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F- Bi NEAR SUBTRACTION SEMIGROUPS

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ABSTRACT

In this paper we introduce the notion of F- bi-near subtraction semigroup. Also we give characterizations of F- bi-near subtraction semigroup.

Mathematical subject classification: *06F35.*

***Key words**: F- bi-near subtraction semigroup, S_1- bi-near subtraction semigroup, S_2- bi-near subtraction semigroup, Nil near subtraction semigroup, idempotent, Nolpotent, Zero devisors.*

1. INTRODUCTION

In 2007, Dheena[1] introduced Near Subtraction Algebra, Throughout his paper by a Near Subtraction Algebra, we mean a Right Near Subtraction Algebra. For basic definition one may refer to Pillz[4]. Zekiye Ciloglu, Yilmaz Ceven [5] gave the notation of Fuzzy Near Subtraction semigroups. Seydali Fathima *et.al* [2, 3] introduced the notation of S_1-near subtraction semigroup and S_2-near subtraction semigroup. In this paper we shall obtained equivalent conditions for regularity in terms of F- Bi near subtraction semigroup.

2. PRELIMINARIES

Definition 2.1: A non-empty subset X together with two binary operations "–"and "." is said to be subtraction semigroup If (i) (X,–) is a subtraction algebra (ii) (X, .) is a semi group (iii) $x(y-z)=xy-xz$ and $(x-y)z= xz-yz$ for every x, y, $z\in X$.

Definition 2.2: A non-empty subset X together with two binary operations "–"and "." is said to be near subtraction semigroup if (i) (X,–) is a subtraction algebra (ii) (X,.) is a semi group and (iii) $(x-y)z= xz-yz$ for every x, y, $z\in X$.

Definition 2.3: A non-empty subset $X=X_1\cup X_2$ together with two binary operations "-" and "." Is said to be **bi-near subtraction semigroup**(right). If (i) $(X_1,-, .)$ is a near-subtraction semigroup (ii) $(X_2,-,.)$ is a subtraction semigroup

Definition 2.4: A non-empty subset X is said to be **S_1-near subtraction semigroup** if for every $a\in X$ there exists $x\in X^*$ such that axa=xa.

Definition 2.5: A non-empty subset X is said to be **S_2-near subtraction semigroup** if for every $a\in X$ there exists $x\in X^*$ such that axa=ax.

Definition 2.6: A **sub commutative near subtraction semigroup** is an intersection of S_1-near subtraction semigroup and S_2-near subtraction semigroup. that is, xa=ax.

Definition 2.7: A non-empty subset X is said to be **nil-near subtraction semigroup** if there exists a positive integer k>1 such that $a^k = 0$ Which implies that xa=0 where $x = a^{k-1}$.

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Definition 2.8: A non-empty subset X is said to be **zero-symmetric**. if 0-x=0 , ox=0 and xo=o for all $x \in X$.

Definition 2.9: A non-empty subset Y of X is closed under "-" and XY strictly contained in Y is called an **X- system**.

3. F-Bi NEAR SUBTRACTION SEMIGROUP

Definition 3.1: A non-empty subset $X=X_1 \cup X_2$ together with two binary operations"-"and "." Is said to be F- **bi near subtraction semigroups**. If (i) for every $a \in X_1$ there exists $x \in X_1^*$ such that axa=xa. (ii) for every $a \in X_2$ there exists $x \in X_2^*$ such that axa=ax.

Example 3.2: Let X_1= {0, a, b, 1} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	1	b	a	0

.	0	a	b	1
0	0	0	0	0
a	0	a	0	0
b	0	0	b	b
1	0	a	b	1

Then X_1 is a s_1-near-subtraction semi group

Let X_1= {0, a, b, 1} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Then X_2 is an S_2-near subtraction semi group.

Hence, $X=X_1 \cup X_2$ is a F-bi near Subtraction Semigroup.

Note 3.3: Obviously, every Bi-near subtraction is a F- bi-near subtraction semi group. But the converse is not true

Example 3.4: Let X_1 = {0, a, b, 1}} in which "-" and "." be defined by

-	0	a	b	1
0	0	0	0	0
a	a	0	1	b
b	b	0	0	b
1	1	0	1	0

.	0	a	b	1
0	0	0	0	0
a	a	a	a	a
b	a	0	1	b
1	0	a	b	1

Thus X_1 is a s_1-near subtraction semi group but not near subtraction semigroup.

Let X_2= {0, a, b, c} in which "-" and "." be defined by

-	0	a	b	C
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
1	c	c	c	0

.	0	a	b	C
0	0	0	0	0
a	0	0	0	a
b	0	0	0	b
c	0	0	0	c

Thus X_2 is an S_2-near subtraction semigroup but not subtraction semigroup

Hence, every F- bi-near subtraction semi group need not be a bi-near subtraction semi group.

4. RESULTS ON *F*-BI NEAR SUBTRACTION SEMIGROUP

Theorem 4.1: The intersection of S_1-near subtraction semigroup and S_2-near subtraction semigroup is sub commutative near subtraction semigroup.

Proof: Let X_1 is an S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa. (1)
Let X_2 is an S_2-near subtraction semigroup, there exists $x \in X_2^*$ such that axa=ax (2)

From (1) and (2), we get xa=ax
Thus, X is a sub commutative near subtraction semigroup.

