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FIXED POINT THEOREMS IN 2-METRIC SPACES

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ABSTRACT

Our object in this paper is to prove some fixed point and common fixed point theorem for expansion mapping in 2-metric space.

Keywords: *Fixed point, Common fixed point, expansion mapping, complete 2-metric space.*

Mathematics Subject Classification: *47H10,54H25.*

2. INTRODUCTION AND PRELIMINARIES

To start the main result first we give some known definition which are helpful to prove of our main result.

Definition 2.1: A 2-metric space is a space X in which for each triple of points x, y, z there exists a real function $d(x, y, z)$ such that
$[M_1]$ to each pair of distinct points x, y, z in X
$d(x, y, z) \neq 0$
$[M_2]$ $d(x, y, z) = 0$ when at least two of x, y, z are equal
$[M_3]$ $d(x, y, z) = d(y, z, x) = d(x, z, y)$
$[M_4]$ $d(x, y, z) = d(x, y, v) + d(x, v, z) + d(v, y, z)$ for all $x, y, z, v \in X$

Function d is called a 2-metric for the space X and (X, d) is called a 2-metric space.

Definition 2.2: A sequence $\{x_n\}$ in 2-metric space (X, d) is said to be convergent at x if $d(x_n, x, z) = 0$ for all z in X.

Definition 2.3: A sequence $\{x_n\}$ in 2-metric space (X, d) is said to be Cauchy sequence if $d(x_n, x, z) = 0$ for all z in X.

Definition 2.4: A 2-metric space (X, d) is said to be complete if every Cauchy sequence is convergent.

Definition 2.5: Let (X, d) be a metric space and $T: X \to X$ be a mapping then T is said to be expansive mapping if for every $x, y \in X$ there exit a number $r > 1$ such that

$$d(Tx, Ty) \geq rd(x, y)$$

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3. MAIN RESULT

Theorem 3.1: Let (X, d) be a complete 2-metric space and let $T: X \to X$ be a mapping satisfying the following condition

$$d(T^{p+1}x, T^{p+2}y, a) \geq \alpha \frac{d(x,T^{p+1}x,a)[1+d(y,T^{p+2}y,a)]}{1+d(x,T^{p+2}y,a)} + \beta \frac{d(x,T^{p+1}x,a)[1+d(y,T^{p+1}x,a)]}{1+d(x,T^{p+1}x,a)} + \gamma \left[\frac{d(x,T^{p+1}x,a)+d(y,T^{p+1}x,a)}{2}\right] + \delta \left[\frac{d(x,T^{p+2}y,a)+d(y,T^{p+2}y,a)}{2}\right] \quad (3.1.1)$$

For all $x, y \in X$, $\alpha, \beta, \gamma, \delta \geq 0, \alpha + \beta + \gamma > 1$, $\gamma + \delta > 2$ and for any non-negative integer p.

Then T has a unique fixed point.

Proof: we prove this theorem for $p = 0$

Now putting $p = 0$ in (3.1.1) then we have

$$d(Tx, T^2y, a) \geq \alpha \frac{d(x,Tx,a)[1+d(y,T^2y,a)]}{1+d(x,T^2y,a)} + \beta \frac{d(x,Tx,a)[1+d(y,Tx,a)]}{1+d(x,Tx,a)} + \gamma \left[\frac{d(x,Tx,a)+d(y,Tx,a)}{2}\right] + \delta \left[\frac{d(x,T^2y,a)+d(y,T^2y,a)}{2}\right] \quad (3.1.2)$$

We define a sequence $\{x_n\} \in X$ as follow:

$x_0 \in X, x_0 = Tx_1, x_1 = Tx_2, x_2 = Tx_3, \dots \dots \dots, x_n = Tx_{n+1}$

