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BI-MONOID MULTIPLICATION ON SUBARTEX SPACES

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ABSTRACT

When we introduced Artex Spaces over bi-monoids, we gave many examples. Many propositions were found and proved. As a development of it we introduced SubArtex Spaces of Artex Spaces over bi-monoids. Many propositions and results were found and proved in SubArtex Spaces of Artex Spaces over bi-monoids. Now for an Artex Space (A, Λ, V) over a bi-monoid (M, +, .) and a SubArtex space S of A and for m∈M, m≠0, we define the bi-monoid multiplication on S. We prove some propositions. The bi-monoid multiplication on S is a subset of S and it is a SubArtex space of A. We give some examples.

Keywords: *Bi-monoids, Bi-commutative monoids, Artex Spaces over bi-monoids, SubArtex Spaces of Artex Spaces over bi-monoids.*

1. INTRODUCTION

Groups of transformations play an important role in geometry The theory of Groups is one of the richest branches of abstract algebra. A more general concept than that of a group is that of a semi-group. The algebraic system Bi-semi-group is more general to the algebraic system ring or an associative ring. We consider the algebraic system Bi-monoid in one side and a lattice in another side. Why don't a bi-monoid act on a lattice? The answer gave the idea to define an Artex space over a bi-monoid. We introduced Artex Spaces over Bi-monoids. We gave many interesting examples. Many propositions were found and proved. This theory was developed from the Lattice theory and Linear Algebra. George Boole introduced Boolean Algebra in 1854. A more general algebraic system is the lattice. A Boolean Algebra is then introduced as a special lattice. Lattices and Boolean algebra have important applications in the theory and design of computers. There are many other areas such as engineering and science to which Boolean algebra is applied.

As the theory of Artex spaces over bi-monoids is developed from lattice theory, this theory will play a good role in many fields especially in science and engineering and in computer fields. In Discrete Mathematics this theory will create a new dimension. A theory will help or will be useful or can lead other theories, if the theory itself is developed in its own way. As a development of Artex Spaces over Bi-monoids, we introduced SubArtex spaces of Artex spaces over bi-monoids. From the definition of a SubArtex space, it is clear that not every subset of an Artex space over a bi-monoid is a SubArtex space. We found and proved some propositions which qualify subsets to become SubArtex Spaces. Now we define the bi-monoid multiplication on S. We prove some propositions. These propositions will have important applications for the development of the theory of Artex spaces over bi-monoids.

2. PRELIMINARIES

2.1 Partial Ordering: A relation $\leq$ on a set P is called a partial order relation or a partial ordering in P if
(i) $a \leq a$, for all $a \in P$ ie $\leq$ is reflexive,
(ii) $a \leq b$ and $b \leq a$ implies $a = b$ ie $\leq$ is anti-symmetric, and
(iii) $a \leq b$ and $b \leq c$ implies $a \leq c$ ie $\leq$ is transitive.

2.2 Partially Ordered Set (POSET): If $\leq$ is a partial ordering in P, then the ordered pair (P, $\leq$) is called a Partially Ordered Set or simply a POSET.

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2.3 Lattice: A lattice is a partially ordered set $(L, \leq)$ in which every pair of elements a, b ϵ L has a greatest lower bound and a least upper bound.

The greatest lower bound of a and b is denoted by $a\Lambda b$ and the least upper bound of a and b is denoted by aVb

2.4 Lattice as an Algebraic System: A lattice is an algebraic system (L, Λ, V) with two binary operations Λ and V on L which are both commutative, associative, and satisfy the absorption laws namely $a\Lambda(aVb) = a$ and $aV(a\Lambda b) = a$, for all a, bϵL

The operations Λ and V are called cap and cup respectively, or sometimes meet and join respectively.

2.5 Semi-group: A non-empty set S together with a binary operation**.** is called a Semi-group if for all a, b, c ϵ S, a.(b . c) = (a.b).c

2.6 Monoid: A non-empty set N together with a binary operation**.** is called a monoid if
(i) for all a, b, c ϵ N, a.(b . c) = (a.b).c and
(ii) there exists an element denoted by e in N such that a.e = a = e.a, for all a ϵ N.
The element e is called the identity element of the monoid N.

