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THE MIDDLE BLICT GRAPH OF A GRAPH

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ABSTRACT

In this paper, we study the concept of the middle blict graph of a graph. We obtain some properties of this graph. We establish a characterization of graphs whose middle blict graphs are eulerian. Also we present characterizations of graphs whose middle blict graphs are planar, outerplanar and k-minimally nonouterplanar.

Keywords: *point block graph, middle blict graph, inner point number, planar, k-minimally nonouterplanar.*

Mathematics Subject Classification: *05C.*

1. INTRODUCTION

In this paper, we consider a graph as finite, undirected without loops and multiple lines. For any undefined term or notation, we refer Kulli [1].

If $B = \{u_1, u_2\ldots\ldots u_r, r\geq 2\}$ is a block of a graph G, then we say that point u_1 and block B are incident with each other, as are u_2 and B, and so on. If $B = \{e_1, e_2,\ldots..e_s, s\geq 1\}$ is a block of G, then we say that line e_1 and block B are incident with each other, as are e_2 and B and so on. If two distinct blocks B_1 and B_2 are incident with a common cut point, then they are adjacent blocks. This idea was introduced in [2]. The points, lines and blocks of a graph are called its members.

The middle graph $M(G)$ of a graph G is the graph whose point set is the union of the set of points and lines of G and in which two points are adjacent if they are adjacent lines of G or one is a point and other is a line incident with it. This concept was introduced in [3] and was studied by Kulli and Patil in [4, 5].

The point block graph $P_b(G)$ of a graph G is the graph whose point set is the union of the set of points and blocks of G and two points are adjacent if the corresponding blocks contain a common cutpoint of G or one corresponds to a block B of G and the other to a point v of G and v is in B. This concept was studied by Kulli and Biradar in [6, 7, 8]. Many other graph valued functions in graph theory were studied, for example, in [9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26].

The following will be useful in the proof of our results.

Theorem A: [8] If G is a connected graph with p points and q lines; and b_i is the number of blocks to which point v_i belongs in G, then the point block graph $P_b(G)$ has $\sum_{i=1}^{p} b_i + 1$ points and $\frac{1}{2}\sum_{i=1}^{p} b_i\left(b_i+1\right)$ lines.

Theorem B: [1, *p.* 50] If G is a (p, q) graph whose points have degree d_i, then the total graph $T(G)$ has $p+q$ points and $2q+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ lines.

Theorem C: [1, *p.* 197] A graph is planar if and only if it has no subgraph homeomorphic to K_5 or $K_{3,\,3}$.

Theorem D: A graph is eulerian if and only if every point is of even degree.

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2. MIDDLE BLICT GRAPHS

The definitions of $P_b(G)$ and M(G) inspired us to introduce the following graph valued function.

Definition 1: The middle blict graph $M_n(G)$ of a graph G is the graph whose set of points is the union of the set of points, lines and blocks of G and in which two points are adjacent if the corresponding lines and blocks of G are adjacent or one corresponds to a point and the other to a line incident with it or one corresponds to a block B of G and other to a point v of G and v is in B.

Example 2: In Figure 1, a graph G and its middle blict graph $M_n(G)$ are shown.

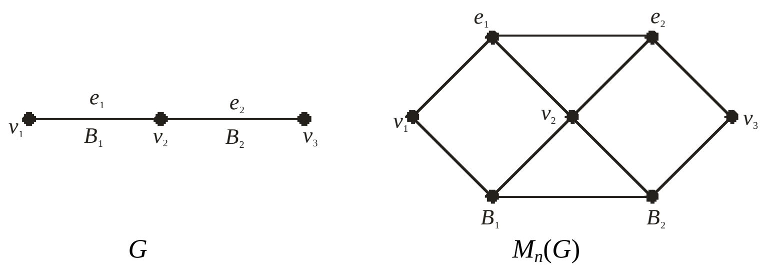


Figure-1

Remark 3: If G is a connected graph, then $M_n(G)$ is also connected and conversely.

Remark 4: The graph $L(G)$ and $B(G)$ are disjoint subgraphs of $M_n(G)$.

It is easy to see the following result.

Theorem 5: For any nontrivial connected graph, $M_n(G) = P_b(G) \cup M(G)$.

The following theorem determines the number of points and lines in the middle blict graph of a graph.