Now consider

$$\begin{aligned} d(x_0, x_1, a) = d(x_1, x_0, a) &= d(Tx_2, T^2x_2, a) \\ &\geq \alpha \frac{d(x_2,Tx_2,a)[1+d(x_2,T^2x_2,a)]}{1+d(x_2,T^2x_2,a)} + \beta \frac{d(x_2,Tx_2,a)[1+d(x_2,Tx_2,a)]}{1+d(x_2,Tx_2,a)} \\ &\quad + \gamma \left[\frac{d(x_2,Tx_2,a)+d(x_2,Tx_2,a)}{2}\right] + \delta \left[\frac{d(x_2,T^2x_2,a)+d(x_2,T^2x_2,a)}{2}\right] \\ &\geq \alpha \frac{d(x_2,x_1,a)[1+d(x_2,x_0,a)]}{1+d(x_2,x_0,a)} + \beta \frac{d(x_2,x_1,a)[1+d(x_2,x_1,a)]}{1+d(x_2,x_1,a)} \\ &\quad + \gamma \left[\frac{d(x_2,x_1,a)+d(x_2,x_1,a)}{2}\right] + \delta \left[\frac{d(x_2,x_0,a)+d(x_2,x_0,a)}{2}\right] \\ &\geq (\alpha + \beta + \gamma) d(x_2, x_1, a) + \delta[d(x_2, x_1, a) - d(x_1, x_0, a)] \end{aligned}$$

$$\Rightarrow d(x_2, x_1, a) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_0, x_1, a)$$

$$\Rightarrow d(x_1, x_2, a) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_0, x_1, a)$$

Similarly we have

$$d(x_2, x_3, a) \leq \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} d(x_1, x_2, a)$$

$$d(x_2, x_3, a) \leq \left[\frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)}\right]^2 d(x_0, x_1, a)$$

In general we can write

$$d(x_n, x_{n+1}, a) \leq \left[\frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)}\right]^n d(x_0, x_1, a)$$

$$d(x_n, x_{n+1}, a) \leq K^n d(x_0, x_1, a) \qquad \text{where } K = \frac{(1+\delta)}{(\alpha+\beta+\gamma+\delta)} < 1$$

Since $0 \leq K < 1$ so for $n \to \infty, K^n \to 0$ we have $d(x_{n+1}, x_n, a) \to 0$.

Hence $\{x_n\}$ is a Cauchy sequence in the complete 2-metric space X. So there is a point $\xi \in X$ such that $\{x_n\} \to \xi$.

Now

$$\begin{aligned} d(\xi, T\xi, a) &= d(T\xi, T^2x_{n+2}, a) \\ &\geq \alpha \frac{d(\xi,T\xi,a)[1+d(x_{n+2},T^2x_{n+2},a)]}{1+d(\xi,T^2x_{n+2},a)} + \beta \frac{d(\xi,T\xi,a)[1+d(x_{n+2},T\xi,a)]}{1+d(\xi,T\xi,a)} \\ &\quad + \gamma \left[\frac{d(\xi,T\xi,a)+d(x_{n+2},T\xi,a)}{2}\right] + \delta \left[\frac{d(\xi,T^2x_{n+2},a)+d(x_{n+2},T^2x_{n+2},a)}{2}\right] \\ &\geq \alpha \frac{d(\xi,T\xi,a)[1+d(x_{n+2},x_n,a)]}{1+d(\xi,x_n,a)} + \beta \frac{d(\xi,T\xi,a)[1+d(x_{n+2},T\xi,a)]}{1+d(\xi,T\xi,a)} \\ &\quad + \gamma \left[\frac{d(\xi,T\xi,a)+d(x_{n+2},T\xi,a)}{2}\right] + \delta \left[\frac{d(\xi,x_n,a)+d(x_{n+2},x_n,a)}{2}\right] \end{aligned}$$

Letting $n \to \infty$ then we have
$d(\xi, T\xi, a) \geq (\alpha+\beta+\gamma) d(\xi, T\xi, a)$
$\Longrightarrow [(\alpha+\beta+\gamma)-1] d(\xi, T\xi, a) \leq 0$

Which gives
$d(\xi, T\xi, a) = 0 \Rightarrow T\xi = \xi.$

Hence ξ is a fixed point of T.

