International Research Journal of Pure Algebra-5(4), 2015, 37-39
Available online through www.rjpa.info ISSN 2248–9037

The Algorithm for Inverse Matrices of Symmetric r-Circulant Matrices over Skew Field

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(Received On: 25-03-15; Revised & Accepted On: 13-04-15)

ABSTRACT

This paper mainly introduces elementary column transformation method of inverse matrix of symmetric r-circulant matrix over skew field by using the elementary column transformation of polynomial matrix.

Keywords: *skew field; symmetric r-circulant matrix; elementary column transformation.*

1. INTRODUCTION

Symmetric r-circulant matrix is an important special matrix. Therefore, the algorithms for inverse matrix of the matrix over skew field cause research of a lot of mathematicians. In this paper, using the elementary column transformation of polynomial matrix is given elementary column transformation method of inverse matrix of symmetric r-circulant matrix. And the algorithm has smaller amount of computations.

2. PREPARATION KNOWLEDGE

In this paper, let K be a skew field and x be unknown, and its primitive formula $f(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_n x^n$ where $a_0, a_1, \cdots, a_n \in K$ is defined to be unary polynomial over skew field K, simply indicating $f(x) = \sum_{t=0}^{n} a_t x^t$. $K[x]$ is a ring, and it is defined to be monadic polynomial ring over skew field K. $K^{m\times n}$ is denoted $m \times n$ -factorial-matrix. $d(x) = (f(x), g(x))$ of leading coefficient being 1 are respectively denoted the greatest left common divisor $f(x)$ and $g(x)$.

Definition 2.1: Let $r \in K$, matrix

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ ra_{n-1} & a_0 & a_1 & \cdots & a_{n-3} & a_{n-2} \\ ra_{n-2} & ra_{n-1} & a_0 & \cdots & a_{n-4} & a_{n-3} \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ ra_1 & ra_2 & ra_3 & \cdots & ra_{n-1} & a_0 \end{pmatrix}$$

is called r-circulant matrix. Referred to as $A = K_r(a_0, a_2, \cdots, a_{n-1}) \in KM_r$.

Definition 2.2: Let $r \in K$, matrix

$$A = \begin{pmatrix} a_0 & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ a_1 & a_2 & a_3 & \cdots & a_{n-1} & ra_0 \\ a_2 & a_3 & a_4 & \cdots & ra_0 & ra_1 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ a_{n-1} & ra_0 & ra_1 & \cdots & ra_{n-3} & ra_{n-2} \end{pmatrix}$$

is called symmetric r-circulant matrix. Referred to as $A = SK_r(a_0, a_2, \cdots, a_n) \in SKM_r$.

Definition 2.3: Introducing n order identity matrix $J_n = (e_n, e_{n-1}, \cdots, e_q)$ and e_i being n order unit column vector of the first i component for 1 and the other component for 0, then AJ_n is called r-circulant matrix.

3. ALGORITHM

Lemma 3.1: Set $A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r$, $B = SK_r(a_{n-1}, a_{n-2} \cdots, a_0) \in SKM_r$, then $BP = A$ and $B = AP$, $P = SK_1(0, \cdots, 0, 1) \in SKM_1$.

Lemma 3.2: Set $A \in K^{m\times n}, A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r \Leftrightarrow A = f(J),\ f(x) = \sum_{i=0}^{n-1} a_i x^i$.

Lemma 3.3: Set $A \in K^{m\times n}, A = SK_r(a_0, a_1, \cdots, a_{n-1}) \in SKM_r \Leftrightarrow A = \left(\sum_{i=1}^{n} a_{n-i} J^{i-1}\right)P$.

Lemma 3.4: Set $A = SK_r(a_0, a_1, \cdots, a_{n-1}) \in SKM_r, B = K_r(a_{n-1}, a_{n-2} \cdots, a_0) \in KM_r(r \neq 0)$, then A is non singular $\Leftrightarrow$ B is non singular.

Lemma 3.5: Set $A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r(r \neq 0)$ be non singular, then $A^{-1} \in KM_r$.

