F- Bi NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT

In this paper we introduce the notion of F- bi-near subtraction semigroup. Also we give characterizations of F- bi-near subtraction semigroup.

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Key words: F- bi-near subtraction semigroup, S_1- bi-near subtraction semigroup, S_2- bi-near subtraction semigroup, Nil near subtraction semigroup, idempotent, Nolpotent, Zero devisors.

1. INTRODUCTION

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[4]. Zekiye Ciloglu, Yilmaz Ceven [5] gave the notation of Fuzzy Near Subtraction semigroups. Seydali Fathima *et.al* [2, 3] introduced the notation of S_1-near subtraction semigroup and S_2-near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of F- Bi near subtraction semigroup.

2. PRELIMINARIES

Definition 2.1: A non-empty subset X together with two binary operations "−"and "." is said to be subtraction semigroup If (i) (X,−) is a subtraction algebra (ii) (X, .) is a semi group (iii) x(y−z)=xy−xz and (x−y)z= xz−yz for every x, y, z∈X.

Definition 2.2: A non-empty subset X together with two binary operations "−"and "." is said to be near subtraction semigroup if (i) (X,−) is a subtraction algebra (ii) (X,.) is a semi group and (iii) (x−y)z= xz−yz for every x, y, z∈X.

Definition 2.3: A non-empty subset $X=X_1 \cup X_2$ together with two binary operations "-" and "." Is said to be **bi-near subtraction semigroup**(right). If (i) $(X_1,-, .)$ is a near-subtraction semigroup (ii) $(X_2,-,.)$ is a subtraction semigroup

Definition 2.4: A non-empty subset X is said to be **S_1-near subtraction semigroup** if for every a∈X there exists x∈X* such that axa=xa.

Definition 2.5: A non-empty subset X is said to be **S_2-near subtraction semigroup** if for every a∈X there exists x∈X* such that axa=ax.

Definition 2.6: A **sub commutative near subtraction semigroup** is an intersection of S_1-near subtraction semigroup and S_2-near subtraction semigroup. that is, xa=ax.

Definition 2.7: A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer k>1 such that $a^k = 0$ Which implies that xa=0 where $x = a^{k-1}$.

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Definition 2.8: A non-empty subset X is said to be **zero-symmetric**. if $0-x=0$, $ox=0$ and $xo=o$ for all $x \in X$.

Definition 2.9: A non-empty subset Y of X is closed under "-" and XY strictly contained in Y is called an **X- system**.

3. F-Bi NEAR SUBTRACTION SEMIGROUP

Definition 3.1: A non-empty subset $X=X_1 \cup X_2$ together with two binary operations"-"and "." Is said to be F- **bi near subtraction semigroups**. If (i) for every $a \in X_1$ there exists $x \in X_1^*$ such that $axa=xa$. (ii) for every $a \in X_2$ there exists $x \in X_2^*$ such that $axa=ax$.

Example 3.2: Let $X_1 = \{0, a, b, 1\}$ in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	0
b	0	0	b	b
1	0	a	b	1

Then X_1 is a s_1-near-subtraction semi group

Let $X_1 = \{0, a, b, 1\}$ in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Then X_2 is an S_2-near subtraction semi group.

Hence, $X=X_1 \cup X_2$ is a F-bi near Subtraction Semigroup.

Note 3.3: Obviously, every Bi-near subtraction is a F- bi-near subtraction semi group. But the converse is not true

Example 3.4: Let $X_1 = \{0, a, b, 1\}\}$ in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Thus X_1 is a s_1-near subtraction semi group but not near subtraction semigroup.

Let $X_2 = \{0, a, b, c\}$ in which "-" and "." be defined by

-	0	a	b	C
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	c	c	c	0

.	0	a	b	C
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
c	0	0	0	c

Thus X_2 is an S_2-near subtraction semigroup but not subtraction semigroup

Hence, every F- bi-near subtraction semi group need not be a bi-near subtraction semi group.

4. RESULTS ON *F*-BI NEAR SUBTRACTION SEMIGROUP

Theorem 4.1: The intersection of S_1-near subtraction semigroup and S_2-near subtraction semigroup is sub commutative near subtraction semigroup.

Proof: Let X_1 is an S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that $axa=xa$. (1)
Let X_2 is an S_2-near subtraction semigroup, there exists $x \in X_2^*$ such that $axa=ax$ (2)

From (1) and (2), we get $xa=ax$
Thus, X is a sub commutative near subtraction semigroup.

