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ENTIRE TOTAL DOMINATING TRANSFORMATION GRAPHS

V. R. KULLI*

Department of Mathematics, Gulbarga University, Gulbarga 585 106, India.

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ABSTRACT

Let $G=(V, E)$ be a graph and let S be the set of all minimal total dominating sets of G. Let x, y, z be three variables each taking value + or –. The entire total transformation graph G^{xyz} is the graph having $V \cup S$ as the vertex set and for any two vertices u and v in $V \cup S$, u and v are adjacent in G^{xyz} if and only if one of the following conditions holds: (i) $u, v \in V$. $x = +$ if $u, v \in D$ where D is a minimal total dominating set of G. $x = –$ if $u, v \notin D$ where D is a minimal total dominating set of G (ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = –$ if $u \cap v = \phi$. (iii)$u \in v$ and $v \in S$. $z=+$ if $u \in v$. $z= –$ if $u \notin v$. In this paper, we initiate a study of entire total dominating transformation graphs in domination theory.

Keywords: *entire total dominating graph, semientire total dominating graph, transformation.*

Mathematics Subject Classification: *05C.*

1. INTRODUCTION

All graphs considered here are finite, undirected without loops and multiple edges. Any undefined term in this paper may be found in Kulli [1].

Let $G=(V, E)$ be a graph. A set $D \subseteq V$ is a dominating set of G if every vertex in $V – D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a dominating set of G. Recently several domination parameters are given in the books by Kulli in [2, 3, 4].

A set $D \subseteq V$ is a total dominating set of G if every vertex in V is adjacent to some vertex in D. The totral domination number $\gamma_t(G)$ of G is the minimum cardinality of a total dominating set of G. A total dominating set D of G is minimal if every $v \in D$, $D – \{v\}$ is not a total dominating set of G.

The total minimal dominating graph $M_t(G)$ is the graph with minimal total dominating sets as its vertices and two vertices in $M_t(G)$ adjacent if the corresponding minimal total dominating sets have a vertex in common. This concept was introduced by Kulli and Iyer in [5]. Many other graph valued functions in domination theory and in graph theory were studied, for example, in [6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21].

The common minimal total dominating graph $CD_t(G)$ of G is the graph having the same vertex set as G with two vertices adjacent in $CD_t(G)$ if there exists a minimal total dominating set in G containing them. This concept was introduced by Kulli in [22].

The total dominating graph $D_t(G)$ of G is the graph with vertex set $V \cup S$, where S is the set of all minimal total dominating sets of G and with two vertices u and v adjacent in $D_t(G)$ if $u \in V$ and v is a minimal total dominating set in G containing u. This concept was introduced by Kulli in [23].

The middle total dominating graph $MD_t(G)$ of a graph G is the graph with vertex set $V \cup S$ where S is the set of all minimal total dominating sets of G with two vertices u, v adjacent in $MD_t(G)$ if u, v are not disjoint minimal total dominating sets in G or $u \in V$ and v is a minimal total dominating set in G containing u. This concept was introduced by Kulli in [24].

Corresponding author: V. R. Kulli
Department of Mathematics, Gulbarga University, Gulbarga 585 106, India.

The semientire total dominating graph $Ed_t(G)$ of G is the graph with vertex set $V \cup S$ where S is the set of all minimal total dominating sets of G with two vertices u, v adjacent in $Ed_t(G)$ if $u, v \in D$ where D is a minimal total dominating set in G or $u \in V$ and v is a minimal total dominating set in G containing u. This concept was introduced by Kulli in [25].

In [26], Kulli introduced the concept of the entire total dominating graph as follows:

The entire total dominating graph $ED_t(G)$ of a graph G is the graph with the vertex set $V \cup S$ where S is the set of all minimal total dominating sets of G with two vertices u, v adjacent in $ED_t(G)$ if u, v are not disjoint minimal total dominating sets in G or $u, v \in D$ where D is a minimal total dominating set in G or $u \in V$ and v is a minimal total dominating set in G containing u.