Let η be another point fixed of T then by condition (3.1.2) we have

$$\begin{aligned} d(\xi,\eta,a) &= d(T\xi, T^2\eta, a) \\ &\geq \alpha \frac{d(\xi,T\xi,a)[1+d(\eta,T^2\eta,a)]}{1+d(\xi,T^2\eta,a)} + \beta \frac{d(\xi,T\xi,a)[1+d(\eta,T\xi,a)]}{1+d(\xi,T\xi,a)} + \gamma \left[\frac{d(\xi,T\xi,a)+d(\eta,T\xi,a)}{2}\right] + \delta \left[\frac{d(\xi,T^2\eta,a)+d(\eta,T^2\eta,a)}{2}\right] \\ &\geq \alpha \frac{d(\xi,\xi,a)[1+d(\eta,\eta,a)]}{1+d(\xi,\eta,a)} + \beta \frac{d(\xi,\xi,a)[1+d(\eta,\xi,a)]}{1+d(\xi,\xi,a)} + \gamma \left[\frac{d(\xi,\xi,a)+d(\eta,\xi,a)}{2}\right] + \delta \left[\frac{d(\xi,\eta,a)+d(\eta,\eta,a)}{2}\right] \end{aligned}$$

$d(\xi,\eta,a) \geq \frac{\gamma+\delta}{2} d(\xi,\eta,a)$

$\left[\left(\frac{\gamma+\delta}{2}\right) - 1\right] d(\xi,\eta,a) \leq 0$

$i.e.\, d(\xi,\eta,a) = 0$

$\Longrightarrow \xi = \eta$

Hence fixed point of T is unique.

Theorem 3.2: Let (X, d) be a complete 2-metric space and let $T: X \to X$ be a mapping satisfying the following condition

$$d(T^{p+1}x, T^{p+2}y, a) \geq \alpha \min\{d(x, T^{p+2}y, a), d(y, T^{p+1}x, a)\} + \beta \left\{\frac{d(x,T^{p+1}x,a)+d(y,T^{p+2}y,a)}{2}\right\} \tag{3.2.1}$$

For all $x, y, a \in X$, $\alpha > 1$, $\beta > 2$ and for any non-negative integer p.

Then T has a unique fixed point.

Proof: we prove this theorem for $p = 0$

Now putting $p = 0$ in (3.2.1) then we have

$$d(Tx, T^2y, a) \geq \alpha \min\{d(x, T^2y, a), d(y, Tx, a)\} + \beta \left\{\frac{d(x,Tx,a)+d(y,T^2y,a)}{2}\right\} \tag{3.2.2}$$

We define a sequence $\{x_n\} \in X$ as follow:
$x_n = Tx_{n+1}, n = 0,1,2, ...$ and $x_0 \in X$.

Now consider

$$\begin{aligned} d(x_n, x_{n-1}, a) &= d(Tx_{n+1}, T^2x_{n+1}, a) \\ &\geq \alpha \min\{d(x_{n+1}, T^2x_{n+1}, a), d(x_{n+1}, Tx_{n+1}, a)\} + \beta \left\{\frac{d(x_{n+1},Tx_{n+1},a)+d(x_{n+1},T^2x_{n+1},a)}{2}\right\} \\ &\geq \alpha \min\{d(x_{n+1}, x_{n-1}, a), d(x_{n+1}, x_n, a)\} + \beta \left\{\frac{d(x_{n+1},x_n,a)+d(x_{n+1},x_{n-1},a)}{2}\right\} \\ &\geq \min \begin{Bmatrix} \left(\alpha + \frac{\beta}{2}\right) d(x_{n+1}, x_{n-1}, a) + \frac{\beta}{2} d(x_{n+1}, x_n, a), \\ \frac{\beta}{2} d(x_{n+1}, x_{n-1}, a) + \left(\alpha + \frac{\beta}{2}\right) d(x_{n+1}, x_n, a) \end{Bmatrix} \\ &\geq \min \begin{Bmatrix} \left(\alpha + \frac{\beta}{2}\right) \{d(x_{n+1}, x_n, a) - d(x_n, x_{n-1}, a)\} + \frac{\beta}{2} d(x_{n+1}, x_n, a), \\ \frac{\beta}{2} \{d(x_{n+1}, x_n, a) - d(x_n, x_{n-1}, a)\} + \left(\alpha + \frac{\beta}{2}\right) d(x_{n+1}, x_n, a) \end{Bmatrix} \\ &\geq \min \begin{Bmatrix} (\alpha+\beta) d(x_{n+1}, x_n, a) - \left(\alpha + \frac{\beta}{2}\right) d(x_n, x_{n-1}, a), \\ (\alpha+\beta) d(x_{n+1}, x_n, a) - \frac{\beta}{2} d(x_n, x_{n-1}, a) \end{Bmatrix} \\ &\geq \min \left\{\frac{(\alpha+\beta)}{1+\left(\alpha+\frac{\beta}{2}\right)}, \frac{(\alpha+\beta)}{1+\frac{\beta}{2}}\right\} d(x_{n+1}, x_n, a) \end{aligned}$$

