 2.21: An element a of a posemigroup S is said to be a *two sided zero*or *zero* of S provided $ab = ba = a$ and $a \leq b$ for all $a, b \in$ S.

Note 2.22: If a is a two sided zero of a posemigroup S, then a is both a left zero and a right zero of S.

Theorem 2.23: If a is a left zero and$b$ is a right zero of a posemigroup S, then $a = b$.

Proof: Since a is a left zero of S, $ab = a$ and $a \leq b$ for all $b, c \in$ S.

Since b is a right zero of S, $ab = b$ and $b \leq a$ for all $a, b \in$ S.

Therefore $a = b$.

Theorem 2.24: Any posemigroup has at most one zero element.

Proof: Let a, b, c be three zeros of a posemigroup S.

Now a can be considered as a left zero and b can be considered as a right zero of S.

By theorem 2.23, $a = b$.

Then S has at most one zero.

Note 2.25: The zero (if exists) of a posemigroup is usually denoted by 0.

3. PARTIALLY ORDERED SUBSEMIGROUPS

Definition 3.1: Let S be a posemigroup. A nonempty subset T of S is said to be a *posubsemigroup* of S if (i) $ab \in$T for all $a, b \in$T, (ii) $s \in$S, $t \in$T, $s \leq t \Rightarrow s \in$T.

Note 3.2: A nonempty subset T of a posemigroup S is a posubsemigroup of S iff (1) TT$\subseteq$T, (2) (T] = T.

Example 3.3: Let S = [0, 1]. Then S is po semigroup under the usual multiplication and usual order relation. Let T = [0, 1/2].

Then T is posubsemigroup of S.

Theorem 3.4: The nonempty intersection of two po subsemigroups of a po semigroup S is a po subsemigroup of S.

Proof: Let S_1, S_2 be two po subsemigroups of S. Let $a, b \in S_1 \cap S_2$.

$a, b \in S_1 \cap S_2 \Rightarrow a, b \in S_1$ and $a, b \in S_2$.

$a, b \in S_1$, S_1 is a po subsemigroup of $S \Rightarrow ab \in S_1$ and $(S_1] = S_1$

$a, b \in S_2$, S_2 is a po subsemigroup of $S \Rightarrow ab \in S_2$ and $(S_2] = S_2$

$ab \in S_1$, $ab \in S_2 \Rightarrow ab \in S_1 \cap S_2$ and $S_1 \cap S_2 \subseteq S_1, S_1 \cap S_2 \subseteq S_2$

$\Rightarrow (S_1 \cap S_2] \subseteq (S_1] = S_1$ and $(S_1 \cap S_2] \subseteq (S_2] = S_2 \Rightarrow (S_1 \cap S_2] \subseteq S_1 \cap S_2 \Rightarrow (S_1 \cap S_2] = S_1 \cap S_2$ and hence $S_1 \cap S_2$ is a po subsemigroup of T.

Theorem 3.5: The nonempty intersection of any family of po subsemigroups of a po semigroup S is a posubsemigroup of S.

Proof: Let $\{S_\alpha\}_{\alpha \in \Delta}$ be a family of posubsemigroups of S and let T = $\bigcap_{\alpha \in \Delta} S_\alpha$.

Let $a, b \in T$.

$a, b \in T \Rightarrow a, b \in \bigcap_{\alpha \in \Delta} S_\alpha \Rightarrow a, b \in S_\alpha$ for all $\alpha \in \Delta$.

$a, b \in S_\alpha$, S_α is a posubsemigroup of $S \Rightarrow ab \in S_\alpha$ and $(S_\alpha] = S_\alpha$

$\Rightarrow ab \in S_\alpha$ for all $\alpha \in \Delta \Rightarrow ab \in \bigcap_{\alpha \in \Delta} S_\alpha$ and $(\bigcap_{\alpha \in \Delta} S_\alpha] \subseteq \bigcap_{\alpha \in \Delta} S_\alpha$

$\Rightarrow ab \in T$ and $(T] \subseteq T$. Therefore T is a po subsemigroup of S.

Definition 3.6: Let S be a po semigroup and A be a nonempty subset of S. The smallest po subsemigroup of S containing A is called a *posubsemigroup of* S *generated by* A. It is denoted by (A).

