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PARTIALLY ORDERED FILTERS IN PARTIALLY ORDERED SEMIGROUPS

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ABSTRACT

In this paper the terms, po filter, po filter of a po semigroup generated by a subset and principle po filter generated by an element a in a po semigroup are introduced. It is proved that in a po semigroup S, the nonempty intersection of a family of po filters is also a po filter of S. It is also proved that a nonempty subset F of a po semigroup S is a po filter if and only if S\F is a completely prime po ideal of S or empty. It is proved that every po filter F of a po semigroup S is a i) po-c-system of S ii) po-m-system of S and iii) po-d-system of S. It is proved that the po filter of a po semigroup S generated by a nonempty subset A is the intersection of all po filters of S containing A. Let S be a po semigroup and a ∈ S, then it is proved that N(a) is the least filter of S containing {a}.

Mathematical subject classification (2010): *20M07; 20M11; 20M12.*

Key Words: *po filter, po filter of a po semigroup generated by a subset and principle po filter generated by an element a in a po semigroup.*

1. INTRODUCTION

The algebraic theory of semigroups was widely studied by CLIFFORD [2], [3], PETRICH [4] and LJAPIN [5]. The ideal theory in general semigroups was developed by ANJANEYULU [1]. In this paper we introduce the notions of po filter, po filter of a po semigroup generated by a subset and principle po filter generated by an element a in a po semigroup and characterize po filters, po filters of a po semigroup generated by a subset and principle po filters generated by an element a in a po semigroup. in po semigroups.

2. PRELIMINARIES

Definition 2.1: A semigroup S is said to be a ***partially ordered semigroup*** if S is a partially ordered set such that $a \le b \Rightarrow ax \le bx$, $xa \le xb$ for all $a, b, x \in$ S.

Definition 2.2: A po (left/right) ideal A of a po semigroup S is said to be a ***completely prime* (*left/right*) *ideal*** of S provided $x, y \in$ S and $xy \in$ A implies either $x \in$ A or $y \in$ A.

Definition 2.3: A po ideal A of a po semigroup S is said to be a ***prime ideal*** of S provided X,Y are ideals of S and XY $\subseteq$ A $\Rightarrow$ X $\subseteq$ A or Y $\subseteq$ A.

Definition 2.4: A nonempty subset A of a po semigroup S is said to be an ***m-system*** provided for any $a, b \in$ A implies that $(S^1aS^1bS^1) \cap A \neq \emptyset$.

Definition 2.5: A po ideal A of a po semigroup S is said to be a ***completely semiprime po ideal*** provided $x \in$ S, $x^n \in$ A for some natural number n implies $x \in$ A.

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Definition 2.6: Let S be a po semigroup. A nonempty subset A of S is said to be a ***d*-system** of S if $a \in A \Rightarrow a^n \in A$ for all natural number *n*.

Definition 2.7: A po ideal A of a po semigroup S is said to be ***semiprime po ideal*** provided X is po ideal of S and $X^n \subseteq A$ for some natural number *n* implies $X \subseteq A$.

Definition 2.8: A non-empty subset A of a po semigroup S is said to be an ***n*-system** provided $a \in A$ implies that $(S^1aS^1) \cap A \neq \emptyset$.

Theorem 2.9: Every completely prime po ideal of a po semigroup S is a prime po ideal of S.

Theorem 2.10: Let S be a commutative po semigroup. A po ideal A of S is a prime po ideal if and only if A is a completely prime po ideal.

Theorem 2.11: Every *m*-system in a po semigroup S is an *n*-system.

Theorem 2.12: A po ideal Q of a po semigroup S is a semiprime po ideal if and only if S\Q is an *n*-system of S (or) empty.

Theorem 2.13: Every completely prime po ideal of a po semigroup S is a completely semiprime po ideal of S.

Theorem 2.14: A po ideal A of a po semigroup S is completely semiprime if and only if S\A is a *d*-system of S or empty.

Theorem 2.15: Let S be a commutative po semigroup. A po ideal A of S is completely semiprime if and only if it is semiprime.

Theorem 2.16: If N is an *n*-system in a po semigroup S and $a \in N$, then there exist an *m*-system M in S such that $a \in M$ and $M \subseteq N$.

