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n^{th} FINITE INTEGRALS INVOLVING PRODUCT OF MULTIVARIABLE JACOBI POLYNOMIAL AND H-FUNCTION OF SEVERAL VARIABLES

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ABSTRACT

In the present paper few finite integrals involving products of multivariable Jacobi polynomial and H function of several variables of generalized arguments have been evaluated. These integrals have been utilized to established the expansion formula for H[Z_1, Z_2…,Z_r] function in series involving product of multivariable Jacobi polynomials since H function of multivariable quite general function in nature. On specializing the parameters of the function involved in results many new as well as known relations may be obtained as particular case.

Key words: *Multivariable Jacobi Polynomials.*

1. INTRODUCTION

The classical Jacobi polynomial for one variable may be defined as [1; equation (1), p. 254]

$$P_n^{(\alpha,\beta)}(x)=\frac{(1+\alpha)_n}{n!}\,2F_1\left(-n;1+\alpha+\beta+n;1+\alpha;\frac{1-x}{2}\right) \tag{1.11}$$

Two variable, three variable and multivariable Jacobi polynomial defined by Shrivastava [2, equ. (1.3) (1.5) (1.7) pg 159-161] in the following manner

$$P_n^{(\alpha_1,\beta_1;\alpha_2,\beta_2)}(x_1,x_2)=\frac{(1+\alpha_1)_n(1+\alpha_2)_n}{(n!)^2}\,3F_2\left(\begin{matrix}-n;1+\alpha_2+\beta_2+n;1+\alpha_1+\beta_1+n;\\ 1+\alpha_2;1+\alpha_1;\frac{1-x_2}{2}.\frac{1-x_1}{2}\end{matrix}\right) \tag{1.12}$$

$$P_n^{(\alpha_1,\beta_1;\alpha_2,\beta_2;\alpha_3,\beta_3)}(x_1,x_2,x_2)=\frac{(1+\alpha_1)_n(1+\alpha_2)_n(1+\alpha_3)_n}{(n!)^2}$$
$$F^{(3)}\left(\begin{matrix}-n;-;-;-;\ 1+\alpha_3+\beta_3+n;\ 1+\alpha_2+\beta_2+n;1+\alpha_1+\beta_1+n;\\ -;-;-;\ 1+\alpha_3;\ 1+\alpha_2;\ 1+\alpha_1;\left(\frac{1-x_3}{2}\right)\left(\frac{1-x_2}{2}\right)\left(\frac{1-x_1}{2}\right)\end{matrix}\right) \tag{1.13}$$

$$P_n^{(\alpha_1,\beta_1;\ \dots\ ;\alpha_p,\beta_p)}(x_1,\dots,x_p)=\prod_{j=1}^{p}\frac{(1+\alpha_p)_n}{(n!)^p}$$
$$F_{\ 0;1;\dots;1}^{\ 1;1;\dots;1}\left(\begin{matrix}-n;1+\alpha_p+\beta_p+n;\dots;1+\alpha_1+\beta_1+n;\\ 1+\alpha_p;\dots;\ 1+\alpha_1;\ \frac{1-x_p}{2}.\frac{1-x_{p-1}}{2}\dots.\frac{1-x_1}{2}\end{matrix}\right) \tag{1.14}$$

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Function F in R.H.S. of equation (1.12) (1.13) (1.14) represents Apples function for two variable, Hypergeometric function for three variables and Kample de fernet for multivariable.

$$H[Z_1, Z_2, ..., Z_r] = H^{O,N;\{M_i,N_i\}}_{A,B;\{A_i,B_i\}}\begin{bmatrix} Z_1 \\ . \\ . \\ Z_r \end{bmatrix}\begin{bmatrix} S\,;T \\ . \\ . \\ S';T' \end{bmatrix}$$