Theorem 6: If G is a (p q) graph whose points have degree d_t and b_i is the number of blocks to which point v_i belongs in G, then the middle blict graph $M_n(G)$ has $q+1+\sum_{i=1}^{p} b_i$ points and $q+\frac{1}{2}\sum_{i=1}^{p}\left[d_i^2+b_i\left(b_i+1\right)\right]$ lines.

Proof: The point set of $M_n(G)$ is $V \cup E \cup B$. But the point set of $P_b(G)$ is $V \cup B$. Therefore the number of points in $M_n(G)$ is the sum of the number of lines in G and the number of points in $P_b(G)$. By Theorem A, $P_b(G)$ has $\sum_{i=1}^{p} b_i+1$ points. Hence the number of points in $M_n(G)$ = $q+\sum_{i=1}^{p} b_i+1.$

By Theorem 5, the number of lines in $M_n(G)$ is the sum of the number of lines in $M(G)$ and in $P_b(G)$. The number of lines in $M(G)$ is the number lines in $T(G) - q$. By Theorem B, $T(G)$ has $2q+\frac{1}{2}\sum_{i=1}^{p} d_i^2$ lines and by Theorem A, $P_b(G)$ has $\frac{1}{2}\sum_{i=1}^{p} b_i\left(b_i+1\right)$ lines. Hence the number of lines in

$$M_n(G) = -q+2q+\frac{1}{2}\sum_{i=1}^{p} d_i^2+\frac{1}{2}\sum_{i=1}^{p} b_i\left(b_i+1\right)$$

$$= q+\frac{1}{2}\sum_{i=1}^{p}\left[d_i^2+b_i\left(b_i+1\right)\right].$$

3. TRAVERSABILITY OF MIDDLE BLICT GRAPHS

The following is the simple result, which we merely state.

Lemma 7: In $M_n(G)$, the point w corresponds to a block B of a graph G. Then $\deg_{Mn(G)} w = n_1+n_2$, where n_1 is the number of blocks adjacent with B and n_2 is the number of points incident to B.

Let c be a cutpoint. The $\deg_B c$ represents the number of blocks incident with c.

We now establish a characterization of graphs whose middle blict graphs are eulerian.

Theorem 8: The middle blict graph $M_n(G)$ of a connected graph G is eulerian if and only if G satisfies the following conditions:

1. degree of every point is odd,
2. for a cutpoint c of G, $\deg_B c$ is odd, and
3. if B is a block of G, then the number of blocks adjacent with B and the number of points incident to B are either all even or odd.

Proof: Suppose $M_n(G)$ is eulerian. Then the degree of every point of $M_n(G)$ is even. Let v be a point of G. Assume $\deg_G v$ is even. We consider the following cases.

Case-1: Suppose v is a noncutpoint of G. Then $\deg_{Mn(G)} v = \deg_G v+1$. Since $\deg_G v$ is even, $\deg_{Mn(G)} v$ is odd, which is a contradiction. Hence the degree of every noncutpoint of G is odd.

Case-2: Suppose v is a cutpoint of G. Let $e = uv$ be the line of G. If u is a noncutpoint of G, then by case 1, $\deg_G u$ is odd. Then by definition, $\deg_{Mn(G)} e = \deg_G u - 1 + \deg_G v - 1+2 = \deg_G u + \deg_G v$. Since $\deg_G v$ is even, $\deg_G u - 1$ is odd, which is a contradiction. Hence the degree of every cutpoint of G is odd.

From the two cases, we conclude that (1) holds.

Let c be a cutpoint of G. On the contrary, assume $\deg_G c$ is even, where $\deg_B c$ is the number of blocks incident with c. Then $\deg_{M_n(G)} c = \deg_G c + \deg_B c$. Since $\deg_{M_n(G)} c$ is even and $\deg_G c$ is odd, we have $\deg_B c$ is odd, which is a contradiction. Thus (2) holds.

Let B be a block of G. Then by Lemma 7, $\deg_{M_n(G)} B = n_1 + n_2$, where n_1 is the number of blocks adjacent with B and n_2 is the number of points incident to B in G. Since $\deg_{M_n(G)} B$ is even, we have n_1 and n_2 are both either even or odd. Thus (3) holds.

