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ON SEMI-MAXIMAL WEAKLY OPEN AND SEMI-MINIMAL WEAKLY CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper new class of sets called semi- maximal weakly open sets and semi-minimal weakly closed sets are introduced in topological spaces. We show that the complement of semi-maximal weakly open set is a semi-minimal weakly closed set and some properties of the new concepts have been studied.

Keywords: *Maximal open set, Minimal closed set, Maximal weakly open set, Minimal weakly closed set, Semi- Maximal weakly open set, Semi- Minimal weakly closed set.*

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1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)[M_aC(X)]$.The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 1963, N.Levine[4] introduced and studied semi-open sets. A subset A of a topological space X is said to be semi-open set if there exist some open set U such that $U\subset A\subset Cl(U)$. The family of all semi-open sets of X is denoted by SO(X). The Complement[5] of semi-open set is called semi-closed set in X. The family of all semi-closed sets are denoted by SC(X). In the year 2009, S.S.Benchalli and B.M.Ittanagi [6] introduced and studied semi-maximal open and semi-minimal closed sets in topological spaces. In the year 2000, M.Sheik john [7] introduced and studied weakly closed sets and weakly open sets in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre[8] introduced and studied maximal weakly open sets and minimal weakly closed sets in topological spaces.

Definition 1.1[1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is φ or U.

Definition 1.2 [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

Definition 1.3[3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is φ or F.

Definition 1.4 [3]:]: A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

Definition 1.5 [4]: A subset A of a topological spaces X is said to be semi-open set if there exist some open set U such that $U\subset A\subset Cl(U)$.

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Definition 1.6 [5]: The complement of semi-open set is called semi-closed set in X.

Definition 1.7 [6]: A set A in a topological space X is said to be semi maximal open set if there exists a maximal open set M such that $M \subset A \subset Cl(M)$.

Definition 1.8 [6]: A subset N of a topological space X is said to be semi-minimal closed set if X-N is semi-maximal open set.

Definition 1.9 [7]: A subset A of (X, τ) is called weakly closed set if $cl(A) \subseteq U$ Whenever $A \subseteq U$ and U is Semi-open in X.

Definition1.10[7]: A subset A in (X, τ) is called weakly open set in X if A^c is weakly closed set in X.

Definition1.11[8]: A proper non-empty weakly open subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

Definition1.12[8]: A proper non-empty weakly closed subset F of X is said to be minimal weakly closed set if any weakly closed set which is contained in F is φ or F.

2. SEMI MAXIMAL WEAKLY OPEN SETS

Definition 2.1: A set A in a topological space X is said to be semi-maximal weakly open set if there exists a maximal weakly open set M Such that $M \subset A \subset S\text{-}Cl(M)$.

The family of all semi-maximal weakly open sets in a topological space X is denoted by $SM_awo(X)$.

Example 2.2: Let X={a,b,c}, τ ={ X, φ ,{a},{b},{a,b}} be a topological space.

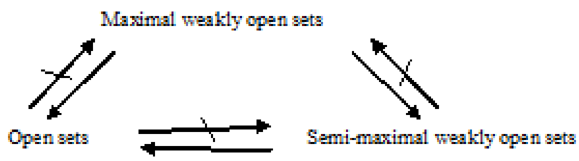
Weakly open sets: {X, φ,{a},{b},{a,b}}

Maximal weakly open sets are : {a,b}

Semi-maximal-weakly-open-sets: {X, {a,b}}

$M_awo(x) \subset SM_awo(x)$.

The above results are given in below implication diagram.



Theorem 2.3: If M is a semi- maximal weakly open set in a topological space X and $M \subset N \subset S\text{-}Cl(M)$ then N is also semi-maximal weakly open in X.

Proof: Let M be a semi-maximal weakly open in X. Then by definition 2.1 there exists a maximal weakly open set U in X such that $U \subset M \subset S\text{-}Cl(U)$. Since $M \subset S\text{-}Cl(U)$ it follows that $S\text{-}Cl(M) \subset Cl(S\text{-}Cl(U)) = S\text{-}Cl(U)$. But from hypothesis $N \subset S\text{-}Cl(M)$ therefore it follows that $U \subset N \subset S\text{-}Cl(U)$. Thus there exists a maximal weakly open set U such that $U \subset N \subset S\text{-}Cl(U)$. Therefore by definition 2.1 it follows that N is semi-maximal weakly open in X.

Theorem 2.4: Let X be a topological space and $M_iwo(x)$ be the class of all maximal weakly open sets in X the following results hold good.
(i) $M_awo(x) \subset SM_awo(X)$
(ii) If $M \in SM_awo(X)$ and $M \subset N \subset S\text{-}Cl(M)$ then $N \subset SM_awo(X)$.

Proof: This follows from theorem 2.3.

Theorem 2.5: : Let X be a topological space. Y be subspace of X and M be a subset of Y. If M is semi-maximal weakly open in X then M is semi-maximal weakly open in Y.

