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GENERALIZED MINIMAL CONTINUOUS MAPS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper a new class of generalized minimal continuous maps in topological spaces are introduced and their related theorems have been proved. A mapping $f: (X, \tau) \rightarrow (Y,\sigma)$ is said to be generalized minimal continuous (briefly g- m_i continuous) map if the inverse image of every minimal closed set in Y is a g- m_i closed set in X. Also, as an analogy of gc- irresolute maps, generalized minimal irresolute maps are introduced and characterized in topological spaces.

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1. INTRODUCTION AND PRELIMINARIES

In 1970 N. Levine [4] introduced generalized closed (g-closed) sets in topological spaces in order to extend many of the important properties of closed sets to a larger family. Further study of g-closed sets in topological spaces was continued by W. Dunham and N. Levine [3]. Recently minimal open sets and maximal open sets in topological spaces were introduced and studied by F. Nakaoka and N. Oda ([6], [7] and [8]). K.Balachandran, P. Sundaram and H. Maki [5] introduced and studied the concept of a new class of continuous maps, namely g-continuous maps that includes a class of continuous maps called gc-irresolute maps and M. Caldas [1] investigated some of the further properties of g-closed sets and g-continuous maps in topological spaces.

In section 2, a new class of mappings called generalized minimal continuous mappings that includes a class of generalized minimal irresolute mappings in topological spaces are introduced in topological spaces. Some properties of such classes of mappings are obtained.

Throughout this chapter (X, τ), (Y,σ) and (Z, η) denote topological spaces on which no separation axioms are assumed unless otherwise explicitly mentioned. For any subset A of a topological space (X, τ), closure of A, interior of A and complement of A is denoted by cl (A), int (A) and A^c respectively. We recall the following definitions, which are prerequisites for our present study.

Definition 1.1: A proper nonempty subset A of a topological space (X, τ) is called
(i) a minimal open (resp. minimal closed) set[6] if any open (resp. closed) subset of X which is contained in A, is either A or ϕ.
(ii) a maximal open (resp. maximal closed) set[7] if any open (resp. closed) set which contains A, is either A or X.

Remark1.2 [8]: Minimal open (resp. minimal closed) sets and maximal closed (resp. maximal open) sets are complements of each other.

Definition 1.3: A subset A of a topological space (X, τ) is called
(i) a generalized closed[1] (briefly g-closed) set if cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is an open set in X.
(ii) a generalized open (briefly g-open) set [1] iff A^c is a g-closed set.

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Definition 1.4[2]: For any subset A of a topological space (X,τ), $cl^*(A)$ is defined to be the intersection of all the g-closed sets containing A in a topological space (X, τ).

Definition 1.5 A mapping f: (X, τ) → (Y,σ) is said to be
- (i) generalized continuous (g-continuous) map [5] if the inverse image of every closed set in Y is g-closed set in X.
- (ii) minimal continuous map if the inverse image of every minimal open (or maximal closed) set in Y is an open (or closed) set in X.
- (iii) maximal continuous map if the inverse image of every maximal open (or minimal closed) set in Y is an open (or closed) set in X.
- (iv) minimal generalized continuous (m_i g-continuous) map if the inverse image of every minimal open (or maximal closed) set in Y is g-open (or g-closed) set in X.
- (v) maximal generalized continuous (m_a g-continuous) map if the inverse image of every maximal open (or minimal closed) set in Y is g-open (or g-closed) set in X.

Definition 1.6: A mapping f : (X, τ) → (Y,σ) is said to be
- (i) gc-irresolute map [5] if the inverse image of every g-closed set in Y is g-closed set in X.
- (ii) minimal irresolute map if the inverse image of every minimal open (or maximal closed) set in Y is minimal open (or maximal closed) set in X.
- (iii) maximal irresolute map if the inverse image of every maximal open (or minimal closed) set in Y is maximal open (or minimal closed) set in X.

2. GENERALIZED MINIMAL CONTINUOUS MAPS

Definition 2.1: A mapping f: (X, τ) → (Y,σ) is said to be generalized minimal continuous (briefly g- m_i continuous) map if the inverse image of every minimal closed set in Y is a g- m_i closed set in X.

Theorem 2.2: Every g-m_i continuous map is m_a g-continuous map.

Proof: Let f: (X, τ) → (Y,σ) be any g-m_i continuous map and U be any maximal open set in Y then U^c is a minimal closed set in Y. Therefore $f^{-1}(U^c)$ is a g- minimal closed set in X. As every g-minimal closed set is a g-closed set, $f^{-1}(U^c) = [f^{-1}(U)]^c$ is a g-closed set in X which implies $f^{-1}(U)$ is a g-open set in X. Hence f is m_a g-continuous map.

Remark 2.3: Converse of the Theorem 2.2 need not be true.

