International Research Journal of Pure Algebra-4(10), 2014, 603-606
Available online through www.rjpa.info ISSN 2248–9037

MINIMAL WEAKLY OPEN MAP AND MAXIMAL WEAKLY OPEN MAPS IN TOPOLOGICAL SPACES

Vivekananda Dembre*
Department of Mathematics,
Rani Channamma University, Belagavi-591156, Karnataka, India.

Jeetendra Gurjar
Department of Mathematics,
Shaikh College of Engineering and Diploma , Belagavi-591156, Karnataka, India.

(Received On: 15-10-14; Revised & Accepted On: 30-10-14)

ABSTRACT

In this paper a new class of maps called minimal weakly open map and maximal weakly open map are introduced and investigated and during this process some properties of the new concepts have been studied.

Keywords: *Minimal open map, Maximal open map, Minimal weakly open map, Maximal weakly open map, Minimal weakly open set, Maximal weakly open set.*

Mathematics subject classification (2000): *54A05.*

1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. Minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)$[$M_aC(X)$].The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000, M.Sheik john [4] introduced and studied weakly closed maps and weakly open maps in topological space. In the year 2008 B.M.Ittanagi [5] introduced and studied minimal open sets and maps in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre[6] [7] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces.

1.1 Definition [1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is ∅ or U.

1.2 Definition [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

1.3 Definition [3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is ∅ or F & A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

1.4 Definition [4]: A map f: $(X, \tau) \rightarrow (Y, \sigma)$ is said to be weakly closed map if f(F) is weakly closed in (Y, σ) for every closed set F of (X, τ) and A map is said to be weakly open map if f(U) is weakly open in Y for every open set U of X.

**Corresponding author: Vivekananda Dembre*, Department of Mathematics, Rani Channamma University, Belagavi-591156, Karnataka, India.*

1.5 Definition [5]: Let X and Y be the topological spaces. A map f : X ⟶ Y is called
(i) minimal open map for every open set U of X, f(U) is minimal open set in Y.
(ii) maximal open map for every open set U of X, f(U) is maximal open set in Y.
(iii) minimal irresolute if $f^{-1}(M)$ is minimal open set in X for every minimal open set M in Y.
(iv) maximal irresolute if $f^{-1}(M)$ is maximal open set in X for every maximal open set M in Y.

1.6 Definition [6]: A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is ∅ or U.

1.7 Definition [7]: A proper non-empty weakly closed subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

2. MINIMAL WEAKLY OPEN MAPS AND MAXIMAL WEAKLY OPEN MAPS

2.1 Definition :
(i) Let X and Y be topological spaces. A map f: X ⟶ Y is called minimal weakly open map for every open set U of X, f(U) is minimal weakly open set in Y.
(ii) Let X and Y be topological spaces. A map f: X ⟶ Y is called maximal weakly open map for every open set U of X, f(U) is maximal weakly open set in Y.

2.2 Theorem: Every open map is minimal weakly open map but not conversely.

Proof: Let f: X ⟶ Y be an open map and let N be any minimal weakly open set in X. Since every minimal weakly open set is an open set, N is an open set in X. Since f is an open map, f(N) is an open set in Y. Hence f is a minimal weakly open map.

2.3 Example: Let X=Y={a,b,c} be with τ = {X, φ, {a}, {b}, {a,b}} and μ = {Y, φ, {a}}. Let f : X ⟶ Y be an identity map then f is a minimal weakly open map but it is not an open map. Since for the open set {a,b} in X, f (a,b)= {a,b} which is not an open set in Y.

2.4 Theorem: Every open map is maximal weakly open map but not conversely

Proof: Similar to that of 2.2

2.5 Example: Let X = Y = {a,b,c} be with τ = {X, φ, {a,b}} and μ = {Y, φ, {a} }. Let f : X⟶Y be an identity map then f is a maximal weakly open map but it not an open map ,since for the open set {a,b} in X; f (a,b)= (a,b) which is not an open set in Y.

2.6 Remark: Minimal weakly open and maximal weakly open maps are independent of each other.

In example 2.3 f is a minimal weakly open map but it is not a maximal weakly open map.

