



INFINTESIMAL C-CONFORMAL MOTIONS OF FINSLRIRAN HYPERSURFACES

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ABSTRACT

Let  $F^{n-1}$  be an hypersurface of a Finsler space  $F^n$ . In this paper we obtain condition for an infinitesimal C-Conformal motion for hypersurface of Finsler space and prove some results relating to the infinitesimal C-Conformal motion. Further a condintion is obtained to preserve \*P-Finsler hypersurface and an infntesimal C-Conformal motion preserves  $C^h$ -recurrent Finsler hypersurface.

**Key Words:** Finsler Space, \*P -Finsler space, P-reducible, C-reducible,  $C^h$ -recurrent Finsler spaces.

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1. INTRODUCTION

The theory of conformal changes of Finsler metrics has been initiated by M. S. Knebelman in 1929. H. Izumi ([1], [2]) gave the condition for a Finsler space to be h-conformally flat and he has also studied \*P-Finsler space. Later S. Kikuchi [3] gave the condition for a Finsler space to be conformally flat. M. Hashiguchi introduced a special change called C-conformal change which satisfies C-conditions. C. Shibata and M. Azuma have studied C-conformal invariant tensors of Finsler metric. The authors S. K. Narasimhamurthy and C. S. Bagewadi have published a paper on infinitesimal C-conformal motions of Special Finsler spaces [6]. In this we study infinitesimal C-conformal motions on some special Finsler hypersurfaces and obtain some results on \*P-Finsler hypersurfaces, C-reducible, P-reducible,  $C^h$ -recurrent Finsler hypersurfaces.

2. PRELIMINARY

Let  $M^n$  be an n-dimensional smooth manifold and  $F^n = (M^n, L)$  be an n-dimensional Finsler space equipped with fundamental function  $L(x, y)$  on  $M^n$ . The metric tensor  $g_{ij}(x, y)$ , angular metric tensor  $h_{ij}$  and Cartan's C-tensor  $C_{ijk}$  are given by

$$g_{ij} = \frac{1}{2} \dot{\partial}_i \dot{\partial}_j L^2, \quad g^{ij} = g_{ij}^{-1}$$

$$h_{ij} = g_{ij} - l_i l_j$$

$$C_{ijk} = \frac{1}{2} \dot{\partial}_k g_{ij}$$

$$C_{ij}^k = \frac{1}{2} g^{km} (\dot{\partial}_m g_{ij}), \quad \text{where } \dot{\partial}_i = \frac{\partial}{\partial x^i}$$

The conformal change of a Finsler metric  $L = \bar{L} = e^{\sigma(x)}L$  then  $\sigma$  is called the conformal factor and depends on point x only. By this change, we have another Finsler space  $\bar{F}^n = (M^n, \bar{L})$  on the same underlying manifold  $M^n$ . M. Hashiguchi introduced the special change called C-conformal which is non-homethetic conformal change satisfying

$$C_{\alpha\beta}^\gamma \sigma^\alpha = 0, \quad \sigma^\alpha = g^{\alpha\beta} \sigma_\beta, \quad \sigma_\alpha = \partial\sigma / \partial x^\alpha, \quad ,$$

$$C_{\alpha\beta}^\gamma \sigma^\alpha = 0, \quad \sigma^\alpha = C_{\gamma}^{\alpha\beta} \sigma_\beta g_{\gamma\beta} = C_{\eta\gamma}^\alpha \sigma^\eta = 0, \text{ also}$$

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$$C_{\alpha\beta\gamma}\sigma^\beta = C_{\alpha\gamma}^\delta g_{\beta\delta}\sigma^\beta = C_{\alpha\gamma}^\delta\sigma_\delta = 0. \tag{2.1}$$

The Berwald connection and the Cartan connection of  $F^n$  are given

$$B\Gamma = (G_{jk}^i, N_{jk}^i, 0), \quad C\Gamma = (F_{jk}^i, N_j^i, C_{jk}^i)$$

respectively.

