International Research Journal of Pure Algebra-5(9), 2015, 138-140
Available online through www.rjpa.info ISSN 2248–9037

ON IDENTIFICATION OF THE NATURE OF TRIANGLE BY NEW APPROACH

GEDEFA NEGASSA FEYISSA, M. P. CHAUDHARY*

Department of Mathematics, School of Mathematical Sciences,
Madawalabu University, Bale Robe, Ethiopia.

(Received On: 01-04-15; Revised & Accepted On: 19-09-15)

ABSTRACT

In this paper, we introduced new technique to explain nature of the triangle. We established necessary procedure to identify right, acute and obtuse angled triangle.

Key words: *Pythagoras Theorem, Acute angled triangle, Obtuse angled triangle.*

AMS Subject Classifications: *51K, 52B.*

INTRODUCTION

Triangle, in Mathematics, plane figure bounded by three straight lines, the sides, which intersect at three points called the vertices. Any one of the sides may be considered the base of the triangle. The perpendicular distance from a base to the opposite vertex is called altitude. The line segment joining the midpoint of a side to the opposite vertex is called Median.

The theorem about right triangles is attributed to and now bears the name a Greek, Pythagoras of Samos, born around 570-540 B.C. Pythagoras is often credited with the first proof of the theorem, however his actual written proof has not been found. Earlier civilizations definitely knew about this geometric fact and perhaps after his travel, Pythagoras took this information back to Greek. It had been discovered by Boudhaayan, an Indian mathematician, three centuries before Pythagoras [9]

The Babylonians discovered the triples (sets of three numbers that satisfy the Pythagoras Theorem) much earlier, approximately 1900-1600 BC, long before Pythagoras time. The Chinese may have proven the Pythagoras Theorem earliest; some estimates as early as 1100 B.C., although 6th Century B.C. is more generally accepted [5]. The Chinese grasped many right- angled triangle principles early on, and applied them to practical problems.

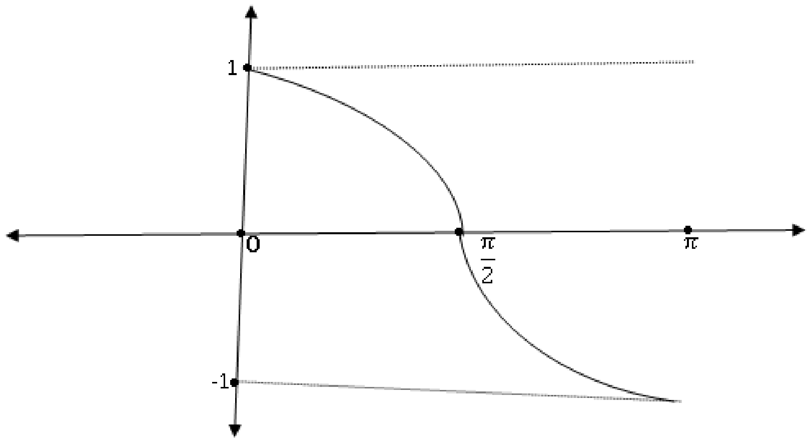
PRELIMINARY

In Euclidean geometry, Cosine function is strictly decreasing on [0, π].The sum of the degree measures of the interior angles of a triangle is 180. The degree measure of an interior angle of a triangle is between 0 and 180. That means if α is the degree measure of an interior angle of a triangle then $0< \alpha < 180$.

Cosine function and its graph

Sketch for the Graph of $f(x) = \text{Cos}x$, for $0 \le x \le \pi$, is given below;

Corresponding Author: M. P. Chaudhary
Department of Mathematics, School of Mathematical Sciences,
Madawalabu University, Bale Robe, Ethiopia. E-mail: mpchaudhary_2000@yahoo.com



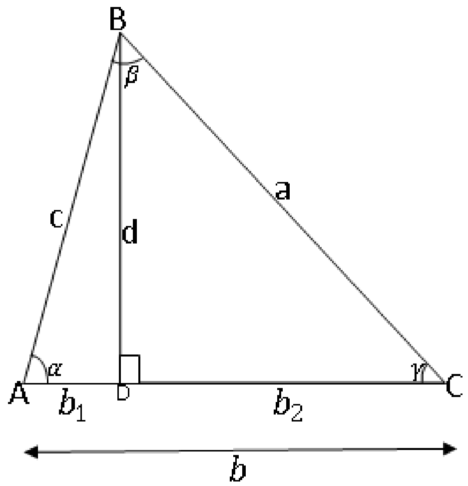
As we see from the graph Cos0=1, $\text{Cos}\frac{\pi}{2} = 0, \text{Cos}\pi = -1$. Thus if $0 < \alpha < \frac{\pi}{2}$, then $\text{Cos}0 > Cos\alpha > \text{Cos}\frac{\pi}{2} \Rightarrow 1 > Cos\alpha > 0$. if $\frac{\pi}{2} < \alpha < \pi$, then $\text{Cos}\frac{\pi}{2} > Cos\alpha > Cos\pi \Rightarrow 0 > cos\alpha > -1$.