3. PO FILTERS IN PO SEMIGROUPS

Definition 3.1: A po sub semigroup F of a po semigroup S is said to be a ***po left filter*** of S if
1. $a, b \in S$, $ab \in F \Rightarrow a \in F$.
2. $a, b \in T$, $a \leq b$ and $a \in F \Rightarrow b \in F$.

Note 3.2: A po subsemigroup F of a po semigroup S is a ***po left filter*** of S iff
1. $a, b \in S$, $ab \in F \Rightarrow a \in F$.
2. $(F] \subseteq F$.

Theorem 3.3: The nonempty intersection of two po left filters of a po semigroup S is also a po left filter of S.

Proof: Let A, B be two po left filters of S.

Let $a, b \in S$, $ab \in A \cap B$.

$ab \in A \cap B \Rightarrow ab \in A$ and $ab \in B$.

$a, b \in S$, $ab \in A$, A is a po left filter of S $\Rightarrow a \in A$.

$a, b \in S$, $ab \in B$, B is a po left filter of S $\Rightarrow a \in B$.

$a \in A$, $a \in B \Rightarrow a \in A \cap B$.

$a, b \in S$, $ab \in A \cap B \Rightarrow a \in A \cap B$.

Let $a, b \in S$, $a \leq b$ and $a \in A \cap B$. Now $a \in A \cap B \Rightarrow a \in A$, $a \in B$.

$a, b \in S$, $a \leq b$, $a \in A$, A is a po left filter of S $\Rightarrow b \in A$.

$a, b \in S$, $a \leq b$, $a \in B$, B is a po left filter of S $\Rightarrow b \in B$.

$b \in$ A, $b \in$ B $\Rightarrow$ $b \in$ A$\cap$B. Thus $a, b \in$ S, $a \leq b$, $a \in$ A $\cap$ B$\Rightarrow$ $b \in$ A $\cap$ B.

Therefore A $\cap$ B is a po left filter of S.

Theorem 3.4: The nonempty intersection of a family of po left filters of a po semigroup S is also a po left filter of S.

Proof: Let $\{F_\alpha\}_{\alpha\in\Delta}$ be a family of po left filters of S and let $F = \bigcap_{\alpha\in\Delta} F_\alpha$.

Let $a, b \in$ S, $ab \in$ F. Now $ab \in$ F $\Rightarrow$ $ab \in \bigcap_{\alpha\in\Delta} F_\alpha \Rightarrow ab \in F_\alpha$ for each $\alpha \in \Delta$.

$ab \in F_\alpha$, F_α is a po left filter of S $\Rightarrow$ $a \in F_\alpha$ for each $\alpha \in \Delta \Rightarrow a \in \bigcap_{\alpha\in\Delta} F_\alpha \Rightarrow a \in$ F.

Let $a, b \in$ S, $a \leq b$ and $a \in$ F. Now $a \in$ F $\Rightarrow$ $a \in \bigcap_{\alpha\in\Delta} F_\alpha \Rightarrow a \in F_\alpha$ for each $\alpha \in \Delta$

$a, b \in$ S, $a \leq b$, $a \in F_\alpha$, F_α is a po left filter of S $\Rightarrow$ $b \in F_\alpha$ for all $\alpha \in \Delta$.

$\Rightarrow b \in \bigcap_{\alpha\in\Delta} F_\alpha \Rightarrow b \in$ F. Therefore F is a po left filter of S.

Theorem 3.5: A nonempty subset F of a po semigroup S is a po left filter if and only if S\F is a completely prime po right ideal of S or empty.

Proof: Suppose that F is a po left filter of S and S\F $\neq \emptyset$. Let $b \in$ S and $a \in$ S\F.

Now $a \in$ S\F $\Rightarrow$ $a \notin$ F.

If $ab \in$ F, then since F is a po left filter of S, $a \in$ F. It is a contradiction.

Therefore $ab \notin$ F. Hence $ab \in$ S\F.

Let $a \in$ S\F and $s \in$ S such that $s \leq a$.

If $s \in$ F then $s \leq a$, F is a po left filter of S $\Rightarrow$ $a \in$ F. It is a contradiction.