In example 2.5 f is a maximal weakly open map but it is not a minimal weakly open map.

2.7 Theorem: A map f: X ⟶ Y is minimal weakly open map iff for any subset S in Y and each maximal weakly closed set M in X containing $f^{-1}(S)$, there is a closed set W in Y such that S⊂W and $f^{-1}(W)$⊂M.

Proof: Suppose f is a minimal weakly open map. Let S be an Subset in Y & M is a maximal weakly closed set in X Such that $f^{-1}(S)$⊂M then W= Y- f (X-M) is a closed set in Y containing S such that $f^{-1}(W)$ ⊂M. Conversely suppose that N is a minimal weakly open set in X, then f^{-1}(Y-f(N)⊂X-N and X-N is a maximal weakly closed set in X; by hypothesis there is a closed set W in Y Such that Y-f(N)⊂W and $f^{-1}(W)$⊂X-N therefore N⊂X- $f^{-1}(W)$.

Hence Y-W⊂f(N)⊂f(X-f^{-1}(W)⊂Y-W which implies f(N) = Y-W; since Y-W is an open set in Y; f(N) is an open set in Y. Hence f is a minimal weakly open map.

2.8 Theorem: A map f: X⟶Y is a maximal weakly open map iff for each subset S in Y and each minimal weakly closed set N in X containing $f^{-1}(S)$ there is a closed set W in Y such that S⊂W and $f^{-1}(W)$⊂N.

Proof: Similar to that of theorem 2.7

2.9 Remark: A map f : X ⟶ Y is minimal weakly open map and A⊂X then f_A:A ⟶ Y need not be a minimal weakly open map.

2.10 Example: Let X=Y={a,b,c} with τ = {X, φ, {a}, {a,b}} and μ = {Y, φ, {a},{a,b}}. Let f : X $\longrightarrow$ Y be an identity map then f is a minimal weakly open map. Let A= {b,c}, τ_A = { A, φ,{b}} then f_A: A $\longrightarrow$ Y is not a minimal weakly open map. Since for the minimal weakly open set {c} in A, f_A{c} = {c} which is not an open set in Y.

2.11 Remark: If f: X$\longrightarrow$Y is maximal weakly open map and A$\subset$X then f_A: A$\longrightarrow$Y need not be a maximal weakly open map.

2.12 Example: Let X=Y={a,b,c} with τ ={X, φ, {a}, {a,b}} and μ = {Y, φ, {a},{a,b}}. Let f: X$\longrightarrow$ Y be an identity map then f is a maximal weakly open map also. Let A= {b,c}, τ_A = { A, φ,{b}} then f_A : A$\longrightarrow$Y is not a maximal weakly open map.Since for the maximal weakly open set {c} in A, f_A{c} = {c} which is not an open set in Y.

2.13 Remark: The composition of minimal weakly open maps need not be a minimal weakly open maps.

2.14 Example: Let X = Y = Z={a,b,c} be with τ = {X, φ, {a}}, μ = {Y, φ, {a},{a,b}} and σ = {Z, φ,{a,b}}.
Let f : (X, τ) $\longrightarrow$(Y, μ) and g :(Y, μ) $\longrightarrow$(Z, σ) be the identity maps then clearly f and g are minimal weakly open maps but gof:(X , τ) $\longrightarrow$ (Z , σ) is not a minimal weakly open map. Since for the minimal weakly open set {a} in X {gof}{a}={a} which is not an open set in Z.

2.15 Theorem: Let f : X $\longrightarrow$Y be a minimal weakly open and g : Y $\longrightarrow$Z be an open maps then gof : X$\longrightarrow$ Z is a minimal weakly open map.

Proof: Let N be any minimal weakly open set in X. Since f is minimal weakly open, f(N) is an open set in Y. Again Since g is an open, Since g is an open; g(f(N))=(gof)(N) is an open set in Z. Hence gof is a minimal weakly open map.