Conversely, suppose (1), (2) and (3) hold. We now prove that $M_n(G)$ is eulerian, so that we have to prove the degree of every point of $M_n(G)$ is even.

Let u be a point of $M_n(G)$. Then u represents either a point or a line or a block of G. We consider the following cases.

Case-1: Suppose u represents a point of G. We consider the following two subcases.

Subcase-1.1: Let u be a noncutpoint of G. Then u lies in a single block. Then by definition,

$$\begin{aligned}\deg_{M_n(G)} u &= \text{number of lines incident with } u \text{ in } G + 1\\ &= \deg_G u + 1.\end{aligned}$$

By condition (1), $\deg_G u$ is odd. Thus $\deg_{M_n(G)} u$ is even.

Subcase-1.2: Let u be a cutpoint of G. Then

$$\begin{aligned}\deg_{M_n(G)} u &= \text{number of lines incident with } u \text{ in } G + \text{number of blocks incident with } u \text{ in G}\\ &= \deg_G u + \deg_B u.\end{aligned}$$

By condition (1), $\deg_G u$ is odd and by condition (2), $\deg_B u$ is odd. Thus $\deg_{M_n(G)} u$ is even.

Case-2: Let $u = v_1v_2$ be a line of G. Then

$$\deg_{M_n(G)} u = \deg_G v_1 + \deg_G v_2.$$

By condition (1), $\deg_G v_1$ and $\deg_G v_2$ are odd. Thus $\deg_{M_n(G)} u$ is even.

Case-3: Let u be a block of G. Then by Lemma 7, $\deg_{M_n(G)} u = n_1 + n_2$, where n_1 is the number of blocks adjacent with u and n_2 is the number of points incident with u. By Condition (3), $\deg_{M_n(G)} u$ is even.

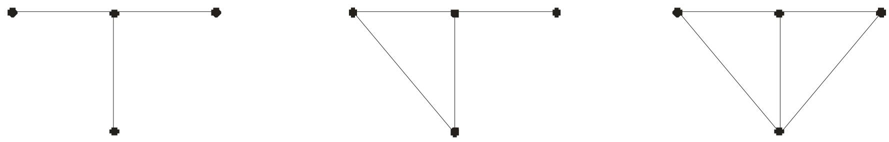
From the above 3 cases, we conclude that if u is a point of $M_n(G)$, then $\deg_{M_n(G)} u$ is even. Thus by Theorem D, $M_n(G)$ is eulerian.

4. PLANARITY OF MIDDLE BLICT GRAPHS

We now establish a characterization of graphs whose middle blict graphs are planar.

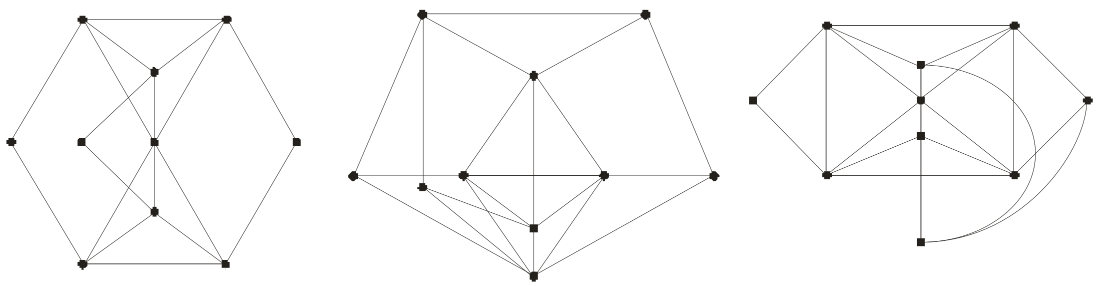
Theorem 9: The middle blict graph $M_n(G)$ of a connected graph G is planar if and only if $\Delta(G) \leq 2$.

Proof: Suppose $M_n(G)$ is planar. Assume $\Delta(G) = 3$. Then there exists a point v of degree 3. Clearly v lies on at most 3 blocks. Then G has a subgraph homeomorphic to G_1 or G_2 or G_3 with respect to the cutpoints as shown in Figure 2. Then $M_n(G_1)$ or $M_n(G_2)$ or $M_n(G_3)$ can be



G_1 G_2 G_3

Figure-2



$M_n(G_1)$ $M_n(G_2)$ $M_n(G_3)$

Figure-3

drawn in the plane as shown in Figure 3. Then $M_n(G_1)$ or $M_n(G_2)$ or $M_n(G_3)$ is a subgraph of $M_n(G)$, $M_n(G)$ has a subgraph homeomorphic to K_5. By Theorem C, $M_n(G)$ is nonplanar, a contradiction.

