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# SYMMETRIC LEFT BI-DERIVATIONS ON SEMIPRIME RINGS

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**_ABSTRACT_**

*Let $R$ be a 2-torsion and 3-torsion free semiprime ring. Let $D: (.,.): R \times R \to R$ and $B(.,.): R \times R \to R$ be a symmetric left bi-derivation and symmetric bi-additive mapping. If $D(d(x), x) = 0$ and $d(d(x)) = f(x)$ holds for all $x$ in $R$, where $d$ be a trace of $D$ and $f$ be a trace of $B$. In this case $D = 0$.*

***Key Words**: Semiprime ring, Symmetric mapping, Trace, Symmetric bi-derivation, Symmetric bi-additive mapping, Symmetric left bi-derivation.*

## INTRODUCTION

The concept of a symmetric bi-derivation has been introduced by Gy. Maksa in [2], [3]. A classical result in the theory of centralizing mappings is a theorem first proved by E. Posner [5]. J. Vukman [6] has studied some results concerning symmetric bi-derivations on prime and semi prime rings. In this paper we proved some results in symmetric left bi-derivations on semiprime rings.

Throughout this paper $R$ will be associative. We shall denote by $Z(R)$ the center of a ring $R$. Recall that a ring $R$ is semiprime if $aRa = (0)$ impliesthat $a = 0$ . We shall write $[x, y]$ for $xy - yx$ and use the identities $[xy, z] = [x, z]y + x[y, z], [x, yz] = [x, y]z + y[x, z]$ .An additive map $d: R \to R$ is called derivation if $d(xy) = d(x)y + xd(y)$ holds for all $x, y \in R$.A mapping $B(.,.): R \times R \to R$ is said to be symmetric if $B(x, y) = B(y, x)$ holds for all $x, y \in R$ .A mapping $f: R \to R$ defined by $f(x) = B(x, x)$ , where $B(.,.): R \times R \to R$ is a symmetric mapping, is called a trace of B. It is obvious that, in case $B(.,.): R \times R \to R$ is symmetric mapping which is also bi-additive (i. e. additive in both arguments) the trace of $B$ satisfies the relation $f(x + y) = f(x) + f(y) + 2B(x, y)$, for all $x, y \in R$.We shall use the fact that the trace of a symmetric bi-additive mapping is an even function. A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric bi-derivation if $D(xy, z) = D(x, z)y + xD(y, z)$ is fulfilled for all $x, y, z \in R$.Obviously, in this case also the relation $D(x, yz) = D(x, y)z + yD(x, z)$ for all $x, y, z \in R$ . A symmetric bi-additive mapping $D(.,.): R \times R \to R$ is called a symmetric left bi-derivation if $D(xy, z) = xD(y, z) + yD(x, z)$ for all $x, y, z \in R$. Obviously, in this case also the relation $D(x, yz) = yD(x, z) + zD(x, y)$ for all $x, y, z \in R$. A mapping $f: R \to R$ is said to be commuting on $R$ if $[f(x), x] = 0$ holds for all $x \in R$. A mapping $f: R \to R$ is said to be centralizing on $R$ if $[f(x), x] \in Z(R)$ is fulfilled for all $x \in R$. A ring $R$ is said to be n-torsion free if whenever $na = 0$, with $a \in R$, then $a = 0$, where $n$ is nonzero integer.

## MAIN RESULTS

**Lemma 1:** [4, Lemma 1] Let $d : R \to R$ be a derivation, where $R$ is a semiprime ring. Suppose that either
(i) $ad(x) = 0$ , for all $x \in R$ or
(ii) $d(x)a = 0$, for all $x \in R$ holds. In both the cases we have $a = 0$ or $d = 0$.

**Lemma 2**: [1, Lemma 3.10] Let $R$ be a semiprime ring of characteristic not two and let $a, b \in R$ be a fixed elements. If $axb + bxa = 0$ is fulfilled for all $x \in R$, then either $a = 0$ or $b = 0$.

**Theorem 1:** Let $R$ be a 2-torsion free semiprime ring. Suppose there exists a symmetric left bi-derivation $D(.,.): R \times R \to R$ such that $D(d(x), x) = 0$ holds for all $x \in R$, where $d$ be a trace of $D$. In this case $D = 0$.