We note that $M_t(G)$, $CD_t(G)$, $D_t(G)$, $MD_t(G)$, $Ed_t(G)$ and $ED_t(G)$ are defined only if G has no isolated vertices.

2. TRANFORMATION GRAPHS

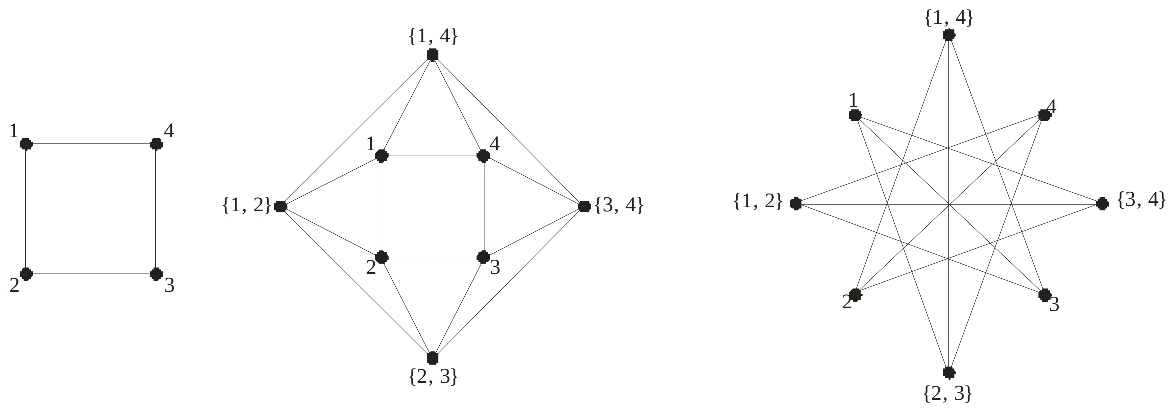
Inspired by the definition of the entire total dominating graph of a graph, we introduce the following transformation graphs in domination theory.

Definition: 1 Let $G = (V, E)$ be a graph and let S be the set of all minimal total dominating sets of G. Let x, y, z be three variables each taking value + and –. The entire total dominating transformation graph G^{xyz} is the graph having $V \cup S$ as the vertex set and for any two vertices u and v in $V \cup S$, u and v are adjacent in G^{xyz} if and only if one of the following conditions holds:

i) $u, v \in V$. $x = +$ if $u, v \in D$ where D is a minimal total dominating set of G. $x = -$ if $u, v \not\in D$ where D is minimal total dominating set of G.
ii) $u, v \in S$. $y = +$ if $u \cap v \neq \phi$. $y = -$ if $u \cap v = \phi$.
iii) $u \in V$ and $v \in S$. $z = +$ if $u \in v$. $z = -$ if $u \not\in v$.

We have eight distinct entire total dominating transformation graphs: G^{+++}, G^{+-+}, G^{++-}, G^{-++}, G^{+--}, G^{-+-}, G^{--+}, G^{---}.

Example: 2 In Figure 1, a graph G, its entire total dominating transformation graphs G^{+++} and G^{---} are shown.



G $\qquad$ G^{+++} $\qquad$ G^{---}

Figure-1

Among entire total dominating transformation graphs one is the entire total dominating graph G^{+++}. Therefore we have

Proposition: 3 For any graph G without isolated vertices, $ED_t(G) = G^{+++}$.

Remark: 4 For any graph G without isolated vertices, $D_t(G)$ is a subgraph of G^{+++}.

Remark: 5 For any graph G without isolated vertices, $M_t(G)$ and $CD_t(G)$ are vertex and also edge disjoint induced subgraphs of G^{+++}.

Remark: 6 For any graph G without isolated vertices, $MD_t(G)$ is a subgraph of G^{+++}.

Remark: 7 For any graph G without isolated vertices, $Ed_t(G)$ is a subgraph of G^{+++}.

Proposition: 8 If G is a graph without isolated vertices, then

i) $\overline{G^{+++}} = G^{---}$ ii) $\overline{G^{++-}} = G^{--+}$

iii) $\overline{G^{+-+}} = G^{-+-}$ iv) $\overline{G^{+--}} = G^{-++}$

Theorem: 9 $G^{+++} = K_{2p+1}$ if and only if $G = pK_2$, $p\geq 1$.