Proof: Suppose M is semi-maximal weakly open in X. By definition 2.1 there exists a maximal weakly open set N in X such that N⊂M⊂S-Cl(N). Now N⊂M⊂Y. Hence Y∩N=N. Since N is maximal weakly open in X. Y ∩ N=N is maximal weakly open in Y. Now we have N⊂M⊂S-Cl(N).Therefore Y∩N⊂Y∩M⊂Y∩S-Cl(N), which implies N⊂M⊂S-Cl γ(N). Thus there exists a maximal weakly open set N in Y Such that N⊂M⊂ S -Clγ(N). Therefore by definition 2.1 it follows that M is semi-maximal weakly open in Y.

Theorem 2.6: Let X be a topological space. Let M,N be maximal weakly open sets in X and U⊂X such that N⊂U⊂S-Cl(N) if M ∩ N = ∅ then U∩W= ∅.

Proof: Since M∩N = ∅, it follows that N⊂X−M therefore S-Cl(N)⊂Cl(X-M)=X-M. Since X-M is minimal weakly closed set and every minimal weakly closed set is closed set . Also we have N⊂U⊂ S-Cl(N). Therefore U⊂ S -Cl(N)⊂X-M .Thus U⊂X-M which means U∩W = ∅.

Theorem 2.7: Intersection of two semi-minimal weakly open sets need not be semi-minimal weakly open . It can be Shown by the following example.

LetX={a,b,c,d}, τ ={X, φ, {a,b},{c,d}} be a topological space.

Semi-maximal-weakly-open-sets:{X,{a,b,c},{a,b,d},{b,c,d},{a,c,d}} take any two semi-maximal weakly open sets {a,b,d} ∩ {b,c,d}={b,d}Which is not a semi-maximal weakly open set.

3. SEMI-MINIMAL WEAKLY CLOSED SETS

Definition 3.1: A subset N of a topological space X is said to be semi-minimal weakly closed set if X-N is semi-maximal weakly open set.

The family of all semi-minimal weakly closed sets in a topological space X is denoted by $Sm_iC(X)$.

Example 3.2: Let X= {a,b,c}, τ = {X, φ,{a},{b},{a,b}} be a topological space.

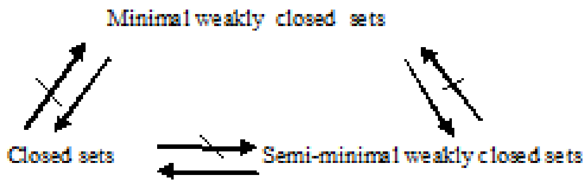
Closed sets are: {X, φ ,{b,c},{a,c}{c}}

Weakly closed sets: {X, φ,{b,c},{a,c},{c}.

Minimal weakly Closed sets: {c}

Semi-minimal-weakly-closed-sets: {φ,{c}}

The above results are given in below implication diagram.



Theorem 3.3: A subset W of a topological space X is semi-minimal weakly closed iff there exists a minimal weakly closed set N in X such that int(N)⊂W⊂N.

Proof: Suppose W is a semi-minimal weakly closed in X then by definition 3.1 X-W is semi-maximal weakly open in X. Therefore by definition 2.1 there exists a maximal weakly open set M such that M⊂X-W⊂S-Cl(M) which implies that X-[S-Cl(M)]⊂X-[X-W]⊂X-M which implies X-[S-Cl(M)] ⊂W ⊂X-M. But it is know that X-[S-Cl(M)]=int(X-M) take X-M=N so,that N is a minimal weakly closed set such that int(N) ⊂W ⊂N.

Conversly, suppose that there exist a minimal weakly closed set N in X such that int(N) ⊂W ⊂N. Therefore it follows that X-N ⊂[X-W]⊂X-int(N).But it is know that X-int(N))=Cl(X-N). Therefore there exists a maximal weakly open set X-N such that X-N ⊂X-W⊂S-Cl(X-N). Thus by definition 2.1 it follows that X-W is semi-maximal weakly open in X. Hence by definition 3.1 it follows that W is Semi-minimal weakly closed set.

Theorem 3.4: If N is semi-minimal weakly closed in X and int(N)⊂W⊂N then W is semi-minimal weakly closed in X.

Proof: Let N be semi-minimal weakly closed in X then by definition of semi-minimal weakly closed sets there exists a minimal weakly closed set F such that int(F) ⊂N ⊂F. Now int(F) ⊂N which implies int(F)=int(int(F))⊂int(N). But int(N) ⊂W, we have int(F) ⊂W. Further since int(F) ⊂ int(N) ⊂W ⊂ N ⊂F. It follows that int(F) ⊂W ⊂ F. Thus there exists a minimal weakly closed set F such that int(F) ⊂W ⊂F therefore W is semi-minimal weakly closed in X.

Theorem 3.5: The following three properties of a subset N of a topological space X are equivalent.
(i) N is semi-minimal weakly closed set in X
(ii) int(cl(N)) ⊂N
(iii) (X-N) is semi-maximal weakly open set in X.

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