$$\geq \frac{(\alpha+\beta)}{1+\left(\alpha+\frac{\beta}{2}\right)} d(x_{n+1}, x_n, a)$$
$$\geq \frac{2(\alpha+\beta)}{2+\alpha+\beta} d(x_{n+1}, x_n, a)$$

$$\Rightarrow d(x_{n+1}, x_n, a) \leq \frac{(2+\alpha+\beta)}{2(\alpha+\beta)} d(x_n, x_{n-1}, a)$$

$$\Rightarrow d(x_{n+1}, x_n, a) \leq K d(x_n, x_{n-1}, a) \qquad \text{where } K = \frac{(2+\alpha+\beta)}{2(\alpha+\beta)} < 1$$

Similarly we can show that
$d(x_n, x_{n-1}, a) \leq K d(x_{n-1}, x_{n-2}, a)$

And $d(x_{n+1}, x_n, a) \leq K^2 d(x_{n-1}, x_{n-2}, a)$

Thus $d(x_{n+1}, x_n, a) \leq K^n d(x_1, x_0, a)$

Since $0 \leq K < 1$ so for $n \to \infty, K^n \to 0$ we have $d(x_{n+1}, x_n, a) \to 0$.

Hence $\{x_n\}$ is a Cauchy sequence in the complete 2-metric space X. So there is a point $z \in X$ such that $\{x_n\} \to z$.

Now
$$d(z, Tz, a) = d(Tz, T^2 x_{n+2}, a)$$
$$\geq \alpha\, min\{d(z, T^2 x_{n+2}, a), d(y, Tz, a)\} + \beta \left\{\frac{d(z,Tz,a)+d(x_{n+2},T^2 x_{n+2},a)}{2}\right\}$$
$$\geq \alpha\, min\{d(z, x_n, a), d(y, Tz, a)\} + \beta \left\{\frac{d(z,Tz,a)+d(x_{n+2},x_n,a)}{2}\right\}$$

Letting $n \to \infty$ then we have
$$d(z, Tz, a) \geq \frac{\beta}{2} d(z, Tz, a)$$
$$\left(\frac{\beta}{2} - 1\right) d(z, Tz, a) \leq 0$$

Which gives
$d(z, Tz, a) = 0 \Rightarrow Tz = z$. Since $\beta > 2$.

Hence z is a fixed point of T.

Let w be another point of T then by condition (3.2.2) we have
$$d(z, w, a) = d(Tz, T^2 w, a)$$
$$\geq \alpha\, min\{d(z, T^2 w, a), d(w, Tz, a)\} + \beta \left\{\frac{d(z,Tz,a)+d(w,T^2 w,a)}{2}\right\}$$
$$\geq \alpha\, min\{d(z, w, a), d(w, z, a)\} + \beta \left\{\frac{d(z,z,a)+d(w,w,a)}{2}\right\}$$
$$\geq \alpha\, min\{d(z, w, a), d(z, w, a)\}$$

$$\Rightarrow (\alpha - 1) d(z, w, a) \leq 0$$

$$\Rightarrow d(z, w, a) = 0 \qquad \text{since } \alpha > 1$$

$$\Rightarrow z = w$$

Hence fixed point of T is unique.

Theorem3.3: Let (X, d) be a complete 2-metric space and let $S, T: X \to X$ are two mappings satisfying the following condition
$$d(S^{p+1}x, T^{p+2}y, a) \geq \alpha\, min\{d(x, T^{p+2}y, a), d(y, S^{p+1}x, a)\}$$
$$+\beta \left\{\frac{d(x,S^{p+1}x,a)+d(y,T^{p+2}y,a)}{2}\right\} + \gamma \left\{\frac{d(x,T^{p+2}y,a)+d(y,S^{p+1}x,a)}{2}\right\} \qquad (3.3.1)$$

For all $x, y \in X$, $\alpha, \beta, \gamma > 1$and for any non-negative integer p.