$$= H^{O,N;\{M_1,N_1;M_2,N_2;...;M_r,N_r\}}_{A,B;\{A_1,B_1;A_2,B_2;...;A_r,B_r\}}\begin{bmatrix} Z_1 \\ . \\ . \\ Z_r \end{bmatrix}\begin{bmatrix} \left(a_j;\alpha_j^{(i)};...;\alpha_j^{(r)}\right)_{1,A}\left(C_j^{(i)},E_j^{(i)}\right)_{1,A_r} \\ . \\ . \\ \left(b_j;\beta_j^{(i)};...;\beta_j^{(r)}\right)_{1,B}\left(d_j^{(i)},\delta_j^{(i)}\right)_{1,B_r} \end{bmatrix}$$

$$= \frac{1}{(2\pi\omega)^r}\int_{L_1}...\int_{L_r}\theta(S_1...S_r)\prod_{i=1}^{r}\left\{\phi_i(S_i)Z_i^{S_i}\,dS_i\right\} \tag{1.21}$$

where $\omega = \sqrt{-1}$

$$S = \left(a_j, \alpha_j^{(i)}...\alpha_j^{(r)}\right)_{1.A} \tag{1.22}$$

$$T = \left(C_j^{(i)}, E_j^{(i)}\right)_{1.A_r} \tag{1.24}$$

$$S' = \left(b_j, \beta_j^{(i)}...\beta_j^{(r)}\right)_{1.B} \tag{1.25}$$

$$T' = \left(d_j^{(i)}, \delta_j^{(i)}\right)_{1.B_r} \tag{1.26}$$

$$\theta(S_1...S_r) = \frac{\prod_{j=1}^{N}\Gamma\left(1-a_j+\sum_{i=1}^{r}\alpha_j S_i\right)}{\prod_{j=N+1}^{A}\Gamma\left(\alpha_j-\sum_{i=1}^{r}\alpha_i^{(i)}S_i\right)\prod_{j=1}^{B}\Gamma\left(1-b_j+\sum_{i=1}^{r}\beta_j S_i\right)}$$

$$\phi_i(S_i) = \frac{\prod_{j=1}^{M_i}\Gamma\left(d_j^{(i)}-\delta_j^{(i)}S_i\right)\prod_{j=1}^{N_i}\Gamma\left(1-C_j^{(i)}+E_j^{(i)}S_i\right)}{\prod_{j=M_i+1}^{B_i}\Gamma\left(1-d_j^{(i)}-\delta_j^{(i)}S_i\right)\prod_{j=N+1}^{A_i}\Gamma\left(C_j^{(i)}-E_j^{(i)}S_i\right)} \tag{1.27}$$

i denotes the no of dashes for example $b_i^{(i)} = b'$, $b_i^{(2)} = b''$ …etc.

For the expansion of multivariable H function and restriction of parameters see the research paper of Saxena, Shrivastava and Panda [3, pg. 271, equ.(4.1)]. In this research paper we will use the following result [4, pg 58]

$$\int_{-1}^{1}\int_{-1}^{1}.....\int_{-1}^{1}\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\alpha_p}(1+x_p)^{\sigma_p}\right]P_n^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\,dx_1.dx_2...dx_r$$

$$=\frac{1}{(n!)^r}\prod_{p=1}^{r}\left[\frac{(-1)^{np}\,2^{\sigma_p+\alpha_p+\frac{1}{r}}\,\Gamma(\alpha_p+n+1)\Gamma(\sigma_p-\beta_p+1)\Gamma(\sigma_p+1)}{\Gamma(\sigma_p-\beta_p-n+1)\Gamma(\sigma_p+\alpha_p+n+2)}\right]$$

$${}_3F_2\left[\begin{matrix}\prod_{p=1}^{r}(-\lambda_p);\prod_{p=1}^{r}(\sigma_p-\beta_p+1);\prod_{p=1}^{r}(\sigma_p+1)\\ \prod_{p=1}^{r}(\sigma_p-\beta_p-n+1);\prod_{p=1}^{r}(\sigma_p+\alpha_p+n+2)\quad;2^r\end{matrix}\right]$$

(1.28)

$$\int_{-1}^{1}\int_{-1}^{1}.....\int_{-1}^{1}\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\rho_p}(1+x_p)^{\beta_p}\right]P_n^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\,dx_1.dx_2...dx_r$$