Theorem 3.7: Let S be a posemigroup and A be a nonempty subset of S. Then (A) = The intersection of all po subsemigroups of S containing A.

***Proof*:** Let Δ be the set of all posubsemigroups of S containing A.

Since S is a po ternary subsemigroup of S containing A, $S \in \Delta$. So $\Delta \neq \emptyset$.

Let $S^* = \bigcap_{S \in \Delta} S$. Since $A \subseteq S$ for all $S \in \Delta$, $A \subseteq S^*$ and hence $S^* \neq \emptyset$.

By theorem 3.5, S^* is a posubsemigroup of S.

Since $S^* \subseteq S$ for all $S \in \Delta$, S^* is the smallest posubsemigroup of S containing A.

Therefore $S^* = (A)$.

4. PARTIALLY ORDERED IDEALS

Definition 4.1: A nonempty subset A of a posemigroup S is said to be *po left ideal*of S if
i) $b \in S$, $a \in A \Rightarrow ba \in A$
ii) $a \in A$ and $s \in S$ such that $s \leq a \Rightarrow s \in A$.

Note 4.2: A nonempty subset A of a posemigroup S is a po left ideal of S if and only if i) $SA \subseteq A$ ii) $(A] \subseteq A$.

Definition 4.3: A nonempty subset A of a posemigroup S is said to be *po right ideal* of S if i) $b \in S$, $a \in A \Rightarrow ab \in A$
ii) $a \in A$ and $s \in S$ such that $s \leq a \Rightarrow s \in A$.

Note 4.4: A nonempty subset A of a posemigroup S is a po right ideal of S if and only if i) $AS \subseteq A$ ii) $(A] \subseteq A$.

Definition 4.5: A nonempty subset A of a po semigroup S is said to be *po two sided ideal*or*po ideal*of S if i) $b \in$ S, $a \in$ A $\Rightarrow ba \in$ A, $ab \in$ A ii) $a \in$ A and $s \in$ S such that $s \leq a \Rightarrow s \in$ A.

Note 4.6: A nonempty subset A of a posemigroup S is a po ideal of S if and only if i) SA $\subseteq$ A, AS $\subseteq$ A ii) (A] $\subseteq$ A.

Note 4.7: A nonempty subset A of a po semigroup S is a po two sided ideal of S if and only if it is both a po left ideal and a po right ideal of S.

Example 4.8: Let $M = \{a, b, c\}$ with the multiplication and the relation $\leq$ on M defined by $xy = \begin{cases} b \text{ if } x, y \in \{a, b\} \\ c \text{ otherwise} \end{cases}$ and $\leq := \{(a, a), (b, b), (c, c), (d, d), (b, c), (b, d), (c, d)\}$. Then M is a po semigroup and $\{b, c\}$ is a poideal of M.

Theorem 4.9: The nonempty intersection of any two po left ideals (or po right ideals or po ideals) of a po semigroup S is a po left ideal (or po right ideals or po ideals) of S.

***Proof*:** Let A, B be two po left ideals of S.

Let $a \in$ A $\bigcap$ B and $b \in$ S

$a \in$ A $\bigcap$ B $\Rightarrow a \in$ A and $a \in$ B

$a \in$ A ; $b \in$ S, A is a po left ideal of S $\Rightarrow ba \in$ A.

$a \in$ B ; $b \in$ S, B is a po left ideal of S $\Rightarrow ba \in$ B.

$ba \in$ A, $ba \in$ B $\Rightarrow ba \in$ A $\bigcap$ B.

Let $a \in$ A $\bigcap$ Band $s \in$S such that $s \leq a$.

$a \in$ A $\bigcap$ B $\Rightarrow a \in$ A and $a \in$ B.

$a \in$ A, $s \in$ S, $s \leq a$, A is a po left ideal of S $\Rightarrow s \in$ A.

$a \in$ B, $s \in$ S, $s \leq a$, B is a po left ideal of S $\Rightarrow s \in$ B.

Therefore $s \in$ A, $s \in$ B $\Rightarrow s \in$ A $\bigcap$ B.