**Proof:** We have $D(d(x), x) = 0$, for all $x \in R$. (1)

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We replace $d(x)$ by $d(x)y$ in (1), we get
$D(d(x)y, x) = 0$
$d(x)D(y, x) + yD(d(x), x) = 0$

By using (1) in the above equation, we get
$d(x)D(y, x) = 0$
$d(x)D(x, y) = 0$, for all $x, y \in R$. (2)

We replace $x$ by $x^2$ in (2), we get
$d(x^2)D(x^2, y) = 0$
$4x^2d(x)2xD(x, y) = 0$
$8x^2d(x)xD(x, y) = 0$

If $x = 0$ it is trivial, if $x \neq 0$ then $d(x)xD(x, y) = 0$, for all $x, y \in R$. (3)

By the linearization of (1), we get
$D(d(x + y), x + y) = 0$
$D(d(x) + d(y) + 2D(x, y), x + y) = 0$
$D(d(x), x) + D(d(x), y) + D(d(y), x) + D(d(y), y) + D(2D(x, y), x) + D(2D(x, y), y) = 0$

By using (1) in the above equation, we get
$D(d(x), y) + D(d(y), x) + D(2D(x, y), x) + D(2D(x, y), y) = 0$
$D(d(x), y) + D(d(y), x) + 2D(D(x, y), x) + 2D(D(x, y), y) = 0$, for all $x, y \in R$. (4)

We replace $x$ by $-x$ in (4), we get
$D(d(-x), y) + D(d(y), -x) + 2D(D(-x, y), -x) + 2D(D(-x, y), y) = 0$
$D(d(x), y) - D(d(y), x) + 2D(D(x, y), x) - 2D(D(x, y), y) = 0$, for all$x, y \in R$. (5)

By adding (4) and (5), we get
$2D(d(x), y) + 4D(D(x, y), x) = 0$
$D(d(x), y) + 2D(D(x, y), x) = 0$, for all $x, y \in R$. (6)

We replace $y$ by $xy$ in (6), we get
$D(d(x), xy) + 2D(D(x, xy), x) = 0$
$xD(d(x), y) + yD(d(x), x) + 2D(xD(x, y) + yD(x, x), x) = 0$
$xD(d(x), y) + yD(d(x), x) + 2D(xD(x, y), x) + 2D(yD(x, x), x) = 0$
$xD(d(x), y) + yD(d(x), x) + 2xD(D(x, y), x) + 2D(x, y)D(x, x) + 2yD(D(x, x), x) + 2D(x, x)D(y, x) = 0$
$xD(d(x), y) + yD(d(x), x) + 2xD(D(x, y), x) + 2D(x, y)d(x) + 2yD(d(x), x) + 2d(x)D(y, x) = 0$

By using (1) and (6) in the above equation, we get
$2D(x, y)d(x) + 2d(x)D(y, x) = 0$
$D(x, y)d(x) + d(x)D(x, y) = 0$, for all $x, y \in R$. (7)

By using (2) in (7), we get
$D(x, y)d(x) = 0$, for all $x, y \in R$. (8)

We replace $y$ by $x$ in (7), we get
$D(x, x)d(x) + d(x)D(x, x) = 0$
$d(x)d(x) + d(x)d(x) = 0$
$2d(x)d(x) = 0$
$d(x)d(x) = 0$, for all $x \in R$. (9)

We replace $y$ by $yx$ in (7), we get
$D(x, yx)d(x) + d(x)D(x, yx) = 0$
$yD(x, x)d(x) + xD(x, y)d(x) + d(x)yD(x, x) + d(x)xD(x, y) = 0$
$yd(x)d(x) + xD(x, y)d(x) + d(x)yd(x) + d(x)xD(x, y) = 0$

By using (3), (8), (9) in above equation, we get
$d(x)yd(x) = 0$, for all $x, y \in R$.

Which implies that $d(x) = 0$, for all $x \in R$, by semiprimeness of $R$, which means that $D(x, y) = 0$, for all $x, y \in R$.

 

**Theorem 2:** Let $R$ be a 2-torsion and 3-torsion free semiprime ring. Let $D(.,.): R \times R \to R$ and $B(.,.): R \times R \to R$ be a symmetric left bi-derivation and symmetric bi-additive mapping respectively. Suppose that $d\big(d(x)\big) = f(x)$ holds for all $x \in R$, where $d$ be a trace of $D$ and $f$be a trace of $B$. In this case $D = 0$.