Third Integral

$$\int_{-1}^{1}\int_{-1}^{1}.....\int_{-1}^{1}\left\{\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\rho_p}(1+x_p)^{\beta_p}\right]\right\}$$

$$P_n^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\,H\left[Z_1x_1^{\mu_1}(1-x_1)^{\delta_1}.......Z_rx_r^{\mu_r}(1-x_r)^{\delta_r}\right]dx_1.dx_2...dx_r$$

$$=\frac{1}{(n!)^r}\prod_{p=1}^{r}\left[\frac{(-1)^{np}\,2^{\rho_p+\beta_p+\frac{1}{r}}\,\Gamma(\beta_p+n+1)\Gamma(\rho_p-\beta_p+1)\Gamma(\rho_p+1)}{\Gamma(\rho_p-\alpha_p-n+1)\Gamma(\rho_p+\beta_p+n+2)}\right]$$

$${}_3F_2\left[\begin{matrix}\prod_{p=1}^{r}(-\lambda_p);\prod_{p=1}^{r}(\rho_p-\beta_p+1);\prod_{p=1}^{r}(\rho_p+1)\\ \prod_{p=1}^{r}(\rho_p-\alpha_p-n+1);\prod_{p=1}^{r}(\sigma_p+\beta_p+n+2)\quad;2^r\end{matrix}\right]$$

(1.29)

2. In this section we evaluate multivariable H function consisting four product of Jacobi function

We will use this integral in the expansion of H [Z_1……Z_r]

$$\int_{-1}^{1}\int_{-1}^{1}.....\int_{-1}^{1}\left\{\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\alpha_p}(1+x_p)^{\sigma_p}\right]\right\}$$

$$P_n^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\,H\left[Z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1}.......Z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right]dx_1.dx_2...dx_r$$

$$=\left\{\prod_{p=1}^{r}\left[(-1)^{np}\,2^{\sigma_p+\alpha_p+\frac{1}{r}}\,\Gamma(\alpha_p+n+1)\left(\sum_{k_p=0}^{\infty}\frac{2^{K_p}}{(k_p)!}\right)\right]\right\}H^{O,N;m_1,n_1+3;m_2,n_2+3;...;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;...;A_r+3,B_r+3}$$

$$\left[\begin{matrix}Z_1\;2^{\mu_1}\\ .\quad .\\ .\quad .\\ Z_r\;2^{\mu_r}\end{matrix}\left|\begin{matrix}\prod_{p=1}^{r}(\lambda_p+1-k_p;\mu_p),\prod_{p=1}^{r}(\beta_p-\sigma_p-k_p;\mu_p),\prod_{p=1}^{r}(-\sigma_p-k_p;\mu_p)\\ \\ \prod_{p=1}^{r}(\lambda_p+1;\mu_p),\prod_{p=1}^{r}(\beta_p-\sigma_p+n-k_p;\mu_p),\prod_{p=1}^{r}(-1-\sigma_p-\alpha_p-n-k_p;\mu_p)\end{matrix}\right.\right] \quad (2.1)$$

Integral (2.1) is valid under the restriction of

Re $(\lambda_i)>-1$,

Re $(\alpha_i)>-1$,

$$\left|\arg Z_i\right| > \frac{\Omega\pi_i}{2},$$

Re $(\sigma_i+\mu_j\,\xi_j)>0$,

Re $(\lambda_i-\mu_j\,\xi_j)>0$

where $\xi_j = \sum_{j=1}^{r} \min_{1\le m_r}\left(\mathrm{Re}\left(\frac{d_j^r}{\delta_j^r}\right)\right)\ \forall\ i,j=1...r$

where all $\mu_1, \mu_2, ...\mu_r$ are positive and are not equal to zero.