Proof: Suppose $G = pK_2$, $p\geq 1$. Then G has exactly one minimal total dominating set D which contains all $2p$ vertices of G. Hence the vertex set of G^{+++} is $V \cup \{D\}$ and has $2p+1$ vertices. Therefore $2p$ vertices together with the corresponding vertex of D form K_{2p+1}. Thus $G^{+++}=K_{2p+1}$.

Conversely suppose $G^{+++}=K_{2p+1}$. We now prove that $G = pK_2$, $p\geq 1$. On the contrary, assume $G \neq pK_2$. Then there exist two minimal total dominating sets D_1 and D_2 in G. We consider the following two cases.

Case-1: Suppose $D_1 \cap D_2 = \phi$. Then the corresponding vertices of D_1 and D_2 are not adjacent in G^{+++}, a contradiction.

Case-2: Suppose $D_1 \cap D_2 \neq \phi$. Then there exists a vertex u in D_1 which is not in D_2. Therefore the corresponding vertices of u and D_2 are not adjacent in G^{+++}, which is a contradiction.

From the above two cases, we conclude that G has exactly one minimal total dominating set. Thus $G=pK_2$.

We characterize graphs G whose entire total dominating transformation graphs G^{+++} are complete.

Theorem: 10 The entire total dominating transformation graph G^{+++} of G is complete if and only if $G=pK_2$, $p \geq 1$.

Proof: This follows from Theorem 9.

We now characterize graphs G for which $Ed_t(G) = G^{+++}$.

Theorem: 11 For any graph G without isolated vertices,

$$Ed_t(G) \subseteq G^{+++}.$$

Furthermore, $Ed_t(G) = G^{+++}$ if and only if one of the following conditions holds:

i) G has exactly one minimal total dominating set which contains all vertices of G.
ii) Every pair of minimal total dominating sets of G are disjoint.

Proof: By Remark 7, $Ed_t(G) \subseteq G^{+++}$.

Suppose $Ed_t(G) = G^{+++}$. We prove (i). On the contrary, assume a vertex u lies in two distinct minimal total dominating sets D_1 and D_2 in G. Then the corresponding vertices of D_1 and D_2 are adjacent in G^{+++} and are not adjacent in $Ed_t(G)$. Hence $G^{+++} \neq Ed_t(G)$, a contradiction. Thus every vertex of G lies in exactly one minimal total dominating set of G. This proves (i). Since $Ed_t(G) = G^{+++}$, it implies that no two minimal total dominating sets in G have a vertex in common. Hence every pair of minimal total dominating sets of G are disjoint. This proves (ii).

Conversely suppose G satisfies (i). Then clearly $Ed_t(G) = G^{+++}$. Now suppose G satisfies (ii). Then two vertices corresponding to minimal total dominating sets cannot be adjacent in G^{+++}. Thus $G^{+++} \subseteq Ed_t(G)$ and since $Ed_t(G) \subseteq G^{+++}$, we see that $Ed_t(G) = G^{+++}$.

Theorem: 12 For any graph G without isolated vertices,

$$D_t(G) \subseteq G^{+++}.$$

Proof: This follows from Remark 4.

Theorem: 13 For any graph G without isolated vertices, $D_t(G)$ and G^{+++} are not isomorphic.

Proof: Let G be a graph without isolated vertices. Since every minimal total dominating set of G contains at least two vertices, the corresponding vertices of these vertices are mutually adjacent in G^{+++} and are not mutually adjacent in $D_t(G)$. Thus $D_t(G) \neq G^{+++}$.

Theorem: 14 For any graph G without isolated vertices,

$$MD_t(G) \subseteq G^{+++}.$$

Proof: This follows from Remark 6.

Theorem: 15 For any G without isolated vertices, $MD_t(G)$ and G^{+++} are not isomorphic.

Proof: This follows from the proof of Theorem 13.