Lemma 3.6: Set $A = SK_r(a_0, a_1, \cdots, a_{n-1}) \in SKM_r(r \neq 0)$ be non singular, then $A^{-1} \in SKM_r$.

Lemma 3.7: Set $A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r(r \neq 0)$, then A is non singular $\Leftrightarrow$ $(f(x), g(x)) = 1$.

Lemma 3.8: Set $A = SK_r(a_0, a_1, \cdots, a_{n-1}) \in SKM_r(r \neq 0)$, then A is non singular $\Leftrightarrow$ $(f(x), g(x)) = 1$.

Theorem 3.1: Set $f(x), g(x)$ be nonzero polynomial of $K[x]$, If $C = (f(x), g(x))$, by a series of the elementary column transformation of C, we have $(d(x), 0)$. Then $(f(x), g(x)) = d(x),\ d(x) = u(x)f(x) + v(x)g(x)$.

Theorem 3.2: Set $f(x), g(x)$ be nonzero polynomial of $K[x]$.If $C = \begin{pmatrix} f(x) & g(x) \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$, by a series of the elementary column transformation of C, we have $\begin{pmatrix} d(x) & 0 \\ u(x) & s(x) \\ v(x) & t(x) \end{pmatrix}$. Then $(f(x), g(x)) = d(x),\ d(x) = u(x)f(x) + v(x)g(x)$.

Corollary 3.1: Set $A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r(r \neq 0)$ and non singular, then we have polynomial $u(x) = \sum_{i=0}^{n-1} b_i x^i$, and $A^{-1} = u(J)$.

Proof: Because $A = K_r(a_0, a_1, \cdots, a_{n-1}) \in KM_r(r \neq 0)$ and non singular, according to lemma 3.7 we have $(f(x), g(x)) = 1$.

By a series of the elementary column transformation for polynomial matrix $C=\begin{pmatrix} f(x) & g(x) \\ 1 & 0 \\ 0 & 1 \end{pmatrix}$, we get $\begin{pmatrix} 1 & 0 \\ u(x) & s(x) \\ v(x) & t(x) \end{pmatrix}$. And $(f(x), g(x))\begin{pmatrix} u(x) & s(x) \\ v(x) & t(x) \end{pmatrix} = (1,0)$,

$A^{-1}=u(J)=\sum_{i=0}^{n-1} b_i J^i$, $u(x)=\sum_{i=0}^{n-1} b_i x^i$. So we have $u(x)f(x)+v(x)g(x)=1$, if $x=J$, then $u(J)f(J)+v(J)g(J)=E$.

Corollary 3.2: Set $B=SK_r(a_0,a_1,\cdots,a_{n-1})\in SKM_r, A=K_r(a_{n-1},a_{n-2}\cdots,a_0)\in KM_r(r\neq 0)$, and $B^{-1}=P(u(J))$.

Proof: According to lemma 3.1 $B=AP$ and $P^{-1}=P$, so $B^{-1}=(AP)^{-1}=P^{-1}A^{-1}=PA^{-1}=P(u(J))$.

According to corollary 3.1 and corollary 3.2 we get elementary transformation algorithm. Follow the steps outlined below :

Step-1: According to $A=K_r(a_0,a_1,\cdots,a_{n-1})\in KM_r(r\neq 0)$, we get $f(x), g(x)$.

Step-2: By a series of the elementary column transformation for $C=\begin{pmatrix} f(x) & g(x) \\ 1 & 0 \end{pmatrix}$, we get $\begin{pmatrix} d(x) & u(x) \\ 0 & s(x) \end{pmatrix}$.

Step-3: If $d(x)=1$, then $A=K_r(a_0,a_1,\cdots,a_{n-1})\in KM_r(r\neq 0)$ and non singular. We get $A^{-1}=u(J)$.

Step-4: We get $B=SK_r(a_0,a_1,\cdots,a_{n-1})\in SKM_r$ and non singular and $A^{-1}=u(J)$, so we get $B^{-1}=P(u(J))$.

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Source of Support: Nil, Conflict of interest: None Declared