Second integral

$$\int_{-1}^{1}\int_{-1}^{1}.....\int_{-1}^{1}\left\{\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\alpha_p}(1+x_p)^{\sigma_p}\right]\right\}$$

$$P_n^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\,H\left[Z_1x_r(1+x_1)^{\mu_1}.......Z_rx_r(1+x_r)^{\mu_r}\right]dx_1.dx_2...dx_r$$

$$=\frac{1}{(n!)^r}\left\{\prod_{p=1}^{r}\left[(-1)^{np}\,2^{\sigma_p+\alpha_p+\frac{1}{r}}\left(\alpha_p+n+1\right)\left(\sum_{k_p=0}^{\infty}\frac{2^{K_p}}{(k_p)!}\right)\right]\right\}H^{O,N;m_1,n_1+3;m_2,n_2+3;...;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;...;A_r+3,B_r+3}$$

$$\left[\begin{matrix} Z_1 & 2^{\delta_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\delta_r}\end{matrix}\left|\begin{matrix}\prod_{p=1}^{r}\left(-\lambda_p+k_p;\delta_p\right),\prod_{p=1}^{r}\left(\beta_p-\sigma_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(-\sigma_p-k_p;\delta_p\right) \\ \\ \prod_{p=1}^{r}\left(\beta_p+n-\sigma_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(-\lambda_p;\mu_p\right)\prod_{p=1}^{r}\left(-\sigma_p-\alpha_p-n-k_p-1;\delta_p\right)\end{matrix}\right.\right]$$

(2.2)

Integral (2.2) is valid under the restriction of

Re $(\alpha_i)>1$,

$$\left|\arg Z_i\right| > \frac{\Omega\pi_i}{2},$$

Re $(\lambda_i-\mu_j\,\xi_j)>0$,

Re $(\sigma_i+\mu_j\,\xi_j)>0$,

where $\xi_j = \sum_{j=1}^{r} \min_{1\le m_r}\left(\mathrm{Re}\left(\frac{d_j^r}{\delta_j^r}\right)\right)\ \forall\ i,j=1...r$

where all $\delta_1, \delta_2, ...\delta_r$ are positive and are not equal to zero.

$$= \frac{1}{(n!)^r}\left\{\prod_{p=1}^{r}\left[(-1)^{np}\, 2^{\rho_p+\beta_p+\frac{1}{r}}\,\Gamma\left(\beta_p+n+1\right)\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)\right]\right\} H^{O,N;m_1,n_1+3;m_2,n_2+3;\ldots;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;\ldots;A_r+3,B_r+3}$$

$$\left[\begin{matrix} Z_1 & 2^{\delta_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\delta_r}\end{matrix}\left|\begin{matrix}\prod_{p=1}^{r}\left(-\lambda_p+k_p;\mu_p\right),\prod_{p=1}^{r}\left(\beta_p-\sigma_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(-\rho_p-k_p;\delta_p\right) \\ \\ \prod_{p=1}^{r}\left(-\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(-\rho_p+\alpha_p+n-k_p;\delta_p\right),\prod_{p=1}^{r}\left(-\rho_p-\beta_p-n-k_p-1;\rho_p\right)\end{matrix}\right.\right] \tag{2.3}$$

Integral (2.3) is valid under the following restrictions of:

Re (λ_i)>-1,

Re (β_i) >-1,

$$\left|\arg Z_i\right| > \frac{\Omega\pi_i}{2}$$

Re $(\rho_i+\delta_j,\ \xi_j)$> 0,

Re $(\lambda_i-\mu_j,\ \xi_j)$> 0

where $\xi_j = \sum_{j=1}^{r}\ \min_{1\le m_r}\left(\mathrm{Re}\left(\frac{d_j^r}{\delta_j^r}\right)\right)\ \forall\ i, j = 1\ldots r$

where all $\mu_1, \mu_2, \ldots\mu_r$ and $\delta_1, \delta_2, \ldots\delta_r$ are positive and are not equal to zero.