Hence A $\bigcap$ B is a poleft ideal of S.

Similarly we can prove the other cases.

Theorem 4.10: The nonempty intersection of any family of po left ideals (or po right ideals or po ideals) of a po semigroup S is a po left ideal (or po right ideals or po ideals) of S.

***Proof*:** Let $\{A_\alpha\}_{\alpha \in \Delta}$ be a family of po left ideals of S and let A = $\bigcap_{\alpha \in \Delta} A_\alpha$ Let $a \in$ A ;$b \in$ S.

Now $a \in$ A $\Rightarrow a \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, $b \in$ S, A_α is a po left ideal of S $\Rightarrow ba \in A_\alpha$.

$ba \in A_\alpha$ for all $\alpha \in \Delta \Rightarrow ba \in \bigcap_{\alpha \in \Delta} A_\alpha = A$.

Let $a \in$ A and $s \in$ S such that $s \leq a$.

$a \in$ A = $\bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for each $\alpha \in \Delta$.

$a \in A_\alpha$, $s \in S$, $s \le a$, A_α is a poleft ideal of S $\Rightarrow s \in A_\alpha$ for each $\alpha \in \Delta$

$s \in A_\alpha$ for each $\alpha \in \Delta \Rightarrow s \in \bigcap_{\alpha \in \Delta} A_\alpha \Rightarrow s \in A$.

Hence A is a po left ideal of S.

Similarly we can prove the other cases.

Theorem 4.11: The union of any two po left ideals (or po right ideals or po ideals) of a po semigroup S is a po left ideal (or po right ideals or po ideals) of S.

Proof: Let A_1, A_2 be two poleft ideals of a po semigroup S.

Let $A = A_1 \cup A_2$. Clearly A is a nonempty subset of S.

Let $a \in A$ and $b \in$ S. Now $a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.

If $a \in A_1$ then $a \in A_1$; $b \in$ S; A_1 is a po left ideal of S $\Rightarrow ba \in A_1$

$ba \in A_1 \subseteq A_1 \cup A_2 = A \Rightarrow ba \in A$.

If $a \in A_2$ then $a \in A_2$; $b \in$ S; A_2 is a po left ideal of S $\Rightarrow ba \in A_2$

$ba \in A_2 \subseteq A_1 \cup A_2 = A \Rightarrow ba \in A$.

Let $a \in A$ and $s \in$ S such that $s \le a$.

$a \in A \Rightarrow a \in A_1 \cup A_2 \Rightarrow a \in A_1$ or $a \in A_2$.

If $a \in A_1$ then $a \in A_1$, $s \in$ S, $s \le a$, A_1 is a poleft ideal of S $\Rightarrow s \in A_1 \subseteq A_1 \cup A_2 = A$

If $a \in A_2$ then $a \in A_2$, $s \in$ S, $s \le a$, A_2 is a poleft ideal of S $\Rightarrow s \in A_2 \subseteq A_1 \cup A_2 = A$

Therefore $a \in$ A and $s \in$ S such that $s \le a \Rightarrow s \in$ A.

Hence A is a po left ideal of S.

Similarly we can prove the other cases.

Theorem 4.12: The union of any family of po left ideals (or po right ideals or po ideals) of a po semigroup S is a po left ideal (or po right ideals or po ideals) of S.

Proof: Let $\{A_\alpha\}_{\alpha \in \Delta}$ be a family of po left ideals of a po semigroup S.

Let $A = \bigcup_{\alpha \in \Delta} A_\alpha$. Clearly A is a non-empty subset of S.

Let $a \in A$ and $b \in$ S. $a \in A \Rightarrow a \in \bigcup_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, $b \in$ T, A_α is a po left ideal of S $\Rightarrow ba \in A_\alpha \subseteq \bigcup_{\alpha \in \Delta} A_\alpha = A \Rightarrow ba \in A$.

Let $a \in$ A and $s \in$ S such that $s \le a$.

$a \in A \Rightarrow a \in \bigcup_{\alpha \in \Delta} A_\alpha \Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta$.