2.7 Doubly Closed Space: A non-empty set D together with two binary operations denoted by + and . is called a Doubly Closed Space if
(i) a.(b+c) = a.b + a.c and
(ii) (a+b).c = a.c + b.c, for all a, b, c ϵ D
A Doubly closed space is denoted by (D, +, .)

Note 1: The axioms (i) a.(b+c) = a.b + a.c and (ii) (a+b).c = a.c + b.c, for all a, b, c ϵ D are called the distributive properties of the Doubly Closed Space.

Note 2: The operations + and **.** need not be the usual addition and usual multiplication respectively.

2.7.1 Example: Let N be the set of all natural numbers.

Then (N, +, .), where + is the usual addition and . is the usual multiplication, is a Doubly closed space.

2.7.2 Example: Let r ϵ N

Let $N_r = \{r, r+1, r+2, r+3, \ldots..\}$

Then $(N_r, +, .)$, where + is the usual addition and . is the usual multiplication, is a Doubly closed space.

2.7.3 Example: (Z, +, –), where + is the usual addition and – is the usual subtraction, is not a Doubly closed space.

Even though + and – are binary operations in Z, (Z, +, –) is not a Doubly closed space because of the distributive properties of the Doubly Closed Space.

Take a = 15, b = 7, c = 4

$$\begin{aligned} \text{Then } a - (b + c) &= 15 - (7 + 4) \\ &= 15 - 11 \\ &= 4 \end{aligned}$$

$$\begin{aligned} \text{But } (a - b) + (a - c) &= (15 - 7) + (15 - 4) \\ &= 8 + 11 \\ &= 19 \end{aligned}$$

Therefore, $a - (b + c) \neq (a - b) + (a - c)$

Therefore, (Z, +, –) is not a Doubly closed space.

2.8 Bi-semi-group: A Doubly closed space (S, +, .) is called a Bi-semi-group if + and . are associative in D.

2.8.1 Example: (N, +, .), (Z, +, .), (Q, +, .), (R, +, .), and (C, +, .), where + is the usual addition and **.** is the usual multiplication, are all Bi-semi-groups.

2.8.2 Example: For $r \geq 1$, (Q_r, +, .), where + is the usual addition and . is the usual multiplication, is a a Bi-semi-group.

2.9 Bi-monoid: A Bi-semi-group (M, +, .) is called a Bi-monoid if there exist elements denoted by 0 and 1 in S such that a+0=a=0+a, for all aϵM and a.1=a=1.a, for all aϵM .

The element 0 is called the identity element of M with respect to the binary operation + and the element 1 is called the identity element of M with respect to the binary operation**.**.

2.9.1 Example: Let W = {0, 1, 2, 3,…}

Then (W, +, .), where + is the usual addition and . is the usual multiplication, is a Bi-monoid.

2.9.2 Example: Let Q'= $Q^+ \cup \{0\}$, where Q^+ is the set of all positive rational numbers. Then (Q', +, .) is a bi-monoid.

2.9.3 Example: R'= $R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers. Then (R', +, .) is a bi-monoid.

2.9.4 Example: (Z, +, .), (Q, +, .), (R, +, .), and (C, +, .), where + is the usual addition and **.** is the usual multiplication, are all Bi-monoids.

2.9.5 Example: Let S be a non-empty set. Consider P(S), the power set of S.

Clearly (P(S), ∪, ∩), where ∪ denotes the union of sets and ∩ denotes the intersection of sets, is a Bi-semi-group. Now for any A ϵ P(S), A∪φ = A = φ∪A

Therefore, the empty set φ acts as the identity element of P(S) with respect to ∪.

Now for any A ϵ P(S), A∩S = A = S∩A

Therefore, the universal set S acts as the identity element of P(S) with respect to ∩

Hence (P(S), ∪, ∩) is a Bi-monoid.

2.10 Bi-commutative monoid: A Bi-monoid (M, +, .) is called a Bi-commutative monoid if a+b = b+a, for all a, b ϵ M and a.b = b.a, for all a,b ϵ M.

2.10.1 Example: Let W={0, 1, 2, 3,…}. Then (W, +, .), where + is the usual addition and . is the usual multiplication, is a Bi-commutative monoid.

2.10.2 Example: Let Q'= $Q^+ \cup \{0\}$, where Q^+ is the set of all positive rational numbers. Then (Q', +, .) is a Bi-commutative monoid.