Fourth Integral

$$\int_{-1}^{1}\int_{-1}^{1}\ldots\ldots\int_{-1}^{1}\left\{\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}(1-x_p)^{\rho_p}(1+x_p)^{\beta_p}\right]\right\}$$

$$P_n^{(\alpha_1,\beta_1;\ldots;\alpha_r,\beta_r)}(x_1\ldots x_r)\, H\left[Z_1 x_1^{\mu_1}\left(\frac{1-x_1}{x_1}\right)^{\delta_1}\ldots\ldots.Z_r x_r^{\mu_r}\left(\frac{1-x_r}{x_r}\right)^{\delta_r}\right]dx_1.dx_2\ldots dx_r$$

$$= \frac{1}{(n!)^r}\left\{\prod_{p=1}^{r}\left[(-1)^{np}\, 2^{\rho_p+\beta_p+\frac{1}{r}}\,\Gamma\left(\beta_p+n+1\right)\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)\right]\right\} H^{O,N;m_1,n_1+3;m_2,n_2+3;\ldots;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;\ldots;A_r+3,B_r+3}$$

$$\left[\begin{matrix} Z_1 & 2^{\mu_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\mu_r}\end{matrix}\left|\begin{matrix}\prod_{p=1}^{r}\left(\lambda_p-k_p;\mu_p\right),\prod_{p=1}^{r}\left(\beta_p-\rho_p-k_p;\mu_p\right),\prod_{p=1}^{r}\left(-\rho_p-k_p;\mu_p\right) \\ \\ \prod_{p=1}^{r}\left(-\lambda_p-1;\mu_p\right),\prod_{p=1}^{r}\left(-\rho_p+n+\alpha_p-k_p;\mu_p\right),\prod_{p=1}^{r}\left(-\rho_p-\beta_p-n-k_p-1;\mu_p\right)\end{matrix}\right.\right] \tag{2.4}$$

Integral (2.4) is valid under the following restrictions of:

Re (λ_i)>1,

Re (β_i) >-1,

$$\left|\arg Z_i\right| > \frac{\Omega\pi_i}{2}$$

$\mathrm{Re}(\rho_i+\mu_j\,\xi_j) > 0$,

$\mathrm{Re}\,(\lambda_i-\mu_j\,\xi_j) > 0$

where $\xi_j = \sum_{j=1}^{r} \min_{1\le m_r} \left(\mathrm{Re}\left(\frac{d_j^r}{\delta_j^r}\right)\right) \;\forall\; i, j = 1\ldots r$

where all $\mu_1, \mu_2, \ldots \mu_r$ are positive and are not equal to zero.

Proof: To establish (2.1), expressing the multivariable H function on L.H.S. as Mellin Barnes Contour Integral (1.21) and then change the order of integration. This change will be valid under the given restriction and this change will be absolutely convergent. We get the following result

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\int_{L_2}\ldots\int_{L_r}\frac{\theta(S_1..S_r)}{(n!)^r}\left\{\prod_{i=1}^{r}\left[\phi(S_i)\,Z_i\right]dS_i\right\}$$
$$\left\{\int_{-1}^{1}\int_{-1}^{1}\ldots\int_{-1}^{1}\prod_{i=1}^{r}\left[(x_p)^{\lambda_p-\mu_pS_p}.(1-x_p)^{\alpha_p}.(1+x_p)^{\sigma_p+\mu_pS_p}\right]P_n^{\alpha_i,\beta_i;\ldots;\alpha_r,\beta_r}(x_1\ldots x_r)\,dx_1.dx_2\ldots dx_r\right\}$$

Evaluate the inner integral with the help of (1.28) we get the following result

$$\frac{1}{(2\pi\omega)^r}\int_{L_1}\int_{L_2}\ldots\int_{L_r}\frac{\theta(S_1..S_r)}{(n!)^r}\left\{\prod_{p=1}^{r}\left[\phi(S_i)\,Z_i^{S_i}\,dS_i\right]\right\}$$
$$\left\{\prod_{p=1}^{r}\left[\frac{(-1)^{n_p}\,2^{\sigma_p+\mu_pS_p+\frac{1}{r}}.\Gamma(\sigma_p+n+1)\,\Gamma(\sigma_p+\mu_pS_p-\beta_p+1).\Gamma(\sigma_p+\mu_pS_p+1)}{\Gamma(\sigma_p+\mu_pS_p-\beta_p-n+1)\Gamma(\sigma_p+\mu_pS_p+\alpha_p+n+2)}\right]\right\}$$
$${}_3F_2\left[\begin{matrix}\prod_{p=1}^{r}(-\lambda_p+\mu_pS_p);\prod_{p=1}^{r}(\sigma_p+\mu_pS_p-\beta_p+1);\prod_{p=1}^{r}(\sigma_p+\mu_pS_p+1);\\ \prod_{p=1}^{r}(\sigma_p+\mu_pS_p-\beta_p-n+1);\prod_{p=1}^{r}(\sigma_p+\mu_pS_p+\alpha_p+n+2);2^r\end{matrix}\right]$$