$a \in A_\alpha$, $s \in$S , $s \le a$, A_α is a po left ideal of S $\Rightarrow a \in A_\alpha$ for some $\alpha \in \Delta \Rightarrow s \in A_\alpha \subseteq \bigcup_{\alpha \in \Delta} A_\alpha$

Therefore $s \in \bigcup_{\alpha \in \Delta} A_\alpha = A$

Therefore A is a po left ideal of S.

Similarly we can prove the other cases.

5. IDEALS GENERATED BY A SUBSET

Definition 5.1: Let S be a po semigroup and A be a nonempty subset of S. The smallest po left ideal of S containing A is called *po left ideal of S generated by A* and it is denoted by L(A).

Theorem 5.2: If S is a po semigroup and A is a nonempty subset of S, then L(A) =(A ∪SA].

***Proof*:** Let $b \in$S and $r \in$ (A∪SA].

$r \in$(A∪ SA]$\Rightarrow r \leq x$ for some $x \in$A∪SA.

$x \in$A∪ SA $\Rightarrow x \in$ A or $x \in$SA.

If $x \in$ A then $br \leq bx \in$SA⊆A∪ SA $\Rightarrow$ $br \in$(A∪ SA].

If $x \in$SA then $x = ya$ where $y \in$S and $a \in$ A.

$br \leq bx = bya \in$SA ⊆A∪ SA $\Rightarrow$ $br \in$(A∪ SA].

Therefore $b \in$S and $r \in$ (A∪ SA] $\Rightarrow$ $br \in$(A∪ SA].

Let $t \in$ (A∪ SA] and $s \in$S such that $s \leq t$.

$t \in$(A∪ SA] $\Rightarrow t \leq x$ for some $x \in$A∪ SA.

$s \leq t$, $t \leq x \Rightarrow s \leq x$. $s \in$S, $s \leq x$, $x \in$A∪ SA $\Rightarrow s \in$(A∪ SA].

Therefore (A∪ SA] is a po left ideal of S.

Let L be a po left ideal of S containing A. A ⊆ L, L is a po left ideal of S⇒SA ⊆SL ⊆ L. A ⊆ L, SA ⊆ L ⇒A∪ SA ⊆ L ⇒ (A∪ SA] ⊆ L

Therefore (A∪ SA] is the smallest po left ideal containing A.

Therefore L(A) = (A∪ SA].

Note 5.3: (A∪SA] is also denoted as (S^1A]

Theorem 5.4: The po left ideal (or po right ideal or po ideal) of a po semigroup S generated by a nonempty subset A is the intersection of all po left ideals ideal (or po right ideal or po ideal) of S containing A.

***Proof*:** Let Δ be the set of all po left ideals of S containing A.

Since S itself is a po left ideal of S containing A, S∈ Δ. So $\Delta \neq \emptyset$.

Let $S^* = \bigcap_{S \in \Delta} S$. Since $A \subseteq S$ for all S∈ Δ, it follows that $A \subseteq S^*$.

By theorem 4.10, S^* is a po left ideal of S.

Let K be a po left ideal of S containing A.

Clearly A ⊆ K and K is a po left ideal of S.

Therefore K $\in \Delta \Rightarrow S^* \subseteq$ K. Therefore S^* is the po left ideal of S generated by A.

Definition 5.5: A po left ideal A of a po semigroup S is said to be the *principal po left ideal generated by an element a* if A is a po left ideal generated by $\{a\}$ for some $a \in S$. It is denoted by L(a).

Theorem 5.6: If S is a po semigroup and $a \in S$ then L(a) = ($a \cup Sa$].

Proof: Proof is similar to theorem 5.2.

Definition 5.11: Let S be a po semigroup and A be a nonempty subset of S. The smallest po right ideal of S containing A is called *po right ideal of S generated by A*and it is denoted by R(A).

Theorem 5.12: Let S be a po semigroup and A is a nonempty subset of S, then R(A) = (A ∪ AS].

Proof*: *Proof is similar to theorem 5.2.

Definition 5.13: A po right ideal A of a po semigroup S is said to be the *principalpo right ideal generated by an element a* if A is a po right ideal generated by $\{a\}$ for some $a \in S$. It is denoted by R(a).

Theorem 5.14: If S is a po semigroup and $a \in S$ then R(a) = ($a \cup aS$].

