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A NOTE ON PRE-OPEN SETS

V. V. S. RAMACHANDRAM*

ASSOCIATE PROFESSOR, B. V. C. COLLEGE OF ENGINEERING, RAJAHMUNDRY

***E-mail:** vvsrch@yahoo.com*

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ABSTRACT

The aim of this paper is to give some results concerning Pre-open sets. We also obtained a necessary and sufficient condition for a topological space to be a Pre-T_1 space.

Keywords: *Pre-open set, Pre-closed set, dense set, Closure of a set, Interior of a set.*

AMS Subject Classification: *54 A 05, 54 A 10.*

INTRODUCTION

The term 'pre-open set' was first used by A. S. Mashhour, M. E. Abd El-Monsef and S.N. El- Deeb In this paper we want to introduce and investigate a necessary and sufficient condition for a topological space to be a Pre-T_1 space.

Throughout this paper X, Y, Z will denote topological spaces. Let A be a subset of X. We will denote the closure of A by cl (A) and Interior of A by Int (A).

PRELIMINARIES

Def 1.1: A subset A of X is pre-open if $A \subseteq \mathrm{Int}\,(\mathrm{cl}(A))$.

Def 1.2: A subset A of X is dense if $\mathrm{cl}(A) = X$.

Def 1.3: If A and B are subsets of X then A is dense in B if $\mathrm{cl}(A) \supseteq B$.

Def 1.4: A topological space X is a Pre-T_1 space if for each pair of distinct elements x, y$\in$ X $\exists$ two pre-open sets A and B such that $x \in A,\ y \notin A$ and $y \in B,\ x \notin B$.

Def 1.5: The complement of a pre-open set is a pre-closed set.

WE START WITH THE FOLLOWING RESULT:

Result 1.6: If A is dense in X then A is a pre-open set.

Proof: Given that **A** is dense in X

$$\Rightarrow \mathrm{cl}(A) = X$$

***Corresponding author:** *V. V. S. RAMACHANDRAM**, ***E-mail:** *vvsrch@yahoo.com*

$$\Rightarrow \mathrm{Int}(\mathrm{cl}(A)) = X$$
$$\Rightarrow A \subseteq \mathrm{Int}(\mathrm{cl}(A))$$

Hence A is a pre-open set.

Remark: A pre-open set need not be dense in X.

Example1.7: X = {a, b, c} and T = { $\varnothing$, X, {a}, {b, c}}
Take the subset A = {b}. Then cl(A) = {b ,c} and Int(cl (A)) = {b ,c}

Hence A is a pre-open set. But A is not dense in X.

Result 1.8: If A is a subset of an open set B in a topological space X such that $A \subseteq B \subseteq cl(A)$ then A is a pre-open set in X.

Proof: Given B is an open set in X so that, $B = Int(B)$.

Now, $A \subseteq B \subseteq cl(A)$
$$\Rightarrow Int(A) \subseteq Int(B) \subseteq Int(cl(A))$$
$$\Rightarrow Int(A) \subseteq B \subseteq Int(cl(A))$$
$$\Rightarrow A \subseteq Int(cl(A)))$$

Hence A is a pre-open set in X.

Theorem 1.9: Arbitrary union of Pre-open sets is a Pre-open set.

Proof: Trivial

Theorem 2: A topological space X is a pre-T_1 space if and only if {x} is a pre-closed set for each $x \in X$.

Proof: Suppose X is a pre-T_1 space. Let $x \in X$.
To prove that {x} is a pre-closed set, it is enough to prove that $\{x\}^1$ is a pre-open set.

If $\{x\}^1 = \varnothing$ then it is clear.

Let $y \in \{x\}' \Rightarrow y \neq x \Rightarrow \exists$ two pre-open sets N_y, N_x such that $x \notin N_y$ and $y \notin N_x$.

If we do the same for each element 'y' in X, we get a family of pre-open sets such that $x \notin N_y$ and $y \notin N_x$.

Clearly $\{x\}' = \bigcup_{y \neq x} \{ N_y \}$

By Theorem (1.9), we get that $\{x\}^1$ is a pre-open set.

Hence {x} is a pre-closed set.

Converse: Suppose that {x} is a pre-closed set for each $x \in X$. Let $x, y \in X$ and $x \neq y$. By assumption {x} and {y} are pre-closed sets.

$\Rightarrow \{x\}^1$ and $\{y\}^1$ are pre-open sets in X.

As $y \neq x$ we get that $x \in \{y\}^1$ and $y \in \{x\}^1$

Hence X is a pre-T_1 space.

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