Therefore $s \in$ S\F. Thus S\F is a po right ideal of S.

Let $a, b \in$ S and $ab \in$ S\F. Now $ab \notin$ F.

Suppose if possible $a \notin$ S\F, $b \notin$ S\F.

Then $a \in$ F, $b \in$ F $\Rightarrow$ $ab \in$ F. It is a contradiction.

Therefore either $a \in$ S\F or $b \in$ S\F.

Hence S\F is completely prime.

Therefore S\F is a completely prime po right ideal of S.

Conversely suppose that S\F is a completely prime po right ideal of S or empty.

If S\F = $\emptyset$, then F = S. Thus F is a po left filter of S.

Assume that S\F $\neq \emptyset$. Let $a, b \in$ S, and $ab \in$ F. If $a \notin$ F then $a \in$ S\F.

$b \in$ S, $a \in$ S\F, S\F is a po right ideal of S $\Rightarrow$ $ab \in$ S\F $\Rightarrow$ $ab \notin$ F. It is a contradiction. Thus $a \in$ F.

Let $a, b \in$ S, $a \leq b$ and $a \in$ F. If $b \notin$ F, then $b \in$ S\F.

$a, b \in$ S, $b \in$ S\F , $a \leq b$, S\F is a po right ideal of S $\Rightarrow$ $a \notin$ F. It is a contradiction.

Thus $b \in$ F. Therefore F is a po left filter of S.

Corollary 3.6: Let S be a po semigroup and F is a po left filter of S. Then S\F is a prime po right ideal of S or empty.

Proof: Since F is a po left filter, by theorem 3.5, S\F is a completely prime po right ideal of S or empty. By theorem 2.9, S\F is a prime po right ideal of S or empty.

Definition 3.7: A subsemigroup F of a po semigroup S is said to be ***po right filter*** of S if

(1) $a, b \in$ S, $ab \in$ F $\Rightarrow b \in$ F
(2) $a, b \in$ S, $a \leq b$ *and* $a \in$ F $\Rightarrow b \in$ F.

Note 3.8: A po subsemigroup F of a po semigroup S is a ***po right filter*** of S if

(1) $a, b \in$ S, $ab \in$ F $\Rightarrow b \in$ F.
(2) (F] $\subseteq$ F.

Theorem 3.9: The nonempty intersection of two po right filters of a po semigroup S is also a po right filter.

Proof: Let A, B be two po right filters of S.

Let $a, b \in$ S, $ab \in$ A∩B.

$ab \in$ A∩B $\Rightarrow ab \in$ A and $ab \in$ B.

$a, b \in$ S, $ab \in$ A, A is a po right filter of S $\Rightarrow$ $b \in$ A.

$a, b \in$ S, $ab \in$ B, B is a po right filter of S $\Rightarrow$ $b \in$ B.

$b \in$ A, $b \in$ B $\Rightarrow b \in$ A ∩B.

$a, b \in$ S, $ab \in$ A∩B $\Rightarrow b \in$ A∩B.

Let $a, b \in$ S, $a \leq b$ and $a \in$ A∩B. Now $a \in$ A∩B $\Rightarrow a \in$ A, $a \in$ B.

$a, b \in$ S, $a \leq b$, $a \in$ A, A is a po right filter of S $\Rightarrow b \in$ A.

$a, b \in$ S, $a \leq b$, $a \in$ B, B is a po right filter of S$\Rightarrow b \in$ B.

$b \in$ A, $b \in$ B $\Rightarrow b \in$ A∩B. Thus $a, b \in$ S, $a \leq b$, $a \in$ A ∩ B$\Rightarrow b \in$ A∩B.

Therefore A∩B is a po right filter of S.

Theorem 3.10: The nonempty intersection of a family of po right filters of a po semigroup S is also a po right filter.

Proof: Let $\{F_\alpha\}_{\alpha \in \Delta}$ be a family of po right filters of S and let $F = \bigcap_{\alpha \in \Delta} F_\alpha$.

Let $a, b \in$ S, $ab \in$ F. Now $ab \in$ F $\Rightarrow ab \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow ab \in F_\alpha$ for each $\alpha \in \Delta$.