Proof: Proof is similar to theorem 5.2.

Definition 5.15: Let S be a po semigroup and A be a nonempty subset of S. The smallest po two sided ideal of S containing A is called *po two sided ideal of S generated by A*and it is denoted by J(A).

Theorem 5.16: Let S be a po semigroup and A is a nonempty subset of S, then J(A) = (A∪SA∪AS∪SAS].

***Proof*:** Let $b \in S$ and $r \in$ (A∪SA∪AS∪SAS].

$r \in$ (A∪ SA∪AS ∪SAS] $\Rightarrow r \leq x$ for some $x \in$ A∪ SA∪AS ∪SAS.

$x \in$ A∪ SA∪AS ∪SAS $\Rightarrow x \in$ A or $x \in$ SA or $x \in$ AS or $x \in$ SAS.

If $x \in$ A then $br \leq bx \in SA \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow br \in$ (A∪ SA∪AS ∪SAS] and $rb \leq xb \in AS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow rb \in$ (A∪ SA∪AS ∪SAS]

If $x \in$ SA then $x = ya$ where $y \in S$ and $a \in A$.
$br \leq bx = bya \in SA \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow br \in$ (A∪ SA∪AS ∪SAS] and $rb \leq xb = yab \in SAS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow rb \in$ (A∪ SA∪AS ∪SAS]

If $x \in$ AS then $x = ay$ where $y \in S$ and $a \in A$.
$br \leq bx = bay \in SAS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow br \in$ (A∪ SA∪AS ∪SAS] and $rb \leq xb = ayb \in AS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow rb \in$ (A∪ SA∪AS ∪SAS]

If $x \in$ SAS then $x = yau$ where $y, u \in S$ and $a \in A$.

$br \leq bx = byau \in SAS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow br \in$ (A∪ SA∪AS ∪SAS] and $rb \leq xb = yaub \in SAS \subseteq$ A∪ SA∪AS ∪SAS $\Rightarrow rb \in$ (A∪ SA∪AS ∪SAS]

Therefore $b \in S$ and $r \in$ (A∪ SA∪AS ∪SAS] $\Rightarrow br, rb \in$ (A∪ SA∪AS ∪SAS].

Let $t \in$ (A∪ SA∪AS ∪SAS] and $s \in S$ such that $s \leq t$.

$t \in$ (A∪ SA∪AS ∪SAS] $\Rightarrow t \leq x$ for some $x \in$ A∪ SA∪AS ∪SAS.

$s \leq t, t \leq x \Rightarrow s \leq x$.

$s \in S, s \leq x, x \in$ A∪ SA∪AS ∪SAS $\Rightarrow s \in$ (A∪ SA∪AS ∪SAS].

Therefore (A∪ SA∪AS ∪SAS] is a po two sided ideal of S.

Let J be a po two sided ideal of S containing A.

A ⊆J, J is a po two sided ideal of S⇒SA ⊆SJ⊆J.

A ⊆J, J is a po two sided ideal of S⇒AS⊆JS⊆J.

A ⊆J, J is a po two sided ideal of S⇒SAS⊆SJS⊆J.

A ⊆J, SA ⊆J, AS ⊆J, SAS⊆J⇒A∪ SA∪AS ∪SAS ⊆J

⇒ (A∪ SA∪AS ∪SAS] ⊆J.

Therefore (A∪ SA∪AS ∪SAS] is the smallest po lateral ideal containing A.

Therefore J(A) = (A∪ SA∪AS ∪SAS].

Definition 5.17: A po two sided ideal A of a po semigroup S is said to be the *principalpo two sided ideal generated by an element a* if A is a po two sided ideal generated by {*a*} for some *a*∈S. It is denoted by J(*a*) or <*a*>.

Theorem 5.18: If S is a po semigroup and *a*∈S then J(*a*) = (*a*∪Sa∪aS∪SaS].

***Proof*:** Proof is similar to theorem 5.16.

Definition 5.19: An ideal A of a po semigroup S is said to be a *proper poideal* of S if A is different from S.

Definition 5.20: An ideal A of a po semigroup S is said to be a *trivial poideal* provided S\ A is singleton.

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