Then S, T has a unique fixed point.m

Proof: we prove this theorem for $p = 0$

Now putting $p = 0$ in (3.3.1) then we have

$$d(Sx, T^2y, a) \geq \alpha \min\{d(x, T^2y, a), d(y, Sx, a)\} + \beta \left\{\frac{d(x,Sx,a)+d(y,T^2y,a)}{2}\right\} + \gamma \left\{\frac{d(x,T^2y,a)+d(y,Sx,a)}{2}\right\} \quad (3.3.2)$$

Let $x_0 \in X$. We define a sequence $\{x_n\} \in X$ as follow:

$x_0 = Tx_1, x_1 = Sx_2, \dots \dots . . x_{2n} = Tx_{2n+1}, x_{2n-1} = Sx_{2n}, \dots$

Consider

$$\begin{aligned} d(x_{2n+1}, x_{2n}, \mathrm{a}) &= d(x_{2n}, x_{2n+1}, \mathrm{a}) = d(Tx_{2n+1}, Sx_{2n+2}, \mathrm{a}) = d(Sx_{2n+2}, T^2x_{2n+2}, \mathrm{a}) \\ &\geq \alpha \min\{d(x_{2n+2}, T^2x_{2n+2}, \mathrm{a}), d(x_{2n+2}, Sx_{2n+2}, \mathrm{a})\} \\ &\quad +\beta \left\{\frac{d(x_{2n+2},Sx_{2n+2},\mathrm{a})+d(x_{2n+2},T^2x_{2n+2},\mathrm{a})}{2}\right\} + \gamma \left\{\frac{d(x_{2n+2},T^2x_{2n+2},\mathrm{a})+d(x_{2n+2},Sx_{2n+2},\mathrm{a})}{2}\right\} \\ &\geq \alpha \min\{d(x_{2n+2}, x_{2n}, \mathrm{a}), d(x_{2n+2}, x_{2n+1}, \mathrm{a})\} \\ &\quad +\beta \left\{\frac{d(x_{2n+2},x_{2n+1},\mathrm{a})+d(x_{2n+2},x_{2n},\mathrm{a})}{2}\right\} + \gamma \left\{\frac{d(x_{2n+2},x_{2n},\mathrm{a})+d(x_{2n+2},x_{2n+1},\mathrm{a})}{2}\right\} \\ &\geq \min \begin{Bmatrix} \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2}\right) d(x_{2n+2}, x_{2n}, \mathrm{a}) + \left(\frac{\beta+\gamma}{2}\right) d(x_{2n+2}, x_{2n+1}, \mathrm{a}), \\ \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2}\right) d(x_{2n+2}, x_{2n+1}, \mathrm{a}) + \left(\frac{\beta+\gamma}{2}\right) d(x_{2n+2}, x_{2n}, \mathrm{a}) \end{Bmatrix} \\ &\geq \min \begin{Bmatrix} \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2}\right) \{d(x_{2n+2}, x_{2n+1}, \mathrm{a}) - d(x_{2n+1}, x_{2n}, \mathrm{a})\} + \left(\frac{\beta+\gamma}{2}\right) d(x_{2n+2}, x_{2n+1}, \mathrm{a}), \\ \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2}\right) d(x_{2n+2}, x_{2n+1}, \mathrm{a}) + \left(\frac{\beta+\gamma}{2}\right) \{d(x_{2n+2}, x_{2n+1}, \mathrm{a}) - d(x_{2n+1}, x_{2n}, \mathrm{a})\} \end{Bmatrix} \\ &\geq \min \begin{Bmatrix} (\alpha + \beta + \gamma) d(x_{2n+2}, x_{2n+1}, \mathrm{a}) - \left(\alpha + \frac{\beta}{2} + \frac{\gamma}{2}\right) d(x_{2n+1}, x_{2n}, \mathrm{a}), \\ (\alpha + \beta + \gamma) d(x_{2n+2}, x_{2n+1}, \mathrm{a}) - \left(\frac{\beta+\gamma}{2}\right) d(x_{2n+1}, x_{2n}, \mathrm{a}) \end{Bmatrix} \\ &\geq \min \left\{\frac{(\alpha+\beta+\gamma)}{1+\left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)}, \frac{(\alpha+\beta+\gamma)}{1+\left(\frac{\beta+\gamma}{2}\right)}\right\} d(x_{2n+2}, x_{2n+1}, \mathrm{a}) \end{aligned}$$