A hypersurface  $F^{n-1}$  of the underlying smooth manifold  $F^n$  may be parametrically represented by the equation  $x^i = x^i(u^\alpha)$ , where  $u^\alpha$  are Gaussian coordinates on  $F^{n-1}$  and Greek indices run from 1 to (n-1). Here we shall assume that the matrix consisting of the projection factors  $B_\alpha^i = \partial x^i / \partial u^\alpha$  is of rank (n-1). The following notations are also employed

$$B_{\alpha\beta}^i = \frac{\partial x^i}{\partial u^\alpha \partial u^\beta}, \quad B_{0\beta}^i = v^\alpha B_{\alpha\beta}^i, \quad B_{\alpha\beta\dots}^{ij\dots} = B_\alpha^i, \quad B_{\beta\dots}^j$$

If the supporting element  $y^i$  at a point  $(u^\alpha)$  of  $M^{n-1}$  is assumed to be tangential to  $M^{n-1}$ , we may then write  $y^i = B_\alpha^i(u)v^\alpha$  i.e  $F^n$  is thought of as the supporting element of  $M^{n-1}$  at a point  $(u_\alpha)$ . Since the function  $\underline{L}(u, v) = L(x(u), y(u, v))$  gives rise to a Finsler matrix of  $M^{n-1}$ , we get a (n-1) dimension Finsler space  $F^{n-1} = (M^{n-1}, \underline{L}(u, v))$ .

At each point  $(u_\alpha)$  of  $F^{n-1}$ , the unit normal vector  $N^i(u, v)$  is defined by

$$g_{ij}B_\alpha^i N^j = 0, \quad g_{ij}N^i N^j = 1.$$

If  $(B_\alpha^i, N_i)$ , is the inverse matrix of  $(B_\alpha^i, N^i)$ , we have

$$B_\alpha^i B_i^\beta = \delta_\alpha^\beta, \quad B_\alpha^i N_i = 0, \quad N^i N_i = 1 \text{ and further } B_i^\alpha B_j^\alpha + N^i N_j = \delta_j^i.$$

Making use of the inverse  $g^{\alpha\beta}$  of  $(g_{\alpha\beta})$ , we get

$$B_i^\alpha = g^{\alpha\beta} g_{ij} B_\alpha^j, \quad N_i = g_{ij} N^j.$$

### 3. LEI DERIVATIVE ON FINSLER HYPERSURFACE

Consider an infinitesimal extended point transformation in a Finsler space generated by the vector  $X = v^i(x)\partial_i$ , i.e.

$$\bar{X}^i = x^i + v^i dt, \quad \bar{y}^i = y^i + (\partial_j v^i) y^j dt. \tag{3.1}$$

The condition for the above transformation to be infinitesimal conformal motion is that there exists a function  $\sigma$  of  $x$  such that

$$L_X g_{\alpha\beta} = 2\sigma(X)g_{\alpha\beta}, \quad L_X g^{\alpha\beta} = -2\sigma(X)g^{\alpha\beta}. \tag{3.2}$$

It is well known that on i.c.m (3.1) satisfies

$$L_X C_{\beta\gamma}^\alpha = 0, \quad \text{and } L_X y^i = B_\alpha^i(u)v^\alpha = 0. \tag{3.3}$$

From this we can easily seen that

- (a)  $L_X C_j B_\beta^j = L_X C_\beta = 0, \quad C_\alpha = C_{\beta\alpha}^\alpha, \quad L_X C^\beta = -2\sigma C^\alpha$
- (b)  $L_X L = \sigma L$
- (c)  $L_X l^\alpha = -\sigma l^\alpha, \quad L_X l_\alpha = \sigma l_\alpha$
- (d)  $L_X h_\beta^\alpha = 0, \quad L_X (g^{\alpha\delta} g_{\beta\gamma}) = 0$

The commutative formulae involving Lie and Covariant derivatives are given by

$$\begin{aligned} (i) \quad L_X T_{\beta||\gamma}^\alpha - (L_X T_\beta^\alpha)_{||\gamma} &= T_\beta^\eta A_{\eta\gamma}^\alpha - T_\eta^\alpha A_{\beta\gamma}^\eta - A_\gamma^\eta \partial_\eta T_\beta^\alpha \\ (ii) \quad L_X H_{\delta\beta\gamma}^\alpha &= A_{\delta\beta||\gamma}^\alpha + A_\beta^\eta G_{\delta\gamma\eta}^\alpha - A_{\delta\gamma\beta}^\alpha - A_{\delta\gamma||\beta}^\alpha - A_\beta^\eta G_{\delta\gamma\eta}^\alpha, \end{aligned} \tag{3.4}$$