**Proof:** We have $d\big(d(x)\big) = f(x)$, for all $x \in R$. (10)

By the linearization of (10), we get

$d\big(d(x+y)\big) = f(x+y)$

$d\big(d(x) + d(y) + 2D(x,y)\big) = f(x) + f(y) + 2B(x,y)$

$d\big(d(x)\big) + d\big(d(y)\big) + d\big(2D(x,y)\big) + 2D\big(d(x), d(y)\big) + 2D\big(d(x), 2D(x,y)\big) + 2D\big(d(y), 2D(x,y)\big)$
$= f(x) + f(y) + 2B(x,y)$

$d\big(d(x)\big) + d\big(d(y)\big) + 4d\big(D(x,y)\big) + 2D\big(d(x), d(y)\big) + 4D\big(d(x), D(x,y)\big) + 4D\big(d(y), D(x,y)\big)$
$= f(x) + f(y) + 2B(x,y)$

By using (10) in the above equation, we get

$4d\big(D(x,y)\big) + 2D\big(d(x), d(y)\big) + 4D\big(d(x), D(x,y)\big) + 4D\big(d(y), D(x,y)\big) = 2B(x,y)$

$2d\big(D(x,y)\big) + D\big(d(x), d(y)\big) + 2D\big(d(x), D(x,y)\big) + 2D\big(d(y), D(x,y)\big) = B(x,y)$, for all $x, y \in R$. (11)

We replace $x$ by $-x$ in (11), we get

$2d\big(D(-x,y)\big) + D\big(d(-x), d(y)\big) + 2D\big(d(-x), D(-x,y)\big) + 2D\big(d(y), D(-x,y)\big) = B(-x,y)$

$2d\big(D(x,y)\big) + D\big(d(x), d(y)\big) - 2D\big(d(x), D(x,y)\big) - 2D\big(d(y), D(x,y)\big) = -B(x,y)$, for all $x, y \in R$. (12)

Subtract (12) from (11), we get

$4D\big(d(x), D(x,y)\big) + 4D\big(d(y), D(x,y)\big) = 2B(x,y)$

$2D\big(d(x), D(x,y)\big) + 2D\big(d(y), D(x,y)\big) = B(x,y)$,for all $x, y \in R$. (13)

We replace $x$ by $2x$ in (13), we get

$2D\big(d(2x), D(2x,y)\big) + 2D\big(d(y), D(2x,y)\big) = B(2x,y)$

$16D\big(d(x), D(x,y)\big) + 4D\big(d(y), D(x,y)\big) = 2B(x,y)$

$8D\big(d(x), D(x,y)\big) + 2D\big(d(y), D(x,y)\big) = B(x,y)$, for all $x, y \in R$. (14)

Subtract (13) from (14), we get

$6D\big(d(x), D(x,y)\big) = 0$

Since $R$ is 2-torison and 3-torison free ring, we get

$D\big(d(x), D(x,y)\big) = 0$, for all $x, y \in R$. (15)

By using (15) and (13), we get

$B(x,y) = 0$, for all $x, y \in R$.

We replace $y$ by $x$ in the above equation, we get $f(x) = 0$, for all $x \in R$. (16)

By using (1) and (16), we get

$d\big(d(x)\big) = 0$, for all $x \in R$. (17)

We replace $y$ by $yz$ in (15), we get

$D\big(d(x), D(x,yz)\big) = 0$

$D\big(d(x), yD(x,z) + zD(x,y)\big) = 0$

$D\big(d(x), yD(x,z)\big) + D\big(d(x), zD(x,y)\big) = 0$

$yD\big(d(x), D(x,z)\big) + D(x,z)D(d(x), y) + zD\big(d(x), D(x,y)\big) + D(x,y)D(d(x), z) = 0$

By using (15) in the above equation, we get

$D(x,z)D(d(x), y) + D(x,y)D(d(x), z) = 0$, for all $x, y, z \in R$. (18)

We replace $z$ by $d(x)$ in (18), we get

$D\big(x, d(x)\big)D(d(x), y) + D(x,y)D\big(d(x), d(x)\big) = 0$

$D\big(x, d(x)\big)D(d(x), y) + D(x,y)d\big(d(x)\big) = 0$

By using (17) in the above equation, we get
$D\big(x, d(x)\big)D(d(x), y) = 0$, for all $x, y \in R$. (19)

We replace $y$ by $xy$ in (19), we get
$D\big(x, d(x)\big)D(d(x), xy) = 0$
$D(d(x), x)\big(xD(d(x), y) + yD(d(x), x)\big) = 0$
$D(d(x), x)xD(d(x), y) + D(d(x), x)yD(d(x), x) = 0$

We replace $y$ by $x$ in the above equation we get $D(d(x), x)xD(d(x), x) = 0$, which implies $D(d(x), x) = 0$ for all $x \in R$ since we have assumed that $R$ is semiprime. Now Theorem 1 completes the proof.

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