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MINIMAL WEAKLY AND MAXIMAL WEAKLY CONTINUOUS FUNCTIONS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of functions called minimal weakly continuous and maximal weakly continuous functions are introduced and investigated and during this process some properties of the new concepts have been studied.

Keywords: *Minimal open set, Maximal open set, Minimal weakly open set, Maximal weakly open set.*

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1. INTRODUCTION

In the year 2001 and 2003, F.Nakaoka and N.oda [1] [2] [3] introduced and studied minimal open [resp. Minimal closed] sets which are subclass of open [resp.closed sets]. The family of all minimal open [minimal closed] in a topological space X is denoted by $m_io(X)$ [$m_ic(X)$]. Similarly the family of all maximal open [maximal closed] sets in a topological space X is denoted by $M_aO(X)$[$M_aC(X)$].The complements of minimal open sets and maximal open sets are called maximal closed sets and minimal closed sets respectively. In the year 2000, M.Sheik john [4] introduced and studied weakly closed sets and weakly open sets in topological space. In the year 2008 B.M.Ittanagi [5] introduced and studied minimal open sets and maps in topological spaces. In the year 2014 R.S.Wali and Vivekananda Dembre[6] [7] introduced and studied minimal weakly open sets and maximal weakly closed sets and maximal weakly open sets and minimal weakly closed sets in topological spaces.

1.1 Definition [1]: A proper non-empty open subset U of a topological space X is said to be minimal open set if any open set which is contained in U is ∅ or U.

1.2 Definition [2]: A proper non-empty open subset U of a topological space X is said to be maximal open set if any open set which is contained in U is X or U.

1.3 Definition [3]: A proper non-empty closed subset F of a topological space X is said to be minimal closed set if any closed set which is contained in F is ∅ or F & A proper non-empty closed subset F of a topological space X is said to be maximal closed set if any closed set which is contained in F is X or F.

1.4 Definition [4]: A subset A of (X, τ) is called weakly closed set if Cl(A) $\subseteq$ U whenever A $\subseteq$ U and U is semi-open in X and A subset A in (X, τ) is called weakly open set in X if A^c is weakly closed set in X.

1.5 Definition [4]: Let X and Y be the topological spaces. A map f : X $\longrightarrow$ Y is called Weakly continuous function if for every weakly open set N in Y, f^{-1}(N) is an open set in X.

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1.6 Definition [5]: Let X and Y be the topological spaces. A map f : X ⟶ Y is called
(i) Minimal continuous function if for every minimal open set N in Y, $f^{-1}(N)$ is an open set in X.
(ii)Maximal continuous function if for every maximal open set N in Y, $f^{-1}(N)$ is an open set in X.

1.7 Definition [6]: A proper non-empty weakly open subset U of X is said to be minimal weakly open set if any weakly open set which is contained in U is ∅ or U.

1.8 Definition [7]: A proper non-empty weakly closed subset U of X is said to be maximal weakly open set if any weakly open set which is contained in U is X or U.

2. MINIMAL WEAKLY CONTINUOUS FUNCTIONS

2.1 Definition : Let X and Y be topological spaces. A function f: X ⟶ Y is said to be minimal weakly continuous if for every minimal weakly open set N in Y, $f^{-1}(N)$ is an open set in X.

2.2 Theorem: Every continuous function is minimal weakly continuous.

Proof: Let f : X ⟶ Y be a continuous function . Let N be any minimal weakly open set in Y, since every minimal weakly open set is an open set. We have N is an open set in Y. Since f is continuous, $f^{-1}(N)$ is an open set in X.Therefore f is a minimal weakly continuous.

2.3 Remark: The converse of the above theorem 2.2 need not be true.

2.4 Example: Let X=Y={a,b,c} be with τ = {X, φ, {a}, {a,b}} and μ = {Y, φ, {a},{b},{a,b}}. Let f : X ⟶ Y be function defined by f(a) = a, f(b) = a, f(c) = c then f is minimal weakly continuous but not continuous.

2.5 Theorem: Let X and Y be topological spaces. A function f : X ⟶ Y is minimal weakly continuous iff the inverse image of each maximal weakly closed set in Y is a closed set.

Proof: Assume that f is minimal weakly continuous and let M be any maximal weakly closed set in Y, Y-M is a minimal weakly open set in Y. Since f is minimal weakly continuous, $f^{-1}(Y-M)$ is an open set in X .