Conversely suppose $\Delta(G) \leq 2$. Then G is either a path or a cycle. If G is either P_n, $n\geq1$ or C_n, $n\geq3$, then clearly $M_n(G)$ is planar, see Figure 4.

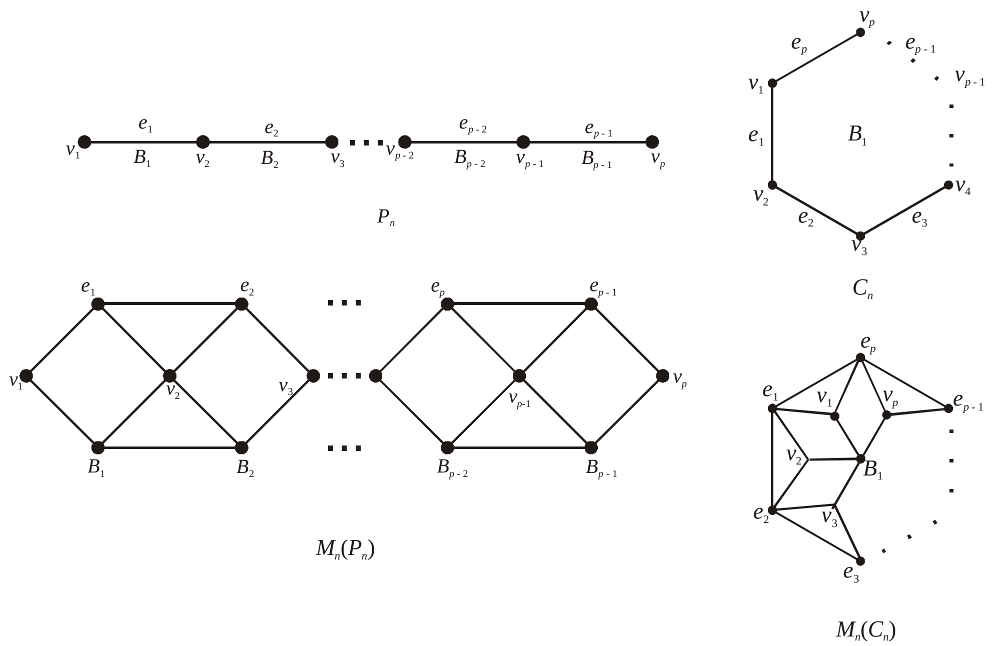


Figure-4

We need the following concept, for our next results. In 1975, Kulli [27] introduced the concept of k-minimally nonouterplanar graph. The inner point number $i(G)$ of a planar graph G is the minimum possible number of points not belonging to the boundary of the exterior region in any embedding of G is the plane. Obviously, G is outerplanar if and only if $i(G) = 0$. A graph G is called minimally nonouterplanar if $i(G) = 1$ and is called k-minimally nonouterplanar if $i(G)=k$, $k\geq 2$ and this concept was studied, for example in [28, 29, 30, 31, 32, 33, 34, 35, 36, 37].

The following results are easy to prove, we omit the proofs.

Theorem 10: The middle blict graph $M_n(G)$ of a connected graph G is outerplanar if and only if $G = P_2$.

Theorem 11: The middle blict graph $M_n(G)$ of a connected graph G is minimally nonouterplanar if and only if $G = P_3$.

Theorem 12: The middle blict graph $M_n(G)$ of a connected graph G is 2-minimally nonouterplanar if and only if $G=P_4$.

Theorem 13: The middle blict graph $M_n(G)$ of a connected graph G is 3-minimally nonouterplanar if and only if $G=P_5$ or C_3.

Theorem 14: The middle blict graph $M_n(G)$ of a connected graph G is 4-minimally nonouterplanar if and only If $G = P_6$.

In the next theorem, we present a characterization of graphs whose middle blict graphs are k-minimally nonouterplanar, $k\geq 5$.