Where  $A_{\beta}^{\eta} = L_X G_{\beta}^{\eta}$ ,  $A_{\beta\gamma}^{\alpha} = L_X G_{\beta\gamma}^{\alpha} = v_{\|\beta\|\gamma}^{\alpha} + H_{\beta\gamma\delta}^{\alpha} v^{\delta} + G_{\beta\gamma\eta}^{\alpha} v_{\|0}^{\eta}$ ,  
 where  $\frac{\beta}{\gamma}$  means the interchange of indices  $\beta$  and  $\gamma$  in the foregoing terms and symbol ‘||’ denotes the covariant derivative in Finsler hypersurface.

Since the Lie derivative is commutative with  $\partial_{\gamma}$ ,  $\hat{\partial}_{\gamma}$  and from (3.1)

$$L_X \gamma_{\beta\gamma}^{\alpha} = \sigma_{\beta} \delta_{\gamma}^{\alpha} + \sigma_{\gamma} \delta_{\beta}^{\alpha} - \sigma^{\alpha} g_{\beta\gamma}, \quad \sigma_{\beta} = \partial_{\beta} \sigma. \quad (3.5)$$

#### 4. AN INFINTESIMAL C-CONFORMAL MOTION ON $F^{n-1}$

If we impose the C-condition on the vector  $\sigma_{\beta}$ , i.e,

$$C_{\alpha\beta}^{\delta} \sigma_{\delta} = 0, \quad (4.1)$$

then the transformation is called an infinitesimal C-conformal motion (denoted by an i. C-c. m). We have

$$\begin{aligned} (i) \quad L \hat{\partial}_k h_j^i &= -h_{jk} l^i - h_k^i l_j \\ (ii) \quad L^2(\hat{\partial}_k C_j^{ih} \alpha_h) &= L \hat{\partial}_k (L C_j^{ih} \alpha_h) - L C_j^{ih} \alpha_h \hat{\partial}_k L = 0, \quad \text{or} \quad (\hat{\partial}_k C_j^{ih}) \alpha_h = 0. \end{aligned} \quad (4.2)$$

Transvecting (4.2) (i) by  $B_{i\beta\gamma}^{\alpha jk}$  we get

$$L \hat{\partial}_k h_j^i B_i^{\alpha} B_{\beta\gamma}^{jk} = (-h_{jk} l^i - h_k^i l_j) B_i^{\alpha} B_{\beta\gamma}^{jk}$$

$$L \hat{\partial}_{\gamma} h_{\beta}^{\alpha} = -h_{\beta\gamma} l^{\alpha} - h_{\gamma}^{\alpha} l_{\beta}.$$

Transvecting (4.2) (ii) by  $B_{ij}^{\alpha\beta} B_{\gamma\delta}^{kh}$

$$L^2(\hat{\partial}_k C_j^{ih} \alpha_h) B_{ij}^{\alpha\beta} B_{\gamma\delta}^{kh} = (L \hat{\partial}_k (L C_j^{ih} \alpha_h) - L C_j^{ih} \alpha_h \hat{\partial}_k L) B_{ij}^{\alpha\beta} B_{\gamma\delta}^{kh} = 0$$

$$L^2(\hat{\partial}_{\gamma} C_{\beta}^{\alpha\delta} \sigma_{\delta}) = L \hat{\partial}_{\gamma} (L C_{\beta}^{\alpha\delta} \sigma_{\delta}) - L C_{\beta}^{\alpha\delta} \sigma_{\delta} \hat{\partial}_{\gamma} L = 0.$$