But $f^{-1}(Y-M) = X- f^{-1}(M)$ is an open set in X.Therefore $f^{-1}(M)$ is a closed set in X. Conversely, Suppose $f^{-1}(M)$ is a closed set in X for every maximal weakly closed set M in Y. Let N be any minimal weakly open set in Y, then Y-N is a maximal weakly closed set in Y, So by hypothesis $f^{-1}(Y-N) = X - f^{-1}(N)$ is a closed set in X. Therefore $f^{-1}(N)$ is an open set in X. Therefore f is minimal weakly continuous.

2.6 Theorem: Let X and Y be topological spaces then f: X⟶ Y is minimal weakly continuous then $f(Cl(M))\subset Cl(f(M))$ for every maximal weakly closed set M in X.

Proof: Let M be any maximal weakly closed set in X and Cl(M)=M. Now $f [Cl(M)] = f(M)\subset Cl(f(M))$, for every maximal weakly closed set M in X.

2.7 Theorem: Let X and Y be topological spaces then f : X ⟶ Y is minimal weakly continuous iff for any point $P\epsilon X$ and for any minimal weakly open set N in Y containing f (P) there exists an open set M in X such that $P\epsilon M$ and $f(M)\subset N$.

Proof: Let N be any minimal weakly open set in Y containing f(P) for any point $P\epsilon M$ where M is an open set in X. Since f is minimal weakly continuous $f^{-1}(N)$ is an open set in X. Take $M = f^{-1}(N)$ we have $f(M)\subset N$. Conversely, Let N be any minimal weakly open set in Y by hypothesis there exists an open set M in X, Such that $P\epsilon M$, $f(P)\epsilon f(M)\subset N$, $P\epsilon f^{-1}(f(M))\subset f^{-1}(N)$ thus $f^{-1}(N)$ is an open set in X ; therefore f is minimal weakly continuous.

2.8 Theorem: Let X and Y be topological spaces and let A be a non-empty subset of X. If f : X ⟶ Y is minimal weakly continuous then the restriction function f_A : A ⟶ Y is minimal weakly continuous where A has the relative topology.

Proof: Let A be a non-empty subset of a topological space X and Let N be any minimal weakly open set in Y. Since f is minimal weakly continuous, $f^{-1}(N)$ is an open set in X. By definition of relative topology $f_A^{-1}(N) = A \cap f^{-1}(N)$. Therefore $A\cap f^{-1}(N)$ is an open set in A ; therefore f_A is minimal weakly continuous.

2.9 Remark: The composition of minimal weakly continuous need not be a minimal weakly continuous.

2.10 Example: Let X = Y = Z={a,b,c} be with τ = { X, φ, {a,b}}, μ ={ Y, φ, {a},{b}{a,b}} and σ = { Z, φ,{a}}. Let f : (X, τ)⟶(Y, μ) be a function defined by f(a)=c, f(b)=c & f(c)=C and g: (Y, , μ) ⟶(Z , σ) be identity function then f and g are minimal weakly continuous but gof : (X, τ) ⟶ (Z, σ) is not minimal weakly continuous where gof : (X, τ)⟶(Z, σ) be identity function.

2.11 Remark: If f: X ⟶Y is continuous and g: Y ⟶Z is minimal weakly continuous then gof : X ⟶Z is minimal weakly continuous.

3. MAXIMAL WEAKLY CONTINUOUS FUNCTIONS

3.1 Definition : Let X and Y be topological spaces. A function f : X ⟶ Y is said to be maximal weakly continuous if for every maximal weakly open set M in Y , f^{-1}(M) is an open set in X.

3.2 Theorem: Every continuous function is maximal weakly continuous.

Proof: Let f : X ⟶Y be a continuous function . Let M be any maximal weakly open set is in Y, since every maximal weakly open set is an open set .We have M is an open set in Y. Since f is continuous, f^{-1}(M) is an open set in X.Therefore f is a maximal weakly continuous.

3.3 Remark: The converse of the above theorem 3.2 need not be true.

3.4 Example: Let X =Y ={a,b,c} with τ = {X, φ, {a,b}} and μ = {Y, φ, {a},{b},{a,b} }. Let f : X ⟶Y be an identity function then f is maximal weakly continuous but not continuous.

3.5 Remark: Maximal weakly continuous and minimal weakly continuous are independent.

3.6 Example: Let X=Y={a,b,c} with τ_1 = {X, φ,{a},{b},{a,b}} and μ_1 = {Y, φ, {a},{a,b}}.

Let f : (X, τ_1) ⟶ (Y, μ_1) be an identity function then f is minimal weakly continuous but not maximal weakly continuous. Let g : (X, τ_2) ⟶ (Y, μ_2) be an identity function then g is maximal weakly continuous but not minimal weakly continuous where X = Y ={a,b,c} with τ_2 = {X, φ,{a,b}} and μ_2 = {Y, φ, {a},{b},{a,b}}.