Theorem 15: The middle blict graph $M_n(G)$ of a connected graph G is k-minimally nonouterplanar, $k\geq 5$ if and only if G is either P_{k+2} or C_{k-1}.

Proof: Suppose G is either P_{k+2} or C_{k-1}, $k\geq 5$. To prove the result we use mathematical induction on k.

Suppose $k = 5$. Then it is easy to see that $M_n(P_7)$ or $M_n(C_4)$ is 5-minimally nonouterplanar.

Assume the result is true for $k=m$. Therefore if G is either P_{m+2} or C_{m-1} then $M_n(G)$ is m-minimally nonouterplanar.

Suppose $k = m+1$. Then G is either P_{m+3} or C_m. We now prove that $M_n(G)$ is $(m+1)$-minimally nonouterplanar.

We consider the following cases.

Case-1: Let $G = P_{m+3}$ and v be an end point of G. Let $G_1 = G - v = P_{m+2}$. By inductive hypothesis, $M_n(G_1)$ is m-minimally nonouterplanar.

Let $e_i = (v_i, v_j)$ be an endline of G_1. Then b_i is an end block incident with the cutpoint v_i, since line and block coincide in a path. The points e_i, b_i and v_j in $M_n(G_1)$ are on the boundary of the exterior region on some cycle C, since $M_n(G_1)$ is m-minimally outerplanar. Now join the point v to the point v_j of G_1 such that the resulting graph is G. Let $e_j = (v_j, v)$ be an endline and $b_j = (v_j, v)$ is an end block of G. The formation of $M_n(G)$ is an extension of $M_n(G_1)$ with additional points e_j, b_j and v such that e_j is adjacent with e_i, v_j and v. Similarly b_j is adjacent with b_i, v_j and v. Clearly v_j is an inner point of $M_n(G)$, but it is not an inner point of $M_n(G)$. Thus $M_n(G)$ is $(m+1)$-minimally nonouterplanar.

Case-2: Let $G = C_m$ and v_m be a point of G. Let $G_1 = C_{m-1.}$ By inductive hypothesis, $M_n(G_1)$ is m-minimally nonouterplanar.

If $e_{m-1} = v_1v_{m-1}$ is a line of G_1 and v_m is not a point of G_1, then it is replaced by the lines $e_m = v_1v_m$ and $e_{m-1} = v_mv_{m-1}$. Then the resulting graph is G. The formation of $M_n(G)$ is an extension of $M_n(G_1)$ with additional points v_m and e_m, where e_m is adjacent to the points, e_{m-1}, v_m, v_1 and e_1; and also v_m is adjacent to the points e_{m-1} and B_1, where B_1 represents the point corresponding to the block of G. Then clearly v_m does not lie on the exterior region of $M_n(G)$. Hence $M_n(G)$ is $(m+1)$-minimally nonouterplanar.

Conversely suppose $M_n(G)$ is k-minimally nonouterplanar. Then by Theorem 9, $\Delta(G) \leq 2$, since $M_n(G)$ is planar. Thus G is either a path or a cycle. We consider the following cases.

Case-1: Suppose G is a path. We consider the following subcases.

Subcase-1.1: Assume $G=P_{k+1}$, $k\geq 5$. In particular, if $k = 5$, then $G = P_6$. By Theorem 14, $M_n(P_6)$ is 4-minimally nonouterplanar, a contradiction.

Subcase-1.2: Assume $G = P_{k+3}$. In particular, if $k = 5$, then $G = P_8$. Then from Figure 4(a) it is easy to observe that $M_n(P_8)$ is 6-minimally nonouterplanar, again a contradiction.

Case-2: Suppose G is a cycle. We consider the following subcases.

Subcase-2.1: Assume $G = C_{k-2}$, $k \geq 5$. In particular if $k=5$, then $G = C_3$. By Theorem 13, $M_n(C_3)$ is 3-minimally nonouterplanar, a contradiction.

Subcase-2.2: Assume $G = C_k$. In particular if $k=5$, then $G = C_5$. Then from Figure 4(b), it is easy to observe that $M_n(C_5)$ is 6-minimally nonouterplanar, again a contradiction.

Thus from the above two cases, we conclude that G is either P_{k+2} or C_{k-1}.

This completes the proof.

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