Using the above calculation, we obtain

$$\begin{aligned} (a) \quad A_{\beta\gamma}^{\alpha} &= L_X G_{\beta\gamma}^{\alpha} = \sigma_{\gamma} \delta_{\beta}^{\alpha} + \sigma_{\beta} \delta_{\gamma}^{\alpha} - \sigma^{\alpha} g_{\beta\gamma} \\ (b) \quad A_{0\beta}^{\alpha} &= A_{\beta 0}^{\alpha} = L_X G_{\beta 0}^{\alpha} = \sigma_{\beta} y^{\alpha} + \sigma_0 \delta_{\beta}^{\alpha} - \sigma^{\alpha} y_{\beta}. \end{aligned} \quad (4.3)$$

Hence we have

**Theorem 4.1:** An infinitesimal C-conformal motion on  $F^{n-1}$  satisfies  $L_X G_{\beta\gamma}^{\alpha} = \sigma_{\gamma} \delta_{\beta}^{\alpha} + \sigma_{\beta} \delta_{\gamma}^{\alpha} - \sigma^{\alpha} g_{\beta\gamma}$

$$L_X G_{\beta}^{\alpha} = \sigma_{\beta} y^{\alpha} + \sigma_0 \delta_{\beta}^{\alpha} - \sigma^{\alpha} y_{\beta}.$$

From (3.4) (i), we have

$$L_X C_{\beta\gamma\|\Gamma}^{\alpha} - (L_X C_{\beta\gamma}^{\alpha})_{\|\Gamma} = A_{\eta\Gamma}^{\alpha} C_{\beta\gamma}^{\eta} - A_{\beta\alpha}^{\eta} C_{\eta\gamma}^{\alpha} - A_{\gamma\beta}^{\eta} C_{\beta\eta}^{\alpha} - A_{\Gamma}^{\eta} \hat{\partial}_{\eta} C_{\beta\gamma}^{\alpha} \quad (4.4)$$

From (3.2), (3.3) and (4.4), we have

$$\begin{aligned} L_X C_{\beta\gamma\|\Gamma}^{\alpha} &= (\sigma_{\eta} \delta_{\Gamma}^{\alpha} + \sigma_{\Gamma} \delta_{\eta}^{\alpha} - \sigma^{\alpha} g_{\eta\Gamma}) C_{\beta\gamma}^{\eta} - (\sigma_{\beta} \delta_{\Gamma}^{\eta} + \sigma_{\Gamma} \delta_{\beta}^{\eta} - \sigma^{\eta} g_{\beta\Gamma}) C_{\eta\gamma}^{\alpha} - (\sigma_{\gamma} \delta_{\Gamma}^{\alpha} + \sigma_{\Gamma} \delta_{\gamma}^{\alpha} - \sigma^{\alpha} g_{\gamma\Gamma}) C_{\beta\eta}^{\alpha} - \\ &\quad (\sigma_{\Gamma} y^{\eta} + \sigma_0 \delta_{\Gamma}^{\eta} - \sigma^{\eta} y_{\Gamma}) \hat{\partial}_{\eta} C_{\beta\gamma}^{\alpha}, \\ L_X C_{\beta\gamma\|\Gamma}^{\alpha} &= \sigma_{\Gamma} C_{\beta\gamma}^{\alpha} - \sigma^{\alpha} C_{\Gamma\beta\gamma} - \sigma_{\beta} C_{\Gamma\gamma}^{\alpha} - \sigma_{\Gamma} C_{\beta\gamma}^{\alpha} - \sigma_{\gamma} C_{\beta\Gamma}^{\alpha} - \sigma_{\Gamma} C_{\beta\gamma}^{\alpha} - \sigma_0 \hat{\partial}_{\Gamma} C_{\beta\gamma}^{\alpha} - \sigma_{\Gamma} y^{\eta} (\hat{\partial}_{\eta} C_{\beta\gamma}^{\alpha}) + \sigma^{\eta} y_{\Gamma} (\hat{\partial}_{\eta} C_{\beta\gamma}^{\alpha}), \\ L_X C_{\beta\gamma\|\Gamma}^{\alpha} &= -\sigma^{\alpha} C_{\Gamma\beta\gamma} - \sigma_{\beta} C_{\Gamma\gamma}^{\alpha} - \sigma_{\gamma} C_{\beta\Gamma}^{\alpha} - \sigma_0 (\hat{\partial}_{\Gamma} C_{\beta\gamma}^{\alpha}). \end{aligned} \quad (4.5)$$