2.10.3 Example: R'= $R^+ \cup \{0\}$, where R^+ is the set of all positive real numbers. Then (R', +, .) is a Bi-commutative monoid.

2.10.4 Example: (Z, +, .), (Q, +, .), (R, +, .), and (C, +, .), where + is the usual addition and **.** is the usual multiplication, are all Bi-commutative monoids.

2.11 Artex Space Over a Bi-monoid: Let (M, +, .) be a bi-monoid with the identity elements 0 and 1 with respect to + and **.** respectively. A non-empty set A together with two binary operations ^ and v is said to be an Artex Space Over the Bi-monoid (M, +, .) if

1. (A, Λ , V) is a lattice and
2. for each mϵM, m≠0, and aϵA, there exists an element ma ϵ A satisfying the following conditions:
   (i) m(a Λ b) = ma Λ mb
   (ii) m(a V b) = ma V mb
   (iii) ma Λ na ≤ (m +n)a and ma V na ≤ (m + n)a
   (iv) (mn)a = m(na), for all m, nϵM, m≠0, n≠0, and a, bϵA
   (v) 1.a = a, for all a ϵ A.

Here, ≤ is the partial order relation corresponding to the lattice (A, Λ, V) and

The multiplication ma is called a **bi-monoid multiplication with an artex element** or simply **bi-monoid multiplication in A.**

Unless otherwise stated A remains as an Artex space with the partial ordering $\leq$ which need not be "less than or equal to" and M as a bi-monoid with the binary operations + and . where + need not be the usual addition and **.** + need not be the usual multiplication.

2.11.1 Example: Let W = {0, 1, 2, 3,…}.

Then (W, +, .) is a bi-monoid, where + and **.** are the usual addition and multiplication respectively.

Let Z be the set of all integers

Then (Z, $\leq$) is a lattice in which Λ and V are defined by a Λ b = minimum of {a, b} and a V b = maximum of {a, b}, for all a, b ϵ Z.

Clearly for each mϵW, m≠0, and for each aϵZ, maϵZ.

Also,

(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na $\leq$ (m +n)a and ma V na $\leq$ (m + n)a
(iv) (mn)a = m(na)
(v) 1.a = a, for all m, nϵW, m ≠ 0, n ≠ 0 and a, bϵZ
Therefore, Z is an Artex Space Over the Bi-monoid (W, +, .)

2.11.2 Example: As defined in Example 2.11.1, Q, the set of all rational numbers is an Artex space over W

2.12 SubArtex Space: Let (A, Λ, V) be an Artex space over a bi-monoid (M, +, .). Let S be a nonempty subset of A. Then S is said to be a SubArtex Space of A if (S, Λ, V) itself is an Artex Space over M.

2.12.1 Example: Z is an Artex Space over W = {0, 1, 2, 3,….} and W is a subset of Z.

Also W itself is an Artex space over W under the operations defined in Z

Therefore, W is a SubArtex space of Z.

2.12.2 Example: Q is an Artex space over W = {0, 1, 2, 3,…..}. Z is a subset of Q.

Clearly Z itself is an Artex Space over W and therefore Z is a SubArtex Space of Q. W is also a SubArtex Space of Q.

2.13 Propositions

Proposition 2.13.1: Let (A, Λ, V) be an Artex Space over a bi-monoid (M, +, .). Then a nonempty subset S of A is a SubArtex Space of A if and only if S is closed under the operations Λ, V and the bi-monoid multiplication in A.

Proposition 2.13.2: Let (A, Λ, V) be an Artex space over a bi-monoid (M, +, .). Then a nonempty subset S of A is a SubArtex space of A if and only if for each m, n ϵ M, m≠0, n≠0, and a, b ϵS, ma Λ nb ϵ S and ma V nb ϵ S

3. BI-MONOID MULTIPLICATION ON SUBARTEX SPACES

3.1 Bi-monoid Multiplication On a SubArtex Space: Let (A, Λ, V) be an Artex space over a bi-monoid (M, +, .). Let S be a SubArtex space of A. Let m ϵ M, m≠0. Then the bi-monoid multiplication on S is defined by mS = {ms / s ϵ S}.

Proposition 3.1.1: Let (A, Λ, V) be an Artex space over a bi- monoid (M, +, .). Let S be a SubArtex space of A. Then for each m ϵ M, m≠0, mS ⊆ S.