$$\Rightarrow d(x_{2n+1}, x_{2n}) \geq \frac{(\alpha+\beta+\gamma)}{1+\left(\alpha+\frac{\beta}{2}+\frac{\gamma}{2}\right)} d(x_{2n+2}, x_{2n+1}, \mathrm{a})$$

$$\Rightarrow d(x_{2n+2}, x_{2n+1}) \leq \frac{(2+2\alpha+\beta+\gamma)}{2(\alpha+\beta+\gamma)} d(x_{2n+1}, x_{2n}, \mathrm{a})$$

$$\Rightarrow d(x_{2n+2}, x_{2n+1}, \mathrm{a}) \leq Kd(x_{2n+1}, x_{2n}, \mathrm{a}) \quad \text{where } K = \frac{(2+2\alpha+\beta+\gamma)}{2(\alpha+\beta+\gamma)} < 1$$

Similarly,

$d(x_{2n+1}, x_{2n}, \mathrm{a}) \leq Kd(x_{2n}, x_{2n-1}, \mathrm{a})$

And

$d(x_{2n+2}, x_{2n+1}, \mathrm{a}) \leq K^2 d(x_{2n}, x_{2n-1}, \mathrm{a})$

Continue in this way we get

$d(x_{2n+2}, x_{2n+1}, \mathrm{a}) \leq K^n d(x_1, x_0, \mathrm{a})$

Since $0 \leq K < 1$ so for $n \to \infty, K^n \to 0$ we have $d(x_{2n+2}, x_{2n+1}) \to 0$.

Hence $\{x_n\}$ is a Cauchy sequence in the complete 2-metric space X. So there is a point $z \in X$ such that $\{x_n\} \to z$.

Now we will show that z is a common fixed point of S and T.

$$\begin{aligned} d(z, Sz, a) = d(x_n, Sz, a) &= d(Sz, T^2x_{n+2}, \mathrm{a}) \\ &\geq \alpha \min\{d(z, T^2x_{n+2}, \mathrm{a}), d(x_{n+2}, Sz, a)\} \\ &\quad +\beta \left\{\frac{d(z,Sz,a)+d(x_{n+2},T^2x_{n+2},\mathrm{a})}{2}\right\} + \gamma \left\{\frac{d(z,T^2x_{n+2},\mathrm{a})+d(x_{n+2},Sz,a)}{2}\right\} \\ &\geq \alpha \min\{d(z, x_n, \mathrm{a}), d(x_{n+2}, Sz, a)\} \\ &\quad +\beta \left\{\frac{d(z,Sz,a)+d(x_{n+2},x_n,\mathrm{a})}{2}\right\} + \gamma \left\{\frac{d(z,x_n,\mathrm{a})+d(x_{n+2},Sz,a)}{2}\right\} \end{aligned}$$

$$\Rightarrow d(z, Sz, a) \geq \frac{\beta+\gamma}{2} d(z, Sz, a)$$

$$\Rightarrow \left(\frac{\beta+\gamma}{2} - 1\right) d(z, Sz, a) \leq 0$$

Which gives $d(z, Sz, a) = 0 \Rightarrow Sz = z$.

Hence z is a fixed point of S.

Similarly we can show that z is a fixed point of T.

Hence z is a common fixed point of S & T.

Let u, v be a common fixed point of S and T then

$$d(u, v, a) = d(Su, Tv, a) = d(Su, T^2v, a)$$

$$\geq \alpha\, min\{d(u, T^2v, a), d(v, Su, a)\} + \beta\left\{\frac{d(u,Su,a)+d(v,T^2v,a)}{2}\right\} + \gamma\left\{\frac{d(u,T^2v,a)+d(v,Su,a)}{2}\right\}$$

$$\geq \alpha\, min\{d(u, v, a), d(v, u, a)\} + \beta\left\{\frac{d(u,u,a)+d(v,v,a)}{2}\right\} + \gamma\left\{\frac{d(u,v,a)+d(v,u,a)}{2}\right\}$$

$\Rightarrow d(u, v, a) \geq (\alpha + \gamma)d(u, v, a)$

$\Rightarrow (\alpha + \gamma - 1)d(u, v, a) \leq 0$

$\Rightarrow d(u, v, a) = 0$

$\Rightarrow u = v$

Hence common fixed point of S and T is unique.

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