Transvecting above by  $y^{\Gamma}$ , we have

$$L_X C_{\beta\gamma|\Gamma}^\alpha y^\Gamma = -\sigma C_{\Gamma\beta\gamma}^\alpha y^\Gamma - \sigma_\beta C_{\Gamma\gamma}^\alpha y^\Gamma - \sigma_\gamma C_{\beta\gamma}^\alpha y^\Gamma - \sigma_0(\partial_\Gamma C_{\beta\gamma}^\alpha) y^\Gamma,$$

$$L_X C_{\beta\gamma|0}^\alpha = \sigma_0 C_{\beta\gamma}^\alpha. \tag{4.6}$$

Thus we have

**Theorem 4.2:** An infinitesimal C-conformal motion satisfies  $L_X C_{\beta\gamma|0}^\alpha = \sigma_0 C_{\beta\gamma}^\alpha$

**Definition [4.1]:** If the tensor  $*P_{jk}^i = p_{jk}^i - \alpha_0(x)C_{jk}^i$

Transvecting above equation by  $B_i^\alpha B_{\beta\gamma}^{jk}$  we get

$$*P_{jk}^i B_i^\alpha B_{\beta\gamma}^{jk} = p_{jk}^i B_i^\alpha B_{\beta\gamma}^{jk} - \alpha_0(x)C_{jk}^i B_i^\alpha B_{\beta\gamma}^{jk},$$

$$*P_{\beta\gamma}^\alpha = P_{\beta\gamma}^\alpha - \sigma_0(x)C_{\beta\gamma}^\alpha. \tag{4.7}$$

vanishes, then the space is called a \* P-Finsler hypersurface,  $\sigma_0(x)$  is a scalar function. Taking Lie derivative on (4.7)

$$L_X *P_{\beta\gamma}^\alpha = L_X P_{\beta\gamma}^\alpha - L_X \sigma_0(x)C_{\beta\gamma}^\alpha$$

By \* P-condition is given by  $P_{\beta\gamma}^\alpha = \sigma_0 C_{\beta\gamma}^\alpha$  and using (4.6) we get

$$L_X *P_{\beta\gamma}^\alpha = \sigma_0 C_{\beta\gamma}^\alpha - L_X(\sigma_0)C_{\beta\gamma}^\alpha - \sigma_0 L_X C_{\beta\gamma}^\alpha,$$

$$L_X *P_{\beta\gamma}^\alpha = \sigma_0 C_{\beta\gamma}^\alpha - \sigma_0 C_{\beta\gamma}^\alpha,$$

$$L_X *P_{\beta\gamma}^\alpha = 0, \text{ where } L_X(\sigma_0) = \sigma_0.$$

Hence we state that

**Theorem 4.3:** An infinitesimal C-conformal motion preserves \* P-Finsler hypersurface.

**Definition [4.2]:** If the tensor  $*P_{ijk} = P_{ijk} - (P_k h_{ij} + P_i h_{jk} + P_j h_{ki})$

Transvecting on both side of the above equation by  $B_i^\alpha B_{\beta\gamma}^{jk}$

$$*P_{ijk} B_i^\alpha B_{\beta\gamma}^{jk} = P_{ijk} B_i^\alpha B_{\beta\gamma}^{jk} - (P_k h_{ij} + P_i h_{jk} + P_j h_{ki}) B_i^\alpha B_{\beta\gamma}^{jk},$$

$$*P_{\alpha\beta\gamma} = P_{\alpha\beta\gamma} - (P_\gamma h_{\alpha\beta} + P_\alpha h_{\beta\gamma} + P_\beta h_{\gamma\alpha}). \tag{4.8}$$

Vanishes, then the Finsler space  $F^n$  is called a P-reducible Finsler hypersurface.