Proof: Let (A, Λ, V) be an Artex space over a bi-monoid (M, +, .).

Let S be a SubArtex space of A.

Let m ϵ M, m≠0.

Now mS = {ms / s ϵ S}.

Since S is a SubArtex space of A, S ≠ ϕ.

Therefore, there exists an element, say s, in S.

=> ms ϵ mS.

Therefore, mS ≠ ϕ.

Let x ϵ mS.

Then x = ms, for some s ϵ S.

Since S is a SubArtex space of A and m ϵ M, m≠0 and sϵ S, msϵ S.

That is, x ϵ mS

Therefore, mS ⊆ S.

Hence for each m ϵ M, m≠0, mS ⊆ S.

Proposition 3.1.2: Let (A, Λ, V) be an Artex space over a bi-commutative monoid (M, +, .). Let S be a SubArtex space of A. Then for each m ϵ M, m≠0, mS is a SubArtex space of A.

Proof: Let (A, Λ, V) be an Artex space over a bi-commutative monoid (M, +, .). Let S be a SubArtex space of A.

Let m ϵ M, m≠0.

Now mS = {ms / s ϵ S }.

Since S is a SubArtex space of A, S ≠ ϕ.

Therefore, there exists an element, say s, in S.

=> ms ϵ mS.

Therefore, mS ≠ ϕ.

By the Proposition 2.13.2 it is enough to show that for each k, n ϵ M, k≠0, n≠0, and x, y ϵ mS, kx Λ ny ϵ mS and kx V ny ϵ mS.

Let k, n ϵ M, k≠0, n≠0, and x, y ϵ mS,

Then x = ms_1 and y = ms_2, for some s_1,s_2 ϵ S.

kx Λ ny = k(ms_1) Λ n(ms_2)
= (km)s_1 Λ (nm)s_2
= (mk)s_1 Λ (mn)s_2, (Since (M, +, .) is a bi-commutative monoid, km = mk and nm = mn)
= (mk)s_1 Λ (mn)s_2,
= m((ks_1) Λ (ns_2))

Since S is a SubArtex space of A, k, n ϵ M, k≠0, n≠0, and s_1, s_2 ϵ S, (ks_1) Λ (ns_2) ϵ S.

Therefore, m((ks_1) Λ (ns_2)) ϵ mS.

That is, kx Λ ny ϵ mS.

Let k, n ϵ M, k≠0, n≠0, and x, y ϵ mS,

Then x = ms_1 and y = ms_2, for some s_1,s_2 ϵ S.

kx V ny = k(ms_1) V n(ms_2)
= (km)s_1 V (nm)s_2
= (mk)s_1 V (mn)s_2, (Since (M, +, .) is a bi-commutative monoid, km = mk and nm = mn)
= (mk)s_1 V (mn)s_2,
= m((ks_1) V (ns_2))

Since S is a SubArtex space of A, k, n ϵ M, k≠0, n≠0, and s_1,s_2 ϵ S, (ks_1) V (ns_2) ϵ S.

Therefore, m((ks_1) V (ns_2)) ϵ mS.

That is, kx V ny ϵ mS.

Hence for each m ϵ M, m≠0, mS is a SubArtex space of A.

Proposition 3.1.3: Let (A, Λ, V) be an Artex space over a bi- monoid (M, +, .).

Let S be a SubArtex space of A. Then for each m, n ϵ M, m≠0, n≠0
m(nS) = (mn)S.

Proof: Let (A, Λ, V) be an Artex space over a bi- monoid (M, +, .).

Let S be a SubArtex space of A.

Let m, n ϵ M, m≠0, n≠0

Let x ϵ m(nS)

Then x = m(ns) , for some s ϵ S
= (mn)s, (since m, n ϵ M and M is a bi-monoid).

Therefore, m(nS) = (mn)S.

Proposition 3.1.4: Let (A, Λ, V) be an Artex space over a bi-commutative monoid (M, +, .). Let S be a SubArtex space of A. Then for each m, n ϵ M, m≠0, n≠0, m(nS) = n(mS).

Proof: Let (A, Λ, V) be an Artex space over a bi- monoid (M, +, .).

Let S be a SubArtex space of A.

Let m,n ϵ M, m≠0, n≠0

Let x ϵ m(nS)

Then x = m(ns), for some sϵS
= (mn)s,
= (nm)s, (since m,n ϵ M and M is a bi-commutative monoid).
= n(ms)

Therefore, x ϵ n(mS)

Therefore, m(nS) ⊆ n(mS).