$ab \in F_\alpha$, F_α is a po right filter of S $\Rightarrow b \in F_\alpha$.

Let $a, b \in$ S, $a \leq b$ and $a \in$ F. Now $a \in$ F$\Rightarrow a \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow a \in F_\alpha$ for each $\alpha \in \Delta$

$a, b \in$ S, $a \leq b$, $a \in F_\alpha$, F_α is a po right filter of S $\Rightarrow b \in F_\alpha$ for all $\alpha \in \Delta$.

$\Rightarrow b \in \bigcap_{\alpha \in \Delta} F_\alpha \Rightarrow b \in$ F.

Therefore F is a po right filter of S.

Theorem 3.11: A nonempty subset F of a po semigroup S is a po right filter if and only if S\F is a completely prime po left ideal of S or empty.

Proof: Suppose that F is a po right filter of S and S\F $\neq \emptyset$. Let $b \in$ S and $a \in$ S\F.

Now $a \in$ S\F $\Rightarrow a \notin$ F.

If $ba \in$ F, then since F is a po right filter of S, $a \in$ F. It is a contradiction.

Therefore $ba \notin$ F. Hence $ba \in$ S\F.

Let $a \in$ S\F and $s \in$ S such that $s \leq a$.

If $s \in$ F then $s \leq a$, F is a po right filter of S $\Rightarrow$ $a \in$ F. It is a contradiction.

Therefore $s \in$ S\F. Thus S\F is a po left ideal of S.

Let $a, b \in$ S and $ab \in$ S\F. Now $ab \notin$ F.

Suppose if possible $a \notin$ S\F, $b \notin$ S\F.

Then $a \in$ F, $b \in$ F $\Rightarrow ab \in$ F. It is a contradiction.

Therefore either $a \in$ S\F or $b \in$ S\F.

Hence S\F is completely prime.

Therefore S\F is a completely prime po left ideal of S.

Conversely suppose that S\F is a completely prime po left ideal of S or empty.

If S\F = $\emptyset$, then F = S. Thus F is a po right filter of S.

Assume that S\F $\neq \emptyset$. Let $a, b \in$ S and $ab \in$ F. If $b \notin$ F then $b \in$ S\F.

$a \in$ S, $b \in$ S\F, S\F is a po left ideal of S $\Rightarrow ab \in$ S\F $\Rightarrow ab \notin$ F. It is a contradiction.

Thus $b \in$ F.

Let $a, b \in$ S, $a \leq b$ and $a \in$ F. If $b \notin$ F, then $b \in$ S\F.

$a, b \in$ S, $b \in$ S\F, $a \leq b$, S\F is a po left ideal of S $\Rightarrow a \in$ S\F

$\Rightarrow a \notin$ F. It is a contradiction.

Therefore F is a po right filter of S.

Corollary 3.12: Let S be a po semigroup and F is a po right filter. Then S\F is a prime po left ideal of S or empty.

Proof: Since F is a po right filter. By theorem 3.11, S\F is a completely prime left po ideal of S or empty. By theorem 2.9, S\F is a prime po left ideal of S or empty.

Definition 3.13: A po subsemigroup F of a po semigroup S is said to be ***po filter*** of S if
(1) $a, b \in$ S, $ab \in$ F $\Rightarrow a, b \in$ F
(2) $a, b \in$ S, $a \leq b$ and $a \in$ F $\Rightarrow b \in$ F

Note 3.14: A po subsemigroup F of a po semigroup S is a ***po filter*** of S iff
(1) $a, b \in$ S, $ab \in$ F $\Rightarrow a, b \in$ F
(2) (F] $\subseteq$ F.

Note 3.15: A po sub semi group F of a po semigroup S is a ***po filter*** of S iff F is a po left filter, a po right filter and po lateral filter of S.

Definition 3.16: A po filter F of a po semigroup S is said to be a ***proper po filter*** if F ≠ S.

Theorem 3.17: The nonempty intersection of two po filters of a po semigroup S is also a po filter of S.

Proof: Let A, B be two po filters of S.

Let $a, b \in$ S, $ab \in$ A∩B.

$ab \in$ A∩B ⇒ $ab \in$ A and $ab \in$ B.