Example 2.4: Let X= Y = {a, b, c, d} with τ= {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X} and σ = {ϕ, {a},{a, b}, {c, d}, {a, c, d}, Y}.

τ-g-open sets: ϕ, {a}, {b}, {c}, {a, b},{a, c}, {b, c}, {b, d}, a, b, c}, {a, b, d}, {b, c, d}, X}. τ g-m_i closed sets: ϕ, {a}, {c}.

σ- m_i closed sets: {b}, {c, d}. σ -m_a open sets: {a, b}, {a, c, d}.

Let f: (X, τ) → (Y,σ) be a mapping defined by f(a)= a, f(b) = c, f(c)=b and f(d)=d. Then f is m_a –g-continuous map but not g-m_i continuous map.

Theorem 2.5:
- (i) If f: (X, τ) → (Y,σ) is any map. Then the following statements are equivalent.
  - (a) f is g-m_i continuous map.
  - (b) The inverse image of each maximal open set in Y is a g-maximal open set in X.
  - (c) For each x∈X and each maximal open set N in Y containing f(x), there exists a g-maximal open set M containing x in X such that $f(M) \subseteq N$.
- (ii) If f: (X, τ) → (Y,σ) is g-m_i continuous map, then for every subset A of X, $f[cl^*(A)] \subseteq cl[f(A)]$.
- (iii) If f: (X, τ) → (Y,σ) is g-m_i continuous map, then for every subset B of Y, $cl^*[f^{-1}(B)] \subseteq f^{-1}[cl(B)]$.

Proof:
- (i) **(a) ⇒ (b):** Let N be any maximal open set in Y. Then N^c is a minimal closed set in Y.
  By (a) $f^{-1}(N^c) = [f^{-1}(N)]^c$ is a g-minimal closed set in X. It follows that $f^{-1}(N)$ is a g-maximal open set in X.

(b) ⇒ (c): For each $x \in X$, let N be any maximal open set in Y containing $f(x)$. So $x \in f^{-1}$ (N) and by (b) f^{-1} (N) is g-maximal open set in X. Let f^{-1} (N) = M. Then f (M) = f [f^{-1} (N) $\subseteq$ N which implies that $f(M) \subseteq N$.

(c) ⇒ (a): For each $x \in X$, let N be any maximal open set in Y containing $f(x)$. Then N^c is a minimal closed set in Y. By (c) there exists a g-maximal open set M such that $f(M) \subseteq N$. Then $f(M) = f[f^{-1}(N)] \subseteq N$. Then M = f^{-1} (N) is a g-maximal open set in X. Therefore $[f^{-1}(N)]^c = f^{-1}(N^c)$ is a g-minimal closed set in X.

Hence f is g-m_i continuous map.

(ii) Let f: (X, τ) → (Y,σ) be g-m_i continuous map such that for any $A \subseteq X$. Let $y \in f[cl^*(A)]$. Let N be a maximal open set in Y containing y. Then there exists a point $x \in X$ and a g-maximal open set M such that $y = f(x) \in f[cl^*(A)]$. So $x \in cl^*(A)$ and $f(M) \subseteq N$. Here M is a g-neighborhood of x. Since $x \in cl^*(A)$, $A \cap M \neq \phi$ holds from [2] and hence $f(A \cap M) = f(A) \cap f(M) = f(A) \cap N \neq \phi$.

Therefore $y = f(x) \in f(A) \subseteq cl\, f(A)$.

Hence $f[cl^*(A)] \subseteq cl\,[f(A)]$ for every subset A of X.

(iii) Let f: (X, τ) → (Y,σ) be g-m_i continuous map and B be any subset of Y. Then $f^{-1}(B) \subseteq X$. Putting $A = f^{-1}(B)$ in (ii) above, we get $f[cl^*(f^{-1}(B))] \subseteq cl\,[f(f^{-1}(B))]$. Therefore $cl^*[f^{-1}(B)] \subseteq f^{-1}[cl(B)]$.

Remark2.6: Converse of the Theorem 2.5 (ii) and (iii) need not be true.

Example 2.7: In Example 2.4, for every $A \subseteq X$, $f[cl^*(A)] \subseteq cl\,[f(A)]$ and for every $B \subseteq Y$, $cl^*[f^{-1}(B)] \subseteq f^{-1}[cl(B)]$, but f is not g-m_i continuous map.

Remark 2.8: The restricted map of a g-m_i continuous map need not be g-m_i continuous map.

Example 2.9: Let X= Y = {a, b, c, d} with τ= {ϕ, {b}, {d}, {b, d},{a, c, d}, X} and σ = {ϕ, {a}, {a, c}, {a, c, d}, Y}. Let f: (X, τ) → (Y,σ) be an identity mapping. Then f is a g-m_i continuous map but for A = {a, c, d} ⊂ X, $f_A : (A, \tau_A) \to (Y, \sigma)$ is not a g-m_i continuous map.