Theorem 4.2: Let $X = X_1 \cup X_2$ be a nil bi- near subtraction semigroup and Let X_1 be a zero symmetric then X is a F-bi near Subtraction Semigroup.

Proof: Since $X = X_1 \cup X_2$ is a nil bi- near subtraction Semigroup. where X_1 and X_2 are nil near subtraction semigroup.

Let $a \in X_1$ Since X_1 is nil, there exists a positive integer $k > 1$ such that $a^k = 0$.

Which implies that $xa = 0$ where $x = a^{k-1}$

therefore $axa - a(xa) = a(0) = 0$ (since X_1 is a zero-symmetric) $= xa$.

Thus X_1 is an S_1-near subtraction semigroup.

Let X_2 be a nil near subtraction semigroup and let $a \in X_2^*$.

Then exists a positive integer $k > 1$ such that $a^k = 0$. We set $x = a^{k-1} \neq 0$

therefore $ax = 0$.

Now, $axa = (ax)a = 0a = 0 = ax$.

that is, $axa = ax$.

Obviously, $0x0 = 0x$ for any $x \in X_2^*$.

Thus X_2 is an S_2-near subtraction semigroup.

Hence X is a F- bi near subtraction semigroup.

Theorem 4.3: Let X be a F- bi near subtraction semigroup and X_1 be a zero symmetric. If X has no non. zero zero devisors then the following are true.
   (i)   Every ideal of X is a F-bi near Subtraction Semigroup.
   (ii) Every X-system of X is a F-bi near Subtraction Semigroup.

Proof: Since $X = X_1 \cup X_2$ is a nil bi- near subtraction Semigroup. where X_1 is an S_1-near subtraction semigroup and X_2 is an S_2-near subtraction semigroup.

Let I_1 be an ideal of X_1 and let $a \in I_1$.

If $a = 0$ then $ana = na$ for any $n \in I_1^*$.

Suppose $a \neq 0$.

Since X_1 is an S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that $axa = xa$.

If $i = ax \in I_1 X_1$, since I_1 is an ideal of X_1 we get $i \in I_1$.

It follows from the hypothesis that $i \neq 0$

Now, $aia - a(ax)a = a(axa) = a(xa) = (ax)a = ia$.

Thus I_1 is an S_1-near subtraction semigroup.

Let I_2 be an ideal of X_2 and let i be a non-zero element of I_2. Since X_2 is an S_2-near subtraction semigroup, there exists $y \in X_2^*$ such that $iyi = iy$ ....(1)

If we take $n = iy$ clearly, $n \in I_2$. Our hypothesis demands that $n \neq 0$.

Now, $ini = i(iy)i = i(iyi) = i(iy)$ (by (1)) $= in$.

that is, $ini = in$.

Consequently I_2 is an S_2-near subtraction semigroup.

Therefore I= $I_1 \cup I_2$ where I_1 is an S_1-near subtraction semigroup and I_2 is an S_2-near subtraction semigroup.

Hence I is a F-bi near Subtraction Semigroup.

Let A_1 be a X-system of X_1 and $A^* = A - \{0\}$.

Let $a \in A^*$

Since X_1 is an S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa.

We take $n = xa \in X_1 A_1$.

Since A_1 is an X-system of X_1, $n \in A_1$

Since X_1 has no non-zero devisors, $n \neq 0$.

Now, ana=a(xa)a=(axa)a+(xa)a=na. If a=o then, since X_1 is zero-symmetric, ana=na for any $n \in A_1^*$.

Thus A_1 an S_1-near subtraction semigroup.

Let A_2 be a X-system of X_2 and let a be a non-zero element of X_2.

Since X_2 is an S_2-near subtraction semigroup, there exists $y \in X_2^*$ such that aya=ay   (2)

If we put c=ya then $c \in X_2 A_2$.

Since A_2 is an X-system, we get $c \in A_2$.

Since X_2 has no non-zero devisors, $c \neq 0$.

Now, aca=a(ya)a= (aya)a=(ay)a (by (2))=a(ya)=ac.

that is, aca = ac.

Consequently A_2 is an S_2-near subtraction semigroup.

Therefore A= $A_1 \cup A_2$ where A_1 is an S_1-near subtraction semigroup and A_2 is an S_2-near subtraction semigroup.

Hence A is a F-bi near Subtraction Semigroup.

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