$a, b \in$ S, $ab \in$ A, A is a po filter of S ⇒ $a, b \in$ A.

$a, b \in$ S, $ab \in$ B, B is a po filter of S, ⇒ $a, b \in$ B.

$a, b \in$ A, $a, b \in$ B ⇒ $a, b \in$ A∩B.

Let $a, b \in$ S, $a \leq b$ and $a \in$ A∩B. Now $a \in$ A∩B ⇒ $a \in$ A, $a \in$ B.

$a, b \in$ S, $a \leq b$, $a \in$ A, A is a po filter of S ⇒ $b \in$ A.

$a, b \in$ S, $a \leq b$, $a \in$ B, B is a po filter of S ⇒ $b \in$ B.

$b \in$ A, $b \in$ B ⇒ $b \in$ A∩B. Thus $a, b \in$ S, $a \leq b$ *and* $a \in$ A ∩B ⇒ $b \in$ A∩B.

Therefore A∩B is a po filter of S.

Theorem 3.18: The nonempty intersection of a family of po filters of a po semigroup S is also a po filter of S.

Proof: Let $\{F_\alpha\}_{\alpha\in\Delta}$ be a family of po filters of S and let $F = \bigcap_{\alpha\in\Delta} F_\alpha$.

Let $a, b \in$ S, $ab \in$ F. Now $ab \in$ F ⇒ $ab \in \bigcap_{\alpha\in\Delta} F_\alpha$ ⇒ $ab \in F_\alpha$ for each $\alpha \in \Delta$.

$ab \in F_\alpha, F_\alpha$ is a po filter of S ⇒ $a, b \in F_\alpha$.

Let $a, b \in$ S, $a \leq b$ and $a \in$ F. Now $a \in$ F ⇒ $a \in \bigcap_{\alpha\in\Delta} F_\alpha$ ⇒ $a \in F_\alpha$ for each $\alpha \in \Delta$

$a, b \in$ S, $a \leq b$ and $a \in F_\alpha$, F_α is a po filter of S ⇒ $b \in F_\alpha$ for all $\alpha \in \Delta$.

$$\Rightarrow b \in \bigcap_{\alpha\in\Delta} F_\alpha \Rightarrow b \in F.$$

Therefore F is a po filter of S.

Theorem 3.19: A nonempty subset F of a po semigroup S is a po filter if and only if S\F is a completely prime po ideal of S or empty.

Proof: Suppose that F is a po filter of S and S\F ≠ ∅. Let $b \in$ S and $a \in$ S\F.

Now $a \in$ S\F ⇒ $a \notin$ F.

If $ab \in$ F, then since F is a po filter of S, $a \in$ F. It is a contradiction.

Therefore $ab \notin$ F. Hence $ab \in$ S\F.

Similarly $ba \in$ S/F.

Let $a \in$ S\F and $s \in$ S such that $s \leq a$.

If $s \in$ F then $s \leq a$, F is a po filter of S ⇒ $a \in$ F. It is a contradiction.

Therefore $s \in S\backslash F$. Thus S\F is a po ideal of S.

Let $a, b \in S$ and $ab \in S\backslash F$. Now $ab \notin F$.

Suppose if possible $a \notin S\backslash F$, $b \notin S\backslash F$.

Then $a \in F$, $b \in F \Rightarrow ab \in F$. It is a contradiction.

Therefore either $a \in S\backslash F$ or $b \in S\backslash F$.

Hence S\F is completely prime.

Therefore S\F is a completely prime po ideal of S.

Conversely suppose that S\F is a completely prime po ideal of S or empty.

If $S\backslash F = \emptyset$, then F = S. Thus F is a po filter of S.

Assume that $S\backslash F \neq \emptyset$. Let $a, b \in S$ and $ab \in F$. If $a \notin F$ then $a \in S\backslash F$. $b \in S$, $a \in S\backslash F$, S\F is a po ideal of S $\Rightarrow ab \in S\backslash F$

$\Rightarrow ab \notin F$. It is a contradiction.

Thus $a \in F$.

Similarly $b \in F$.

Let $a, b \in S$, $a \leq b$ and $a \in F$. If $b \notin F$, then $b \in S\backslash F$.