Taking Lie derivative on both side of the above equation, we have

$$L_X *P_{\alpha\beta\gamma} = L_X P_{\alpha\beta\gamma} - L_X(P_\gamma h_{\alpha\beta} + P_\alpha h_{\beta\gamma} + P_\beta h_{\gamma\alpha}),$$

$$\text{using } L_X P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma}, \quad L_X P_\gamma = -\sigma P_\gamma,$$

$$L_X *P_{\alpha\beta\gamma} = L_X P_{\alpha\beta\gamma} - (L_X P_\gamma h_{\alpha\beta} + P_\gamma L_X h_{\alpha\beta} + L_X P_\alpha h_{\beta\gamma} + P_\alpha L_X h_{\beta\gamma} + L_X P_\beta h_{\gamma\alpha} + P_\beta L_X h_{\gamma\alpha}), \quad L_X *P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [(-\sigma P_\gamma h_{\alpha\beta}) + (-\sigma P_\beta h_{\gamma\alpha}) + (-\sigma P_\alpha h_{\beta\gamma}) + 2\sigma(P_\alpha h_{\beta\gamma} + P_\beta h_{\gamma\alpha} + P_\gamma h_{\alpha\beta})],$$

$$L_X *P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [-\sigma(P_\gamma h_{\alpha\beta} + P_\alpha h_{\beta\gamma} + P_\beta h_{\gamma\alpha}) + 2\sigma(P_\gamma h_{\alpha\beta} + P_\alpha h_{\beta\gamma} + P_\beta h_{\gamma\alpha})],$$

$$L_X *P_{\alpha\beta\gamma} = \sigma P_{\alpha\beta\gamma} - [-\sigma P_{\alpha\beta\gamma} + 2\sigma P_{\alpha\beta\gamma}],$$

$$L_X *P_{\alpha\beta\gamma} = 0.$$

Thus we state

**Theorem 4.4:** An infinitesimal C-conformal motion preserves P-reducible Finsler hypersurface.

**Definition [4.3]:** If the tensor

$$* C_{jkl}^i = (C_{jk;l}^i - \alpha_l C_{jk}^i),$$

Transvecting above equation by  $B_l^\alpha B_{\beta\gamma}^{jkl}$  we get

$$* C_{jkl}^i B_l^\alpha B_{\beta\gamma}^{jkl} = (C_{jk;l}^i - \alpha_l C_{jk}^i) B_l^\alpha B_{\beta\gamma}^{jkl},$$

$$* C_{\beta\gamma}^\alpha = (C_{\beta\gamma||\Gamma}^\alpha - \sigma_\Gamma C_{\beta\gamma}^\alpha) \tag{4.9}$$

vanishes, then the space  $F^n$  is called  $C^h$ -recurrent Finsler hypersurface here  $\sigma_\Gamma = \sigma_\Gamma(x, y)$  is a covariant vector field. Transvecting (4.5) by  $\sigma^\Gamma$  and using (2.1) we get

$$(L_X C_{\beta\gamma||\Gamma}^\alpha) \sigma^\Gamma = -\sigma C_{\Gamma\beta\gamma} \sigma^\Gamma - \sigma_\beta C_{\Gamma\gamma}^\alpha \sigma^\Gamma - \sigma_\gamma C_{\beta\Gamma}^\alpha \sigma^\Gamma - \sigma_0 (\dot{\partial}_\Gamma C_{\beta\gamma}^\alpha) \sigma^\Gamma,$$

$$(L_X C_{\beta\gamma||\Gamma}^\alpha) \sigma^\Gamma = -\sigma^\alpha C_{\beta\gamma} - \sigma_\beta C_\gamma^\alpha - \sigma_\gamma C_\beta^\alpha - \sigma_0 (C_{\beta\gamma}^\alpha),$$

$$(L_X C_{\beta\gamma||\Gamma}^\alpha) \sigma^\Gamma = 0. \tag{4.10}$$

Taking Lie-derivative on (4.9), we obtain

$$L_X * C_{\beta\gamma}^\alpha = (L_X C_{\beta\gamma||\Gamma}^\alpha - L_X \sigma_\Gamma C_{\beta\gamma}^\alpha),$$

$$L_X * C_{\beta\gamma}^\alpha = L_X C_{\beta\gamma||\Gamma}^\alpha - \sigma_\Gamma L_X C_{\beta\gamma}^\alpha - L_X (\sigma_\Gamma) C_{\beta\gamma}^\alpha,$$

$$L_X * C_{\beta\gamma}^\alpha = L_X C_{\beta\gamma||\Gamma}^\alpha - L_X (\sigma_\Gamma) C_{\beta\gamma}^\alpha, \tag{4.11}$$