Conversely, let x ϵ n(mS)

Then x = n(ms) , for some s ϵ S
= (nm)s,
= (mn)s, (since m, n ϵ M and M is a bi-commutative monoid).
= m(ns)

Therefore, x ϵ m(nS)

Therefore, n(mS) ⊆ m(nS).

Therefore, for each m,n ϵ M, m≠0, n≠0

Hence m(nS) = n(mS).

3.2 Examples

3.2.1 Example: Let W = {0, 1, 2, 3,…}.
Then (W, +, .) is a bi-commutative monoid, where + and **.** are the usual addition and multiplication respectively.

Let Q be the set of all rational numbers

Then (Q, $\leq$) is a lattice in which Λ and V are defined by a Λ b = minimum of {a, b} and a V b = maximum of {a, b}, for all a, b ϵ Q.

Clearly for each mϵW, m$\neq$0, and for each a ϵ Q, ma ϵ Q.

Also,

(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na $\leq$ (m +n)a and ma V na $\leq$ (m + n)a
(iv) (mn)a = m(na)
(v) 1.a = a, for all m, nϵW, m $\neq$ 0, n $\neq$ 0 and a, bϵ Q

Therefore, Q is an Artex Space Over the Bi-commutative monoid (W, +, .)

Let Z be the set of all integers

Then Z itself is an Artex Space Over the Bi-commutative monoid (W, +, .) and hence is a SubArtex space of Q.

For any m ϵ W, m$\neq$0, mZ $\subseteq$ Z

Also mZ is a SubArtex space of Q.

3.2.2 Example: Let R' = $R^+ \cup\{0\}$, where R^+ is the set of all positive real numbers and let w = {0, 1, 2,...}(R', $\leq$) is a lattice in which Λ and V are defined by a Λ b = mini {a, b} and
a V b = maxi {a, b}, for all a, b ϵ R'.

Here ma is the usual multiplication of a by m.

Clearly for each m ϵ W, m$\neq$0, and for each a ϵ R', ma ϵ R'.

Also,

(i) m(a Λ b) = ma Λ mb
(ii) m(a V b) = ma V mb
(iii) ma Λ na $\leq$ (m +n)a and ma V na $\leq$ (m + n)a
(iv) (mn)a = m(na) , for all m, n ϵ W, m$\neq$0, n$\neq$0, and a, b ϵ R'
(v) 1.a = a, for all a ϵ R'

Therefore, R' is an Artex Space Over the bi-monoid (W, +, .)

Generally, if Λ_1, Λ_2, and Λ_3 are the cap operations of A, B and C respectively and if V_1, V_2, and V_3 are the cup operations of A , B and C respectively, then the cap of A×B×C denoted byΛ and the cup of A×B×C denoted by V are defined by
x Λ y =(a_1,b_1,c_1) Λ (a_2,b_2, c_2) = $(a_1 \Lambda_1 a_2 , b_1 \Lambda_2 b_2 \ c_1 \Lambda_3 c_2)$ and
x V y =(a_1,b_1,c_1) V (a_2,b_2, c_2) = $(a_1 V_1 a_2, b_1 V_2 b_2, c_1 V_3 c_2)$

Here, Λ_1, Λ_2, and Λ_3 denote the same meaning minimum of two elements in R' and V_1, V_2, and V_3 denote the same meaning maximum of two elements in R'

R'^3 = R'×R'×R' is an Artex Space over W, where cap and cup operations are denoted by Λ and V respectively.

Let S = {(a, 0, 0) / aϵR'}

Claim: S is a SubArtex Space of R'^3

Let m, nϵW, and m$\neq$0, n$\neq$0, and x, y ϵ S, where x = $(a_1, 0, 0)$ and y = $(a_2, 0, 0)$, a_1, $a_2 \epsilon$R'

Now, mx Λ ny = m$(a_1, 0, 0)$ Λ n$(a_2, 0, 0)$
= $(ma_1, m0, m0)$ Λ $(na_2, n0, n0)$
= $(ma_1, 0, 0)$ Λ $(na_2, 0, 0)$
= $(ma_1 \Lambda_1 na_2, 0 \Lambda_2 0, 0 \Lambda_3 0)$
= $(ma_1 \Lambda_1 na_2, 0, 0)$

