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ON T-FUZZY Bi-IDEALS IN NEAR- RINGS WITH RESPECT TO t-NORM

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ABSRACT

In this paper we introduce the concept of T-fuzzy bi-ideals using t-norm in zero-symmetric near-ring and investigate some of their properties.

Key words: *Near-ring, fuzzy subnear-ring, fuzzy bi-ideal, T-fuzzy bi-ideal with respect to t-norm.*

1. INTRODUCTION

The theory of fuzzy set was first inspired by Zadeh[6]. Triangular norms were introduced by Schweizer and Sklar [4, 5] to model the distances in probabilistic metric spaces. P.Dheena, G.Mohanraj [3] and M.Akram [2] have studied several properties of T-fuzzy ideals of rings and T-fuzzy ideals of near-rings. In [1] Abou-zaid introduced the notion of a fuzzy subnear-ring. In this paper we introduce the notion of fuzzy bi-ideals in near-rings with respect to t-norm T and investigate some of their properties. Also we prove that every T-fuzzy bi-ideals of a regular near-ring N is a T-fuzzy subnear-ring of N.

2. PRELIMINARIES

Definition 2.1: An algebra (N, +, .) is said to be a near-ring if it satisfies the following conditions:
(1) (N,+) is a group (not necessarily abelian),
(2) (N, .) is a semi group,
(3) For all $x, y, z \in N$, $x.(y+z) = x.y+x.z$.

Definition 2.2: A mapping $f:N\rightarrow N'$ is called a near-ring homomorphism if $f(x+y)=f(x)+f(y)$ and $f(xy)=f(x) f(y)$ for all $x, y\in N$.

Definition 2.3: [6]. A mapping $\mu: X\rightarrow[0,1]$,where X is an arbitrary nonempty set and is called a fuzzy set in X.

Definition 2.4: [1]. A fuzzy subset μ in a near-ring N is said to be a fuzzy subnear-ring of N if it satisfies the following conditions:
(1) $\mu(x-y) \geq \min\{\mu(x), \mu(y)\}$,
(2) $\mu(xy) \geq \min\{\mu(x), \mu(y)\}$ for all $x, y\in N$.

Lemma 2.5:.If μ is a fuzzy bi-ideal of N, then $\mu(0) \geq \mu(x)$ for all $x\in N$.

Definition 2.6.[4]: A t-norm is a function $T:[0,1]x[0,1]\rightarrow[0,1]$ that satisfies the following conditions for all $x, y, z\in[0,1]$,
(1) $T(x,1) =x$,
(2) $T(x, y) =T(y, x)$,
(3) $T(x, T(y, z)) = T(T(x, y), z)$,
(4) $T(x, y) \leq T(x, z)$ whenever $y \leq z$.

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A simple example of such defined t-norm is a function T(x, y) = min(x, y). In general case, $T(x, y) \leq \min(x, y)$ and T(x, 0) = 0 for all x, y∈[0,1].

Definition 2.7: A subgroup B of N is said to be bi-ideal if BNB ⊆ B.

Definition 2.8: Let μ, λ be the fuzzy subsets of a set X. A fuzzy subset (μ∩λ) (x) = min{μ(x), λ(x)}.

Definition 2.9:.Let μ, λ be the fuzzy subsets of a set X. A fuzzy subset (μ∧λ) (x) = T(μ(x), λ(x)).

Definition 2.10: A fuzzy subset μ of a near-ring N is called fuzzy bi-ideal if
(1) μ(x-y) ≥ min{μ(x),μ(y)}
(2) μ(xyz) ≥ min{μ(x),μ(z)} for all x, y, z∈N.

Definition 2.11: A fuzzy bi-ideal μ of a near-ring N is said to be normal if μ(0)=1.

Definition 2.12:.Let N and N′ be two near-rings and 'f 'a function of N into N′.
(1) If λ is a fuzzy set in N′ , then the preimage of λ under ' f ' is the fuzzy set in N defined by μ(x) = (λof) (x) = λ(f(x)) for each x∈N,
(2) If μ is a fuzzy set of N ,then the image of μ under f is the fuzzy set in N′ defined by

$$f(\mu)(y) = \begin{cases} \sup_{x \in f^{-1}(y)} \mu(x) & \text{if } f^{-1}(y) \neq \phi, \\ 0 & \text{otherwise for each } y \in N'. \end{cases}$$

3. SOME THEOREMS ON T-FUZZY BI-IDEALS IN NEAR-RINGS

Definition 3.1: A fuzzy subset μ of a near-ring N is called T- fuzzy bi-ideal if
(1) μ(x-y) ≥ T(μ(x),μ(y))
(2) μ(xyz) ≥ T(μ(x),μ(z)) for all x, y, z ∈N.