Transvecting (4.11) by  $\sigma^\Gamma$  and using (4.10), we have

$$L_X * C_{\beta\gamma}^\alpha \sigma^\Gamma = L_X C_{\beta\gamma||\Gamma}^\alpha \sigma^\Gamma - L_X (\sigma_\Gamma) C_{\beta\gamma}^\alpha \sigma^\Gamma,$$

$$L_X * C_{\beta\gamma}^\alpha = L_X C_{\beta\gamma||\Gamma}^\alpha \sigma^\Gamma - L_X (\sigma_\Gamma) C_{\beta\gamma}^\alpha \sigma^\Gamma,$$

$$L_X * C_{\beta\gamma}^\alpha = -\sigma^2 C_{\beta\gamma}^\alpha. \tag{4.12}$$

Thus we obtain in view of Deicke's theorem

**Theorem 4.5:** An infinitesimal C-conformal motion preserves  $C^h$ -recurrent Finsler hypersurface if and only if it is Riemannian.

**Definition [4.4]:** If the tensor

$$* C_{ijk} = C_{ijk} - \frac{1}{(n+1)} (h_{ij} C_k + h_{jk} C_i + h_{ki} C_j),$$

Contracting on both side of the above equation by  $B_\alpha^i B_{\beta\gamma}^{jk}$  we get

$$* C_{ijk} B_\alpha^i B_{\beta\gamma}^{jk} = C_{ijk} B_\alpha^i B_{\beta\gamma}^{jk} - \frac{1}{n} (h_{ij} C_k + h_{jk} C_i + h_{ki} C_j) B_\alpha^i B_{\beta\gamma}^{jk},$$

$$* C_{\alpha\beta\gamma} = C_{\alpha\beta\gamma} - \frac{1}{n} (h_{\alpha\beta} C_\gamma + h_{\beta\gamma} C_\alpha + h_{\gamma\alpha} C_\beta) \tag{4.13}$$

vanishes, then the Finsler space is called a C-reducible Finsler hypersurface.

Taking Lie derivative of (4.13) and using

$$L_X C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma}, \quad L_X h_{ij} = 2\alpha h_{ij}, \quad L_X C_\alpha = 0, \text{ we get}$$

$$L_X * C_{\alpha\beta\gamma} = L_X C_{\alpha\beta\gamma} - \frac{1}{n} (L_X h_{\alpha\beta} C_\gamma + L_X h_{\beta\gamma} C_\alpha + L_X h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n}(L_X h_{\alpha\beta} C_\gamma + L_X h_{\beta\gamma} C_\alpha + L_X h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n}(2\sigma h_{\alpha\beta} C_\gamma + 2\sigma h_{\beta\gamma} C_\alpha + 2\sigma h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - \frac{1}{n}2\sigma(h_{\alpha\beta} C_\gamma + h_{\beta\gamma} C_\alpha + h_{\gamma\alpha} C_\beta),$$

$$L_X * C_{\alpha\beta\gamma} = 2\sigma C_{\alpha\beta\gamma} - 2\sigma C_{\alpha\beta\gamma},$$

$$L_X * C_{\alpha\beta\gamma} = 0.$$

Thus, we have

**Theorem 4.6:** *An infinitesimal C-conformal motion preserves C-reducible Finsler hypersurface.*

**CONCLUSION:**

In this paper, we have applied the Lie derivatives on Finsler hypersurface to obtain the condition for the transformation (3.1) to be infinitesimal Conformal motion and ensures (3.5) as Lie derivative is commutative with  $\partial_r, \dot{\partial}_r$ . Further we proved that an infinitesimal C-conformal motion preserves \*P-Finsler hypersurface, P-reducible Finsler hypersurface and C-reducible Finsler hypersurface. Also by the view of Deicks's theorem, on the condition of Riemannian we proved that infinitesimal C-conformal motion preserves  $C^h$ -recurrent Finsler hypersurface.

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