Exact value of $\gamma_t(G^{+++})$ for pK_2 is given below.

Proposition: 16 For any graph pK_2, $p \geq 1$,

$$\gamma_t((pK_2)^{+++}) = 2.$$

REFERENCES

1. V.R.Kulli, *College Graph Theory*, Vishwa International Publications, Gulbarga, India (2012).
2. V.R.Kulli, *Theory of Domination in Graphs*, Vishwa International Publications, Gulbarga, India (2010).
3. V.R.Kulli, *Advances in Domination Theory I*, Vishwa International Publications, Gulbarga, India (2012).
4. V.R.Kulli, *Advances in Domination Theory II*, Vishwa International Publications, Gulbarga, India (2013).
5. V.R.Kulli and R.R.Iyer, *The total minimal dominating graph*. In Advances in Domination Theory I, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, 121-126 (2012).
6. V.R.Kulli and B Janakiram, The minimal dominating graph, *Graph Theory Notes of New York, New York Academy of Sciences*, 28, 12-15 (1995).
7. V.R.Kulli and B Janakiram, The common minimal dominating graph, *Indian J.Pure Appl. Math*, 27(2), 193-196 (1996).
8. V.R.Kulli, B Janakiram and K.M. Niranjan, The vertex minimal dominating graph *Acta Ciencia Indica*, 28, 435-440 (2002).
9. V.R.Kulli, B Janakiram and K.M. Niranjan, The dominating graph, *Graph Theory Notes of New York, New York Academy of Sciences*, 46, 5-8 (2004).
10. V.R. Kulli, The middle edge dominating graph, *Journal of Computer and Mathematical Sciences*, 4(5), 372-375 (2013).
11. V.R. Kulli, The semientire edge dominating graph, *Ultra Scientist*, 25(3)A, 431-434 (2013).
12. B.Basavanagoud, V.R. Kulli and V.V.Teli, Equitable total minimal dominating graph, *International Research Journal of Pure Algebra*, 3(10), 307-310 (2013).
13. V.R. Kulli, On common edge graphs, *J. Karnatak University Sci.*, 18, 321-324 (1973).
14. V.R. Kulli, The block point tree of a graph, *Indian J. Pure Appl. Math.*, 7, 620-624 (1976).
15. V.R. Kulli, The semitotal block graph and the total block graph of a graph, *Indian J. Pure Appl. Math.*, 7, 625-630 (1976).
16. V.R. Kulli, On the plick graph and the qlick graph of a graph, *Research Journal*, 1, 48-52 (1988).
17. V.R. Kulli and M.S. Biradar, The point block graph of a graph, *Journal of Computer and Mathematical Sciences*, 5(5), 476-481 (2014).
18. V.R. Kulli and M.S. Biradar, The blict graph and blitact graph of a graph, *J. Discrete Mathematical Sciences and Cryptography*, 4(2-3), 151-162 (2001).
19. V.R. Kulli and M.S. Biradar, The line splitting graph of a graph, *Acta Ciencia Indica*, 28, 57-64 (2001).
20. V.R. Kulli and M.H. Muddebihal, Lict and litact graph of a graph, *J. Analysis and Computation*, 2, 33-43 (2006).
21. V.R. Kulli and N.S. Warad, On the total closed neighbourhood graph of a graph, *J. Discrete Mathematical Sciences and Cryptography*, 4, 109-114 (2001).
22. V.R. Kulli, The common minimal total dominating graph, *J. Discrete Mathematical Sciences and Cryptography*, 17, 49-54 (2014).
23. V.R.Kulli, *The total dominating graph of a graph*. In Advances in Domination Theory I, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, 148 (2012).
24. V.R.Kulli, *The middle total dominating graph*. In Advances in Domination Theory I, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, 149 (2012).
25. V.R.Kulli, *The semientire total dominating graph*. In Advances in Domination Theory II, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, 75-80 (2013).
26. V.R.Kulli, *The entire total dominating graph of a graph*. In Advances in Domination Theory I, V.R.Kulli., ed., Vishwa International Publications, Gulbarga, India, 151 (2012).

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