MAIN THEOREM

Our objective is to find out necessary conditions, which enable to identify a triangle, whether it is right angled triangle, acute angled triangle or obtuse angled triangle.

Derivation of Main Theorem

Let us consider a triangle ΔABC as given below,



b = $b_1 + b_2$; $b_1 \neq b_2$.

Let us assume that 'a' is a longest side in the ΔABC. Let'd' be the length of the unique perpendicular segment from B to AC. From above ΔABC, we have the following conditions.

$\alpha \geq \beta, \alpha \geq \gamma$; b_1= c Cosα; d = cSinα; AC = b = $(b_1 + b_2)$; $b_2 = (b - b_1)$.

Now fromΔBDC, since line BD is perpendicular on the line AC, hence we have the following:
$d^2 + {b_2}^2 = a^2 \Rightarrow {b_2}^2 = a^2 - d^2 \Rightarrow {b_2}^2 = a^2 - (c\text{Sin}\alpha)^2 \Rightarrow {b_2}^2 = a^2 - c^2\text{Sin}^2\alpha$
$\Rightarrow (b - b_1)^2 = a^2 - c^2\text{Sin}^2\alpha \Rightarrow (b - cCos\alpha)^2 = a^2 - c^2\text{Sin}^2\alpha$
$\Rightarrow b^2 - 2bc\text{Cos}\alpha + c^2\text{Cos}^2\alpha = a^2 - c^2\text{Sin}^2\alpha$
$\Rightarrow b^2 - 2bc\cos\alpha + c^2\cos^2\alpha + c^2\sin^2\alpha = a^2$
$\Rightarrow b^2 + -2bc\text{Cos}\alpha + c^2(\text{Sin}^2\alpha + \text{Cos}^2\alpha) = a^2$
$\Rightarrow a^2 = b^2 + c^2 - 2bc\text{Cos}\alpha$ -----(1)

From equation (1), we have following conditions;

[I]. Let us assume that
$a^2 = b^2 + c^2 \Rightarrow b^2 + c^2 - 2bcCos\alpha = b^2 + c^2 \Rightarrow -2bcCos\alpha = 0 \Rightarrow 2bcCos\alpha = 0$
$\Rightarrow Cos\alpha = 0$, (since $2 \neq 0, b \neq 0, c \neq 0$) $\Rightarrow \alpha = 90$, $\Rightarrow \alpha$ is right angle .
Therefore ΔABC is right angled triangle.

[II]. Let us assume that
$a^2 < (b^2 + c^2) \Rightarrow (b^2 + c^2 - 2bcCos\alpha) < (b^2 + c^2) \Rightarrow -2bcCos\alpha < 0 \Rightarrow 2bcCos\alpha > 0 \Rightarrow Cos\alpha > 0$,
($since$ $2 \neq 0, b \neq 0, c \neq 0$)$\Rightarrow \alpha < 90$, because Cosine function is decreasing on $[0,\pi]$, hence α is acute angle.
Therefore ABC is acute angled triangle.

[III]. Again, let us assume that
$a^2 > (b^2 + c^2) \Rightarrow (b^2 + c^2 - 2bcCos\alpha) > (b^2 + c^2) \Rightarrow -2bcCos\alpha > 0 \Rightarrow 2bcCos\alpha < 0 \Rightarrow Cos\alpha < 0$,
($since$ $2 \neq 0, b \neq 0, c \neq 0$) $\Rightarrow \alpha > 90$, because Cosine function is decreasing on $[0, \pi]$, hence α is obtuse angle.
Therefore ABC is obtuse angled triangle.

Results: Thus we have following conditions from the theorem;
[i]. If, $a^2 = b^2 + c^2$, then ΔABC is right angled triangle.
[ii]. If, $a^2 < (b^2 + c^2)$, then ΔABC is acute angled triangle.
[iii]. If, $a^2 > (b^2 + c^2)$, then ΔABC is obtuse angled triangle.

SIGNIFICANCE OF THE THEOREM

Triangle is the simplest polygon; any polygon can be decomposed into triangles. Thus the properties of a Polygon can be studied based on properties of a triangle. Among similar properties of a triangle the one which save a time is preferable. This triangle formula saves time.

ACKNOWLEDGEMENT

One author (M.P.Chaudhary) is thankful to the Department of Mathematics of Julius-Maximilians Wurzburg University (Julius-Maximilians-Universitat Wurzburg), Germany for providing hospitality and necessary facilities during his stay in July 2014.

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Source of Support: Nil, Conflict of interest: None Declared