Note: If we take T-norm as min-norm T-fuzzy bi-ideal coincides with fuzzy bi-ideal.

Example 3.2: Let N= {0, a, b, c} be the klein's four group. Define multiplication in N as follows.

+	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

.	0	a	b	c
0	0	0	0	0
a	0	b	0	b
b	0	0	0	0
c	0	b	0	b

Then (N, +, .) is a near-ring ((see[6], p.408) scheme 15). Define a fuzzy set μ: N→[0,1] by μ(0)=μ(a)=0.3, μ(b) = μ(c) = 0.2. Let T be a t-norm defined by T(α,β) = max(α+β-1) for all α, β∈[0,1]. Then it can be easily verified that N is a T-fuzzy bi-ideal of N.

Theorem 3.3: Every fuzzy bi-ideal of a near-ring N is a T-fuzzy bi-ideal of N.

Proof: Let μ be fuzzy bi-ideal. Let x, y, z ∈ N.

Then μ(x-y) ≥ min{μ(x), μ(y)} ≥ T(μ(x), μ(y)) and μ(xyz) ≥ min{μ(x),μ(z)}≥T(μ(x), μ(z)). Thus μ is a T-fuzzy bi-ideal of a near-ring N.

Theorem 3.4: If μ and λ are T-fuzzy bi-ideal of a Near-ring N, then μ∧λ is a T-fuzzy bi-ideal of a Near-ring N.

Proof: Let μ and λ be a T-fuzzy bi-ideal of a Near-ring N.
For let x, y, z∈N,
(1) (μ∧λ)(x-y) = T(μ(x-y), λ(x-y))
≥ T[T(μ(x),μ(y)), T(λ(x),λ(y))]
= T(T(T(μ(x), μ(y)),λ(x)), λ(y))

$= T(T(\lambda(x), T(\mu(x), \mu(y)), \lambda(y))$
$= T(T(T(\lambda(x), \mu(x)), \mu(y)), \lambda(y))$
$= T(T(\mu(x), \lambda(x)), T(\mu(y), \lambda(y)))$
$= T((\mu\wedge\lambda)(x), (\mu\wedge\lambda)(y))$.

Therefore $(\mu\wedge\lambda)(x-y) \geq T((\mu\wedge\lambda)(x), (\mu\wedge\lambda)(y))$.

(2) $(\mu\wedge\lambda)(xyz) = T(\mu(xyz), \lambda(xyz))$
$\geq T(T(\mu(x), \mu(z)), T(\lambda(x), \lambda(z)))$
$= T\{T[T(\mu(x), \mu(z)), \lambda(x)], \lambda(z)\}$
$= T\{T[\lambda(x), T(\mu(x), \mu(z)], \lambda(z)\}$
$= T\{T[T(\lambda(x), \mu(x)), \mu(z)], \lambda(z)\}$
$= T\{T(\lambda(x), \mu(x)), T(\mu(z), \lambda(z))\}$
$= T((\mu\wedge\lambda)(x), (\mu\wedge\lambda)(z))$

Therefore $(\mu\wedge\lambda)(xyz) \geq T((\mu\wedge\lambda)(x), (\mu\wedge\lambda)(z))$.

Hence $\mu\wedge\lambda$ is a T-fuzzy bi-ideal of N.

Corollary 3.5:.If μ and λ are fuzzy bi-ideals of a near-ring N, then $\mu\cap\lambda$ is a fuzzy bi-ideal of N.

Proof: By taking min T-norm in Theorem 3.4 we get the result.

Theorem 3.6: Every T-fuzzy bi-ideal of a regular near-ring N is a T-fuzzy Subnear-ring of N.

Proof: Let μ be a T-fuzzy bi-ideal of a near-ring N. Let a, $b\in N$. Then $\mu(a-b) \geq T(\mu(a), \mu(b))$.It is enough to prove that $\mu(ab) \geq T(\mu(a), \mu(b))$. Since N is regular, there exists $x\in N$ such that a = axa.

Now, $\mu(ab) = \mu((axa)b) = \mu(a(xa)b) \geq T(\mu(a), \mu(b))$. Hence μ is a T-fuzzy subnear-ring of N.

Theorem 3.7: A fuzzy set μ in a near-ring N is a T-fuzzy bi-ideal of N iff the level set $U(\mu;\alpha)=\{x\in N/\mu(x) \geq \alpha\}$ is a bi-ideal of N when it is non-empty.