Theorem 2.10: If f: (X, τ) → (Y,σ) is a g-m_i continuous map and h : (Y, σ) →(Z, η) is a maximal irresolute map, then h o f: (X, τ) →(Z, η) is g-m_i continuous map.

Proof: Let N be a minimal closed set in Z. Then N^c is a maximal open set in Z. Since h: (Y, σ) →(Z, η) is a maximal irresolute map, $h^{-1}(N^c) = [h^{-1}(N)]^c$ is a maximal open set in Y. Therefore h^{-1} (N) is a minimal closed set in Y. → (Y,σ) is a But f: (X, τ) g-m_i continuous map. Therefore $f^{-1}[h^{-1}(N)] = (ho f)^{-1}(N)$ is a g-minimal closed set in X. Hence hof: (X, τ) →(Z, η) is g-m_i continuous map.

Definition 2.11: A mapping f: (X, τ) → (Y,σ) is said to be generalized minimal irresolute (briefly g- m_i irresolute) map if the inverse image of every g-minimal closed set in Y is a g- minimal closed set in X.

Remark 2.12: gc-irresolute maps and g-m_i irresolute maps are independent of each other.

Example 2.13: Let X= Y = {a, b, c, d} with τ= {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {b, c, d}, X} and σ = {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, Y}.

Let f: (X, τ) → (Y,σ) be a mapping defined by $f(a) = c$, $f(b) = a$, $f(c) = d$ and $f(d) = c$. Then f is gc-irresolute map but not g-m_i irresolute map.

Again let X= Y = {a, b, c, d} with τ= {ϕ, {a}, {c}, {d}, {a, c}, {a, d}, {b, d}, {c, d}, {a, b, d}, {a, c, d}, {b, c, d}, X} and σ = {ϕ, {a}, {c, d}, {a, c, d}, {b, c, d}, Y}.

Let h: (X, τ) → (Y,σ) be a mapping defined by h (a)= a, h(b) = d, h(c)= c and h (d)= b. Then h is g-m_i irresolute map but not gc-irresolute map.

Theorem 2.14:

(i) If f: (X, τ) → (Y,σ) is any mapping. Then the following statements are equivalent.
  (a) f is g-m_i irresolute map.
  (b) The inverse image of each g-maximal open set in Y is a g-maximal open set in X.
  (c) For each x∈X and each g-maximal open set N in Y containing f(x), there exists g-maximal open set M containing x in X such that $f(M) \subseteq N$.

(ii) If f: (X, τ) → (Y,σ) is g-m_i irresolute map, then for every subset A of X, $f[cl^*(A)] \subseteq cl[f(A)]$.

(iii) If f: (X, τ) → (Y,σ) is g-m_i irresolute map, then for every subset B of Y, $cl^*[f^{-1}(B)] \subseteq f^{-1}[cl(B)]$.

Proof: Follows from the Theorem 2.5.

Remark 2.15: The restricted map of a g-m_i irresolute map need not be a g-m_i irresolute map.

Example 2.16: Let X= Y = {a, b, c, d} with τ= {ϕ, {a}, {a, b}, {c, d}, {a, c, d}, X} and σ = {ϕ, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {b, d},{a, b, c}, {a, b, d}, {b, c, d}, Y}.

Let f: (X, τ) → (Y,σ) be a mapping defined by f(a)= d, f(b) = b, f(c)= c and f(d) = a. Then f is a g-m_i irresolute map but for A = {a, b, c} ⊂ X, $f_A : (A, \tau_A) \to (Y, \sigma)$ is not a g-m_i irresolute map.

Theorem 2.17: If f: (X, τ) → (Y,σ) is a g-m_i irresolute map and h: (Y, σ) →(Z, η) is a g-m_i irresolute map, then h o f: (X, τ) →(Z, η) is g-m_i irresolute map.

Proof: Let N be a g-minimal closed set in Z. Then by hypothesis $h^{-1}(N)$ is a g-minimal closed set in Y. But f: (X, τ) → (Y,σ) is a g-m_i irresolute map.

Therefore $f^{-1}[h^{-1}(N)] = (h \circ f)^{-1}(N)$ is a g-minimal closed set in X. Hence ho$f$: (X, τ) →(Z, η) is g-m_i irresolute map.

Theorem 2.18: If f: (X, τ) → (Y,σ) is a g-m_i irresolute map and h: (Y, σ) →(Z, η) is a g-m_i continuous map, then ho$f$: (X, τ) →(Z, η) is g-m_i continuous map.

Proof: Let N be a minimal closed set in Z. Then by hypothesis $h^{-1}(N)$ is a g-minimal closed set in Y. But f: (X, τ) → (Y,σ) is a g-m_i irresolute map. Therefore $f^{-1}[h^{-1}(N) = (h o f)^{-1}(N)$ is a g-minimal closed set in X. Hence h o f: (X, τ) →(Z, η) is g-m_i continuous map.

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