Now we represent the hypergeometric function into series and addition and integration [4, p 176 (75)] which is valid under the given restriction (2.1). At last using the definition of multivariable H which is defined in (1.21) we get the result (2.1).

EXPANSION

The objective of this section is to apply the result of (2.1) to (2.4) then to express it in form H [$Z_1\ldots Zr_1$] and established four expansion formulas

First expansion

$$\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}.(1-x_p)^{\sigma_p}\right]\;H\left[Z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1}\ldots.Z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right]$$
$$=\left\{\prod_{p=1}^{r}\left[2^{\sigma_p}\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)\right]\sum_{l=0}^{\infty}(1+\alpha_p+\beta_p+l)\,(1+\alpha_p+\beta_p+2l)\,(-1)^{l_p}\right\}$$
$$P_l^{(\alpha_1,\beta_1;\ldots;\alpha_r,\beta_r)}(x_1\ldots x_r)\;H_{A,B;A_1+3,B_1+3;\ldots;A_r+3,B_r+3}^{O,N;m_1,n_1+3;m_2,n_2+3;\ldots;m_r,n_r+3;}$$

$$\left[\begin{matrix} Z_1 & 2^{\delta_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\delta_r} \end{matrix}\left|\begin{matrix} \prod_{p=1}^{r}\left(1+\lambda_p-k_p;\mu_p\right),\prod_{p=1}^{r}\left(-k_p-\alpha_p;\mu_p\right),\prod_{p=1}^{r}\left(\sigma_p+\beta_p-k_p;\mu_p\right) \\ \\ \prod_{p=1}^{r}\left(1+\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(-1-\sigma_p-k_p;\mu_p\right),\prod_{p=1}^{r}\left(1-\sigma_p-k_p;\mu_p\right) \end{matrix}\right.\right] \tag{3.1}$$

Expansion (3.1) which is valid under the given restriction of (2.1) is also satisfied.

Second expansion

$$\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}.(1+x_p)^{\sigma_p}\right] \quad H\left[Z_1x_1^{\mu_1}\left(1+x_1\right)^{\delta_1}....Z_rx_r^{\mu_r}\left(1+x_r\right)^{\delta_r}\right]$$

$$=\left\{\prod_{p=1}^{r}\left[2^{\sigma_p}\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)(-1)^{l_p}\right]\sum_{l=0}^{\infty}(1+\alpha_p+\beta_p+l)(1+\alpha_p+\beta_p+2l)\right\}$$

$$P_l^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\ \ H^{O,N;m_1,n_1+3;m_2,n_2+3;...;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;...;A_r+3,B_r+3}$$

$$\left[\begin{matrix} Z_1 & 2^{\delta_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\delta_r} \end{matrix}\left|\begin{matrix} \prod_{p=1}^{r}\left(-\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(-\sigma_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(-k_p-\sigma_p-\beta_p;\delta_p\right) \\ \\ \prod_{p=1}^{r}\left(k_p-\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(1-\sigma_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(1-l-2-\rho_p;\delta_p\right) \end{matrix}\right.\right] \tag{3.2}$$

Expansion (3.2) which is valid under the given restriction of (2.2) is also satisfied.