Since m, nϵW, m$\neq$0, n$\neq$0, and a_1, $a_2 \epsilon$R', and R' is an Artex space over W, $ma_1 \Lambda_1 na_2$ ϵ R'

Therefore, $(ma_1 \Lambda_1 na_2, 0, 0) \in S$

That is, $mx \Lambda ny \in S$

Now $mx V ny = m(a_1, 0, 0) V n(a_{2,} 0, 0)$
$= (ma_1, m0, m0) V (na_{2,} n0, n0)$
$= (ma_1, 0, 0) V (na_{2,} 0, 0)$
$= (ma_1 V_1 na_2, 0 V_2 0, 0 V_3 0)$
$= (ma_1 V_1 na_2, 0, 0)$

Since m, n$\in$W, m$\neq$0, n$\neq$0, and a_1, $a_2 \in$R', and R' is an Artex space over W, $ma_1 V_1 na_2 \in$ R'

Therefore, $(ma_1 V_1 na_2, 0, 0) \in S$

That is, $mx V ny \in S$

Therefore by Proposition 2.13.2, S is a SubArtex Space of R'^3.

Hence Claim.

For any m $\in$ W, m$\neq$0, mS = m {(a, 0, 0) / a$\in$R'}
= {m(a, 0, 0) / a$\in$R'}
= {(ma, 0, 0) / a$\in$R'}$\subseteq$S.

Also mS is clearly a SubArtex Space of R'^3.

REFERENCES

1. K.Muthukumaran and M.Kamaraj, "Artex Spaces Over Bi-Monoids", "*Research Journal Of Pure Algebra*", 2(5), May 2012, Pages 135-140.
2. K.Muthukumaran and M.Kamaraj, "Subartex Spaces of An Artex Space Over A Bi-Monoid", "*Mathematical Theory And Modeling*" (with IC Impact Factor 5.53), An USA Journal of "*International Institute For Science, Technology And Education*", Vol.2, No.7, 2012, Pages 39 – 48.
3. K.Muthukumaran and M.Kamaraj, "Bounded Artex Spaces Over Bi-Monoids And Artex Space Homomorphisms", "*Research Journal of Pure Algebra*", 2(7), July 2012, Pages 206 – 216.
4. K.Muthukumaran and M.Kamaraj, "Some Special Artex Spaces Over Bi-Monoids", "*Mathematical Theory And Modeling*" (with IC Impact Factor 5.53), An USA Journal of "*International Institute For Science, Technology And Education*", Vol.2, No.7, 2012, Pages 62 – 73.
5. K.Muthukumaran and M.Kamaraj, "Boolean Artex Spaces Over Bi-Monoids", *Mathematical Theory and Modeling*" (with IC Impact Factor 5.53), an USA Journal of "*International Institute for Science, Technology And Education*", Vol.2, No.7, 2012, Pages 74 – 85.
6. K.Muthukumaran and M.Kamaraj, "Cap-Cosets and Cup-Cosets of A Subset S in An Artex Space A Over A Bi-Monoid M", *"IOSR Journal of Mathematics", A Journal Of "International Organization of Scientific Research", Volume 2, Issue 1(July – August 2012) Pages 17 - 22.*
7. K.Muthukumaran and M.Kamaraj, "M-Artex Spaces Over Monoids and Cap-Quotient and Cup-Quotient M-Artex Spaces", *"IOSR Journal of Mathematics", A Journal of "International Organization of Scientific Research" Volume 2, Issue 4(Sep –Oct. 2012) Pages 22 - 28.*
8. John T.Moore, The University of Florida /The University of Western Ontario, Elements of Abstract Algebra, Second Edition, The Macmillan Company, Collier-Macmillan Limited, London, 1967.
9. J.P.Tremblay and R.Manohar, Discrete Mathematical Structures with Applications to Computer Science, Tata McGraw-Hill Publishing Company Limited, New Delhi, 1997.
10. Garrett Birkhoff & Thomas C.Bartee, Modern Applied Algebra, CBS Publishers & Distributors, 1987.
11. J.Eldon Whitesitt, Boolean algebra And Its Applications, Addison-Wesley Publishing Company, Inc., U.S.A., 1961.
12. I.N.Herstein, Topics in Algebra, Wiley Eastern Limited.

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