International Research Journal of Pure Algebra-5(5), 2015, 85-87
Available online through www.rjpa.info ISSN 2248–9037

ON SEMI-ESSENTIAL SUBSEMIMODULES

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(Received On: 20-05-15; Revised & Accepted On: 31-05-15)

ABSTRACT

The notion of essential subsemimodule was introduced by Pawar in [6]. In this paper we define semi-essential subsemimodule and extend some results of essential subsemimodule to semi-essential subsemimodule over a semiring.

AMS Mathematics Subject Classification* 2000*:* *16Y60.

Keywords: *Semiring, semimodule, essential subsemimodule, semi-essential subsemimodule.*

1. INTRODUCTION

The notion of semi-essential submodule was given by Ali S. Mijbass and Nada K. Abdullah in [4]. The notion of essential subsemimodule was introduced by Pawar in [6] and in [5] Pawar and Deore discussed some basic results for essential ideals. In this paper we define semi-essential subsemimodule and extend some results of essential subsemimodule and essential ideal of [5] and [6] to semi-essential subsemimodule over a semiring on the line of [4].

2. PRELIMINARIES

For preliminary definitions and properties of semirings, ideals, semimodules etc. the reader is referred to [2].

Definition: 2.1 A semiring is a set R together with two binary operations called addition $(+)$ and multiplication $(\cdot)$ such that $(R,+)$ is a commutativemonoid with identity element 0_R; $(R,\ \cdot)$ is a monoid with identity element 1, multiplication distributes over addition from either side and 0 is multiplicative absorbing, that is, $a \cdot 0 = 0 \cdot a = 0$ for each $a \in R$.A semiring R is said to have a unity if there exists $1_R \in R$ such that $1_R \cdot a = a \cdot 1_R = a$ for each $a \in R$.

For e.g.: The set $\mathbb{N}$of non-negative integers with the usual operations ofaddition and multiplication of integers is a semiring with$1_{\mathbb{N}}$.

Definition: 2.2 Let R be a semiring. A left R-semimodule is a commutative monoid $(M,+)$ with additive identity 0_M for which we have a function $R \times M \to M$ defined by $(r,m) \hookrightarrow r \cdot m$ and called scalar multiplication which satisfies the following conditions for all r and $r'$of R and all elements m and m' of M,

1. $(r \cdot r')m = r(r' \cdot m)$
2. $r \cdot (m + m') = r \cdot m + r \cdot m'$
3. $(r + r') \cdot m = r \cdot m + r' \cdot m$
4. $1_R \cdot m = m$ (If exists)
5. $r \cdot 0_M = 0_M = 0_R \cdot m.$

Convention: In this paper all semirings considered will be assumed to be commutative semirings with unity.

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3. ESSENTIAL SUBSEMIMODULES

The notion of essential subsemimodule was introduced by Pawar in [6].

Definition: 3.1 [6] A nonzero R-subsemimodule N of M is called essential subsemimodule of M if $N \cap K \neq 0$ for each nonzero R-subsemimodule K of M

Proposition: 3.2 [6] Let M be a left R-semimodule. Any subsemimodule of M which contains an essential subsemimodule of M is itself essential in M.

Proposition: 3.3 [6] Let M be a left R-semimodule. If K is an essential subsemimodules of L and L is an essential subsemimodule of M then K is essential in M.

4. SEMI-ESSENTIAL SUBSEMIMODULES

Definition: 4.1 [3] Let R be a semiring and M be an R-semimodule. A subsemimodule N of M is called prime if

i) N is proper subsemimodule of M and
ii) If for any $m \epsilon M, r \in R,\ mr \in N \ \Rightarrow m \in N$ or $r \in A_N(M) = \{a \in R \mid aM \subseteq N\}$.

Definition: 4.2 A nonzero R-subsemimodule N of M is called semi-essential if $N \cap P \neq 0$ for each nonzero prime R-subsemimodule P of M.

Note: Any essential R-subsemimodule is semi-essential subsemimodule.

Proposition: 4.3 If M is a semi-simple R-semimodule, then M is the only semi-essential R-subsemimodule of M.

Proposition: 4.4 A nonzero R-subsemimodule N of M is semi-essential if and only if for each nonzero prime R-subsemimodule P of M there exists $x \in P$ and there exists $r \in R$ such that $0 \neq rx \in N$.

Proposition: 4.5 Let M be an R-semimodule and let N_1, N_2 be R-subsemimodules of M such that N_1 is an R-subsemimodule of N_2. If N_1 is a semi-essential R-subsemimodule of M, then N_2 is a semi-essential R-subsemimodule of M.

Corollary: 4.6 Let N_1 and N_2 are R-subsemimodules of M. If $N_1 \cap N_2$ is a semi-essential R-subsemimodule of M, then N_1 and N_2 are semi-essential.

Proposition: 4.7 Let N_1 and N_2 are R-subsemimodules of M such that N_1 is essential and N_2 is semi-essential. Then $N_1 \cap N_2$ is a semi-essential R-subsemimodule of M.

Lemma: 4.8 Let N be an R-subsemimodule of M and let P be a prime subsemimodule of M. If $(N \cap P: x) = ann(M)$, for each $x \in M$and $x \notin N \cap P$, then $N \cap P$ is a prime R-subsemimodule of M.

Proof: Let $rm \in N \cap P$, where $r \in R$ and $m \in M$ and suppose that $m \notin N \cap P$.Now **s**ince $rm \in N \cap P$ then $r \in (N \cap P: m)$. This implies that $r \in ann(M)$, and hence $r \in (N: M) \cap (P: M)$.Therefore $r \in r\ (N \cap P: M))$. Thus $N \cap P$ is a prime R-subsemimodule of M.

Proposition: 4.9 Let N_1 and N_2 are semi-essential R-subsemimodules of M. If $(N_1 \cap P: x) = an\,n(M)$, for each prime R-subsemimodule P of M, for each $x \in M$ and $x \notin N_1 \cap P$, then $N_1 \cap N_2$ is semi-essential.

Proof: Let P be a nonzero prime R-subsemimodule of M. Now by Lemma 4.8, $N_1 \cap P$ is a prime R-subsemimodule of M. Therefore $(N_1 \cap N_2) \cap P = N_2 \cap (N_1 \cap P) \neq 0$. Thus $N_1 \cap N_2$ is semi-essential.

Definition: 4.10 Let M and N be R-semimodules. An R-homomorphism $f: M \to N$ is called semi-essential if $f(M)$ is a semi-essential R-subsemimodule of N.

Proposition: 4.11 N is a semi-essential R-subsemimodule of M if and only if the inclusion function $i: N \longrightarrow M$ is semi-essential R-momomorhism.

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Source of Support: Nil, Conflict of interest: None Declared