$a, b \in S$, $b \in S\backslash F$, $a \leq b$, S\F is a po ideal of S $\Rightarrow a \in S\backslash F \Rightarrow a \notin F$. It is a contradiction.

Therefore F is a po filter of S.

Corollary 3.20: Let S be a po semigroup. If F is a po filter, then S\F is a prime po ideal of S or empty.

Proof: Since F is a po filter of S. By theorem 3.19, S\F is a completely prime po ideal of S or empty. By theorem 2.9, S\F is a prime po ideal of S or empty.

Corollary 3.21: A nonempty subset F of a commutative po semigroup S is a po filter if and only if S\F is a prime po ideal of S or empty.

Proof: Suppose that S\F is po filter of commutative po semigroup S.

By corollary 3.20, S\F is prime po ideal of S or empty.

Conversely suppose that S\F is a prime po ideal of S or empty.

If $S\backslash F = \emptyset$, then F = S. Thus F is a po filter of S.

Assume that S\F is a prime po ideal of S.

By theorem 2.10, S\F is a completely prime po ideal of S or empty.

By theorem 3.19, F is a po filter of S.

Theorem 3.22: Every po filter F of a po semigroup S is a po-*c*-system of S.

Proof: Suppose that F is a po filter.

By theorem 3.19, S\F is a completely prime po ideal of S.

By theorem 2.12, F is a po-*c*-system of S.

Theorem 3.23: A po semigroup S does not contain proper po filters if and only if S does not contain proper completely prime po ideals.

Proof: Suppose that a po semigroup S does not contain proper po filters.

Let A be a completely prime po ideal of S and $A \subset S$.

Then $\emptyset \neq S\backslash A \subseteq S$ and $S\backslash (S\backslash A)$ (= A) is a completely prime po ideal of S.

Since $S\backslash A$ is the complement of A to S, by theorem 3.19, $S\backslash A$ is a po filter of S.

Then $S\backslash A = S$ and hence $A = \emptyset$. It is a contradiction.

Therefore S does not contain proper completely prime po ideals.

Conversely suppose that S does not contain proper completely prime po ideals.

Let F be a po filter of S and $F \subset S$.

Since $S\backslash F \neq \emptyset$, by theorem 3.19, $S\backslash F$ is a completely prime po ideal of S.

Then $S\backslash F = S$ and hence $F = \emptyset$. It is a contradiction.

Therefore S does not contain proper po filters.

Theorem 3.24: Every po filter F of a po semigroup S is a po-*m*-system of S.

Proof: Suppose that F is a po filter of a po semigroup S.

By corollary 3.20, $S\backslash F$ is a prime po ideal of S.

By theorem 2.12, $S\backslash(S\backslash F) = F$ is a po-*m*-system of S or empty.

Corollary 3.25: Let S be a po semigroup. If F is a po filter, then $S\backslash F$ is a completely semiprime po ideal of S.

Proof: Suppose that F is a po filter of a po semigroup S.

By theorem 3.20, $S\backslash F$ is a completely prime po ideal of S.

By theorem 2.13, $S\backslash F$ is a completely semiprime po ideal of S.

Corollary 3.26: Every po filter F of a po semigroup S is a po-*d*-system of S.

Proof: Suppose that F is a po filter of a po semigroup S.

By corollary 3.25, $S\backslash F$ is a completely semiprime po ideal of S.

By theorm 2.14, $S\backslash(S\backslash F) = F$ is a po-*d*-system of S or empty.

Corollary 3.27: Let S be a po semigroup. If F is a po filter, then $S\backslash F$ is a semiprime po ideal of S.

Proof: Suppose that F is a po filter of a po semigroup S.

By theorem 3.19, $S\backslash F$ is a completely prime po ideal of S.

By theorem 2.13, $S\backslash F$ is a completely semiprime po ideal of S.

By theorem 2.15, $S\backslash F$ is a semiprime po ideal of S.

Corollary 3.28: Every po filter F of a po semigroup S is a po-*n*-system of S.

Proof: Suppose that F is a po filter of a po semigroup S.

By corollary 3.27, S\F is a semiprime po ideal of S.