2.16 Remark: The composition of maximal weakly open maps need not be a maximal weakly open maps.

2.17 Example: Let X=Y=Z={a,b,c} be with τ = {X, φ, {a}}, μ = {Y, φ, {a},{a,b}} and σ = {Z, φ,{a,b}}.

Let f : (X, τ) $\longrightarrow$(Y, μ) be a function defined by f(a) = b, f(b)=c, f(c) = b and g :(Y, μ)$\longrightarrow$ (Z, σ) be the identity maps then clearly f and g are maximal weakly open maps but gof : (X, τ)$\longrightarrow$(Z, σ) is not a maximal weakly open map. Since for the maximal weakly open set {a,c} in X, {gof}{a,c}={a,c} which is not an open set in Z,
where gof : (X , τ)$\longrightarrow$(Z, σ) be identity map.

2.18 Theorem: Let f: X$\longrightarrow$ Y be a maximal weakly open and g :Y$\longrightarrow$ Z be an open maps then gof: X$\longrightarrow$ Z is a maximal weakly open map.

Proof: Similar to that of theorem 2.15

2.19 Defintion: Minimal Weakly Irresolute and Maximal Weakly Irresolute

(i) Minimal weakly irresolute if f^{-1}(M) is minimal weakly open set in X for every minimal weakly open set M in Y.
(ii) Maximal weakly irresolute if f^{-1}(M) is maximal weakly open set in X for every maximal weakly open set M in Y.

2.20 Theorem: Let f: X $\longrightarrow$Y and g: Y$\longrightarrow$ Z be the maps then gof: X $\longrightarrow$Z be a minimal weakly open map then

(i) g is minimal weakly open if f is minimal weakly irresolute and surjective.
(ii) f is minimal weakly open if g is continuous and injective.

Proof:

(i) Let N be any minimal weakly open set in Y. Since f is minimal weakly open set in Y. Since f is minimal weakly irresolute and Surjective. f^{-1}(N) is a minimal weakly open set in X. Again since gof is minimal weakly open.
(gof)f^{-1}(N) is an open set in Z but (gof)(f^{-1}(N) = g(f(f^{-1}(N)) = g(N) is an open set in Z. Hence g is a minimal weakly open map.
(ii) Let N be any minimal weakly open set in X. Since gof is minimal weakly open (gof)(N) is an open set in Z. Again since g is continuous and injective, g^{-1}(gof)(N) is an open set in Y.
But g^{-1}(gof)(N) = g^{-1}(g(f(N)) = f(N) is an open set in Y. Hence f is a minimal weakly open map.

2.21 Theorem: Let f: X $\longrightarrow$Y and g: Y$\longrightarrow$ Z be maps and let gof : X$\longrightarrow$Z be a maximal weakly open map then

(i) g is maximal weakly open if f is maximal weakly irresolute and surjective.
(ii) f is maximal weakly open if g is continuous and injective.

Proof: Similar to that of 2.20.

REFERENCES

1. F.Nakaoka and F.oda, Some Application of minimal open sets, int.j.math.math.sci.vol 27, No.8, 471-476 (2001).
2. F.Nakaoka and F.Oda, Some Properties of Maximal open sets, Int.J.Math.Math.sci.Vol 21, No.21, 1331-1340 (2003).
3. F.Nakaoka and F.Oda, on Minimal closed sets, Proceeding of topological spaces and it's application, 5, 19-21 (2003).
4. M.Shiek John, A study on generalizations of closed sets on continuous maps in topological and bitopological spaces, ph.d thesis Bharathiar university, Ciombatore.(2002).
5. B.M.Ittanagi, on some advanced topics in General Topological Spaces, Ph.d thesis, Karnatak University Dharwad(2008).
6. R.S.Wali and Vivekananda Dembre Minimal weakly open sets and Maximal weakly closed sets in topological spaces, International journal of Mathematical Archieve – Vol-5(9), Sept (2014).
7. R.S.Wali and Vivekananda Dembre, Minimal Weakly Closed sets and Maximal Weakly Open Sets in Topological Spaces, Intenational Research Journal of Pure Algebra – Vol- 4(9), Sept-2014.

Source of Support: Nil, Conflict of interest: None Declared