Third expansion

$$\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}.(1-x_p)^{\rho_p}\right] \quad H\left[Z_1x_1^{\mu_1}\left(1-x_1\right)^{\delta_1}....Z_rx_r^{\mu_r}\left(1-x_r\right)^{\delta_r}\right]$$

$$=\left\{\prod_{p=1}^{r}\left[2^{\rho_p}\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)(-1)^{l_p}\right]\sum_{l=0}^{\infty}(1+\alpha_p+\beta_p+l)(1+\alpha_p+\beta_p+2l)\right\}$$

$$P_l^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\ \ H^{O,N;m_1,n_1+3;m_2,n_2+3;...;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;...;A_r+3,B_r+3} \tag{3.3}$$

$$\left[\begin{matrix} Z_1 & 2^{\delta_1} \\ \cdot & \cdot \\ \cdot & \cdot \\ Z_r & 2^{\delta_r} \end{matrix}\left|\begin{matrix} \prod_{p=1}^{r}\left(-\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(k_p-\rho_p;\delta_p\right),\prod_{p=1}^{r}\left(-k_p-\rho_p-l;\delta_p\right) \\ \\ \prod_{p=1}^{r}\left(k_p-\lambda_p;\mu_p\right),\prod_{p=1}^{r}\left(l-\rho_p-k_p;\delta_p\right),\prod_{p=1}^{r}\left(1-\alpha_p-\beta_p-l-k_p-\rho_p;\delta_p\right) \end{matrix}\right.\right]$$

Expansion (3.3) which is valid under the given restriction of (2.3) is also satisfied.

Fourth expansion

$$\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}.(1-x_p)^{\rho_p}\right]\ H\left[Z_1\left(\frac{1-x_1}{x_1}\right)^{\mu_1}....Z_r\left(\frac{1-x_r}{x_r}\right)^{\mu_r}\right]$$

$$=\left\{\prod_{p=1}^{r}\left[2^{\rho_p}\left(\sum_{k_p=0}^{\infty}\frac{2^{k_p}}{(k_p)!}\right)(-1)^{l_p}\right]\sum_{l=0}^{\infty}(1+\alpha_p+\beta_p+l)(1+\alpha_p+\beta_p+2l)\right\}$$

$$P_l^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r)\ H^{O,N;m_1,n_1+3;m_2,n_2+3;...;m_r,n_r+3;}_{A,B;A_1+3,B_1+3;...;A_r+3,B_r+3}$$

$$\left[\begin{matrix}Z_1 & 2^{\delta_1}\\ \cdot & \cdot\\ \cdot & \cdot\\ Z_r & 2^{\delta_r}\end{matrix}\left|\begin{matrix}\prod_{p=1}^{r}(1+\lambda_p-k_p;\mu_p),\prod_{p=1}^{r}(\rho_p-k_p;\mu_p),\prod_{p=1}^{r}(\rho_p+\alpha_p-k_p;\mu_p)\\ \\ \prod_{p=1}^{r}(1+\lambda_p;\mu_p),\prod_{p=1}^{r}(1-k_p-\rho_p;\mu_p),\prod_{p=1}^{r}(1-\rho_p-k_p-\beta_p-l;\mu_p)\end{matrix}\right.\right] \qquad (3.4)$$

Proof: To establish (3.1) let

$$\prod_{p=1}^{r}\left[(x_p)^{\lambda_p}.(1-x_p)^{\rho_p}\right]\ H\left[Z_1\left(\frac{1+x_1}{x_1}\right)^{\mu_1}....Z_r\left(\frac{1+x_r}{x_r}\right)^{\mu_r}\right]=\sum_{l=0}^{\infty}m_l\,P_l^{(\alpha_1,\beta_1;...;\alpha_r,\beta_r)}(x_1...x_r) \qquad (3.5)$$

where m_l is a constant. For finding this we multiply $\prod_{p=1}^{r}\left[(1-x_p)^{\alpha_p}.(1+x_p)^{\beta_p}\right]$ on both sides of the equation (3.5) and then integrate in the limit –l to +l. Lastly in the left side result (2.1) is recalculated and on right side orthogonal property of Jacobi polynomial is applied to get the desired result [5, pg 285(5) and (9)]

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