By theorem 2.16, S\ (S\F) = F is a po-*n*-system of S.

Definition 3.29: Let S be a po semigroup and A be a nonempty subset of S. The smallest po left filter of S containing A is called ***po left filter of* S *generated by A*** and it is denoted by F_l (A).

Theorem 3.30: The po left filter of a po semigroup S generated by a nonempty subset A of S is the intersection of all po left filters of S containing A.

Proof: Let Δ be the set of all po left filters of S containing A.

Since S itself is a po left filter of S containing A, S ∈ Δ. So Δ ≠ ∅.

Let $F^* = \bigcap_{F \in \Delta} F$. Since $A \subseteq F$ for all F ∈ Δ, $A \subseteq F^*$. So $F^* \neq \emptyset$.

By theorem 3.4, F^* is a po left filter of S.

Let K be a po left filter of S containing A.

Clearly A ⊆ K and K is a po left filter of S.

Therefore K ∈ Δ ⇒ $F^* \subseteq$ K. Therefore F^* is the smallest po left filter of S containing A and hence F* is the po left filter of S generated by A.

Definition 3.31: Let S be a po semigroup and A be a nonempty subset of S. The smallest po right filter of S containing A is called ***po right ideal of* S *generated by A*** and it is denoted by F_r (A).

Theorem 3.32: The po right filter of a po semigroup S generated by a nonempty subset A is the intersection of all po right filters of S containing A.

Proof: Let Δ be the set of all po right filters of S containing A.

Since S itself is a po right filter of S containing A, S ∈ Δ. So Δ ≠ ∅.

Let $F^* = \bigcap_{F \in \Delta} F$. Since $A \subseteq F$ for all F ∈ Δ, $A \subseteq F^*$. So $F^* \neq \emptyset$.

By theorem 3.11, F^* is a po right filter of S.

Let K be a po right filter of S containing A.

Clearly A ⊆ K and K is a po right filter of S.

Therefore K ∈ Δ ⇒ $F^* \subseteq$ K. Therefore F^* is the smallest po right filter of S containing A and hence F* is the po right filter of S generated by A.

Definition 3.33: Let S be a po semigroup and A be a nonempty subset of S. The smallest po filter of S containing A is called ***po filter of* S *generated by A*** and it is denoted by N (A).

Theorem 3.34: The po filter of a po semigroup S generated by a nonempty subset A is the intersection of all po filters of S containing A.

Proof: Let Δ be the set of all po filters of S containing A.

Since S itself is a po filter of S containing A, S ∈ Δ. So Δ ≠ ∅.

Let $F^* = \bigcap_{F \in \Delta} F$. Since $A \subseteq F$ for all F ∈ Δ, $A \subseteq F^*$. So $F^* \neq \emptyset$.

By theorem 3.15, F^* is a po filter of S.

Let K be a po filter of S containing A.

Clearly A ⊆ K and K is a po filter of S.

Therefore $K \in \Delta \Rightarrow F^* \subseteq K$.

Therefore F^* is the po filter of S generated by A.

Definition 3.35: A po filter F of a po semigroup S is said to be a ***principal po filter*** provided F is a po filter generated by $\{a\}$ for some $a \in$ S. It is denoted by N(a).

Corollary 3.36: Let S be a po semigroup and $a \in$ S. Then N(a) is the least filter of S containing $\{a\}$.

Note 3.37: For every $a \in$ S, the intersection of all po filters containing $\{a\}$ is again a po filter and thus the least po filter containing $\{a\}$.

Theorem 3.38: If N(b) ⊆ N(a), then N(a)\N(b), if it is nonempty, is a completely prime po ideal of N(a).

Proof: Clearly N(b) is a po filter of N(a), By theorem 3.19, N(a)\N(b) is a completely prime po ideal of N(a).

Theorem 3.39: If $a, b \in$ S and $b \in$ N(a), then N(b) ⊆ N(a).

Proof: From the definition of the principal po filter, it is clear.

Corollary 3.40: If $a, b \in$ S and $a \leq b$ then N(b) ⊆ N(a).

Proof: Since $a \leq b$ then it is clear that $b \in$ N(a).

By theorem 3.39, we have N(b) ⊆ N(a).

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