3.7 Theorem: Let X and Y be topological spaces. A function f : X ⟶ Y is maximal weakly continuous iff the inverse image of each minimal weakly closed set in Y is a closed set in X.

Proof: Assume that f is maximal weakly continuous and let N be any minimal weakly closed set in Y, Y-N is a maximal weakly open set in Y. Since f is maximal weakly continuous, f^{-1}(Y-N) is an open set in X .

But f^{-1}(Y-N) = X- f^{-1}(N) is an open set in X.Therefore f^{-1}(N) is a closed set in X. Conversely, Suppose f^{-1}(N) is a closed set in X for every minimal weakly closed set N in Y. Let M be any maximal weakly open set in Y, then Y-M is a minimal weakly closed set in Y, So by hypothesis f^{-1}(Y-M) = X – f^{-1}(M) is a closed set in X. Therefore f^{-1}(M) is an open set in X. Therefore f is maximal weakly continuous.

3.8 Theorem: Let X and Y be topological spaces. A function f : X ⟶Y is maximal weakly continuous then f[Cl(N)]⊂Cl(f(N)) for every minimal weakly closed set N in X.

Proof: Let N be any minimal weakly closed set in X and Cl(N)=N. Now f(Cl(N))=f(N), Now we know that f(Cl(N))=f(N)⊂Cl(f(N)), for every minimal weakly closed set N in X.

3.9 Theorem: Let X and Y be topological spaces then f : X ⟶Y is maximal weakly continuous iff for any point P∈X and for any maximal weakly open set M in Y containing f(P) there exists an open set N in X such that P∈N and f(N)⊂M.

Proof: Let M be any maximal weakly open set in Y containing f(P) for any point P∈N, where N is an open set in X. Take N = f^{-1}(M) we have f(N)⊂M. Conversely, Let M be any maximal weakly open set in Y; by hypothesis there exists an open set N in X, Such that P∈N, f(P)∈f(N)⊂M ; thus f^{-1}(M) is an open set in X ; therefore f is maximal weakly continuous.

3.10 Theorem: Let X and Y be topological spaces and A be a non-empty subset of X. If f : X ⟶Y is maximal weakly continuous then the restriction function f_A : A ⟶Y is maximal weakly continuous where A has the relative topology.

Proof: Let A be a non-empty subset of a topological space X and Let M be any maximal weakly open set in Y. Since f is maximal weakly continuous, $f^{-1}(M)$ is an open set in X. By definition of relative topology $f_A^{-1}(M) = A \cap f^{-1}(M)$. Therefore $A \cap f^{-1}(M)$ is an open set in A, Therefore f_A is maximal weakly continuous.

3.11 Remark: The composition of maximal weakly continuous need not be a maximal weakly continuous.

3.12 Example: Let X = Y = Z={a,b,c} be with τ = {X, φ, {a}}, μ ={Y, φ, {a},{b}{a,b}} , σ = {Z, φ,{a,b}}
Let f :(X, τ) ⟶ (Y, μ) be a function defined by f(a) = c, f(b) = c & f(c) = c and g : (Y, μ) ⟶ (Z, σ) be identity function then f and g are maximal weakly continuous but gof : (X, τ) ⟶ (Z , σ) is not maximal weakly continuous where gof : (X, τ) ⟶ (Z, σ) be identity function.

3.13 Remark: If f : X ⟶ Y is continuous and g :Y ⟶ Z is maximal weakly continuous then gof: X ⟶ Z is maximal weakly continuous.

REFERENCES

1. F.Nakaoka and F.oda, Some Application of minimal open sets, int.j.math.math.sci.vol 27, No.8, 471-476 (2001).
2. F.Nakaoka and F.Oda, Some Properties of Maximal open sets, Int.J.Math.Math.sci.Vol 21, No.21, 1331-1340 (2003).
3. F.Nakaoka and F.Oda, on Minimal closed sets, Proceeding of topological spaces and it's application, 5, 19-21 (2003).
4. M.Shiek John, A study on generalizations of closed sets on continuous maps in topological and bitopological spaces, ph.d thesis Bharathiar university, Ciombatore.(2002).
5. B.M.Ittanagi, on some advanced topics in General Topological Spaces, Ph.d thesis, Karnatak University Dharwad(2008).
6. R.S.Wali and Vivekananda Dembre Minimal weakly open sets and Maximal weakly closed sets in topological spaces, International journal of Mathematical Archieve –Vol- 5(9), Sept (2014).
7. R.S.Wali and Vivekananda Dembre, Minimal Weakly Closed sets and Maximal Weakly Open Sets in Topological Spaces, Intenational Research Journal of Pure Algebra – Vol- 4(9), Sept-2014.

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