Proof: Let x, y $\in U(\mu ; \alpha)$. Then $\mu(x) \geq \alpha$ and $\mu(y) \geq \alpha$.Now, $\mu(x-y) \geq T(\mu(x), \mu(y)) \geq \alpha$ we get $x-y\in U(\mu; \alpha)$. Hence $U(\mu; \alpha)$ is a subgroup of N. Let x, z $\in U(\mu; \alpha)$ and $y\in N$. Then $\mu(x)\geq \alpha$ and $\mu(z)\geq \alpha$.Therefore $\mu(xyz)\geq T(\mu(x), \mu(z)) \geq \alpha$ we get $xyz\in U(\mu; \alpha)$. Hence $U(\mu; \alpha)$ is a bi-ideal of N.

Conversely: suppose that x, $y\in N$ and $\mu(x-y) < T(\mu(x), \mu(y))$. choose α such that $\mu(x-y) < \alpha < T(\mu(x), \mu(y))$ we get x, $y\in U(\mu; \alpha)$. But $x-y\notin U(\mu; \alpha)$,a contradiction. Therefore $\mu(x-y)\geq T(\mu(x), \mu(y))$.similarly we can prove that $\mu(xyz) \geq T(\mu(x), \mu(z))$.Hence μ is a T-fuzzy bi-ideal of N.

Theorem 3.8:.Let f: $N\to N'$ be an onto homomorphism of near-rings. If μ is a T-fuzzy bi-ideal of N, then $f(\mu)$ is a T-fuzzy bi-ideal in N′.

Proof: Let μ be a T-fuzzy bi-ideal of N. Then $\{x/x\in f^{-1}(y_1 - y_2)\}\supseteq\{x_1 - x_2 / x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}$.

(i) $f(\mu)(y_1 - y_2) = \sup\{\mu(x)/x\in f^{-1}(y_1 - y_2)\}$
$\geq \sup\{\mu(x_1 - x_2)/ x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}$
$\geq \sup\{T(\mu(x_1), \mu(x_2)) / x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2)\}$
$= T(\sup\{\mu(x_1) / x_1\in f^{-1}(y_1)\}, \sup\{\mu(x_2) / x_2\in f^{-1}(y_2)\})$
$= T(f(\mu)(y_1), f(\mu)(y_2))$

Therefore $f(\mu)(y_1 - y_2) \geq T(f(\mu)(y_1), f(\mu)(y_2))$.

(ii) $f(\mu)(y_1y_2y_3) = \sup\{\mu(x)/x\in f^{-1}(y_1y_2y_3)\}$
$\geq \sup\{\mu(x_1x_2x_3)/x_1\in f^{-1}(y_1), x_2\in f^{-1}(y_2), x_3\in f^{-1}(y_3)\}$
$\geq \sup\{T(\mu(x_1), \mu(x_3)) /x_1\in f^{-1}(y_1), x_3\in f^{-1}(y_3)\}$
$= T(\sup\{\mu(x_1)/ x_1\in f^{-1}(y_1)\}, \sup\{\mu(x_3)/ x_3\in f^{-1}(y_3)\})$
$= T(f(\mu)(y_1), f(\mu)(y_3))$.

Therefore $f(\mu)(y_1y_2y_3)\geq T(f(\mu)(y_1), f(\mu)(y_3))$.

Hence $f(\mu)$ is a T-fuzzy bi-ideal of N′.

Theorem 3.9: Let μ be a T-fuzzy bi-ideal of a near-ring N and let μ^* be a fuzzy set in N defined by $\mu^*(x) = \mu(x)+1-\mu(0)$ for all $x\in N$. Then μ^* is a normal T-fuzzy bi-ideal of N containing μ.

Proof: Let μ be a T-fuzzy bi-ideal of a near-ring N.

For any $x, y\in N$,

$$\begin{aligned}\mu^*(x-y) &= \mu(x-y)+1-\mu(0)\\ &\geq T(\mu(x), \mu(y))+1-\mu(0)\\ &= T(\mu(x)+1-\mu(0), \mu(y)+1-\mu(0))\\ &= T(\mu^*(x), \mu^*(y))\end{aligned}$$

Therefore $\mu^*(x-y) \geq T(\mu^*(x), \mu^*(y))$.

For any $x, y, z \in N$,

$$\begin{aligned}\mu^*(xyz) &= \mu(xyz)+1-\mu(0)\\ &\geq T(\mu(x), \mu(z))+1-\mu(0)\\ &= T(\mu(x)+1-\mu(0), \mu(z)+1-\mu(0))\\ &= T(\mu^*(x), \mu^*(z)).\end{aligned}$$

Therefore $\mu^*(xyz) \geq T(\mu^*(x), \mu^*(z))$.

Hence μ^* is a T-fuzzy bi-ideal of a near-ring N. Clearly $\mu^*(0) = 1$ and $\mu\subset\mu^*$. This ends the proof.

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