Theorem 4.2: Let $X=X_1 \cup X_2$ be a nil bi- near subtraction semigroup and Let X_1 be a zero symmetric then X is a F-bi near Subtraction Semigroup.

Proof: Since $X=X_1 \cup X_2$ is a nil bi- near subtraction Semigroup. where X_1 and X_2 are nil near subtraction semigroup.

Let a€ X_1 Since X_1 is nil, there exists a positive integer k >1 such that $a^k=0$.

Which implies that xa=0 where $x=a^{k-1}$

therefore axa-a(xa)=a(0)=0(since X_1 is a zero-symmetric)=xa.

Thus X_1 is an S_1-near subtraction semigroup.

Let X_2 be a nil near subtraction semigroup and let $a \in X_2^*$.

Then exists a positive integer k>1 such that $a^k = 0$. We set $x=a^{k-1} \neq 0$

therefore ax=0.

Now, axa=(ax)a=0a=0=ax.

that is, axa=ax.

Obviously, 0x0=0x for any $x \in X_2^*$.

Thus X_2 is an S_2-near subtraction semigroup.

Hence X is a F- bi near subtraction semigroup.

Theorem 4.3: Let X be a F- bi near subtraction semigroup and X_1 be a zero symmetric. If X has no non. zero zero devisors then the following are true.

(i) Every ideal of X is a F-bi near Subtraction Semigroup.
(ii) Every X-system of X is a F-bi near Subtraction Semigroup.

Proof: Since $X=X_1 \cup X_2$ is a nil bi- near subtraction Semigroup. where X_1 is an S_1-near subtraction semigroup and X_2 is an S_2-near subtraction semigroup.

Let I_1 be an ideal of X_1 and let $a \in I_1$.

If a=0 then ana=na for any $n \in I_1^*$.

Suppose $a \neq 0$.

Since X_1 is an S_1-near subtraction semigroup. there exists $x \in X_1^*$ such that axa=xa.

If $i=ax \in I_1X_1$, since I_1 is an ideal of X_1 we get $i \in I_1$.

It follows from the hypothesis that $i \neq 0$

Now, aia-a(ax)a=a(axa)= a(xa)= (ax)a=ia.

Thus I_1 is an S_1-near subtraction semigroup.

Let I_2 be an ideal of X_2 and let i be a non-zero element of I_2. Since X_2 is an S_2-near subtraction semigroup, there exists $y \in X_2^*$ such that iyi=iy (1)

If we take n=iy clearly, $n \in I_2$. Our hypothesis demands that $n \neq 0$.

Now, ini=i(iy)i=i(iyi)=i(iy)(by (1))=in.

that is, ini=in.

Consequently I_2 is an S_2-near subtraction semigroup.

Therefore I= $I_1 \cup I_2$ where I_1 is an S_1-near subtraction semigroup and I_2 is an S_2-near subtraction semigroup.

Hence I is a F-bi near Subtraction Semigroup.

Let A_1 be a X-system of X_1 and A^* =A-{0}.

Let $a \in A^*$

Since X_1 is an S_1-near subtraction semigroup. there exists x€ X_1^* such that axa=xa.

We take $n=xa \in X_1A_{1.}$

Since A_1 is an X-system of X_1, $n \in A_1$

Since X_1 has no non-zero devisors, $n \neq 0$.

Now, ana=a(xa)a=(axa)a+(xa)a=na. If a=o then, since X_1 is zero-symmetric, ana=na for any $n \in A_1^{*.}$

Thus A_1 an S_1-near subtraction semigroup.

Let A_2 be a X-system of X_2 and let a be a non-zero element of X_2.

Since X_2 is an S_2-near subtraction semigroup, there exists $y \in X_2^*$ such that aya=ay (2)

If we put c=ya then $c \in X_2A_2$.

Since A_2 is an X-system, we get $c \in A_2$.

Since X_2 has no non-zero devisors, $c \neq 0$.

Now, aca=a(ya)a= (aya)a=(ay)a (by (2))=a(ya)=ac.

that is, aca = ac.

Consequently A_2 is an S_2-near subtraction semigroup.

Therefore A= $A_1 \cup A_2$ where A_1 is an S_1-near subtraction semigroup and A_2 is an S_2-near subtraction semigroup.

Hence A is a F-bi near Subtraction Semigroup.

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REFERENCES

1. Dheena.P and Sathesh kumar.G *On Strongly Regular Near-Subtracttion semi groups*, Commun. Korean Math. Soc.22 (2007), No.3, pp.323-330.
2. Seyadali Fathima.S and. Balakrishnan.R,, S_1-near subtraction semigroups, Ultra Scientist Vol.24(2012), (3)A, 578-584.
3. Seyadali Fathima.S and. Balakrishnan.R,, S_2-near subtraction semigroups, Research Journal of Pure Algebra-2 (2012), (12) A,382-386.
4. Pilz Gunter, *Near-Rings*, North Holland,Amsterdam,1983
5. Zekiye Ciloglu, Yilmaz Ceven, *On Fuzzy Near Subtraction semigroups*, SDU Journal of Science (E-Journal), 2014, 9(1):193-202.

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